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Layout and yaw optimisation of an offshore wind farm through analytical modelling

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Abstract. In this work, a computational tool for analytical large scale modelling of offshore wind farms is presented. Using Horns Rev 1 as a reference baseline, an optimal layout and yaw angle setting for one wind direction based on aerodynamic considerations is proposed. Based on a wake model presented by Qian & Ishihara, flow velocities as well as turbulence intensities are predicted when several wake flows are superimposed. Subsequently, the aerodynamic performance of the wind turbines is calculated. The proposed tool shows good agreement with measurement and LES data. The influence of performance-driving parameters, namely turbine spacing, ambient turbulence intensity and yaw angle are investigated. The axial spacing and ambient turbulence are identified as the most significant factors. Based on these results, the proposed optimal layout solution suggests a higher power yield of up to 57 %, while the optimal yaw angles found in this work show a power increase of over 6 % compared to the baseline layout.

1. Introduction

The planned expansion of offshore wind power requires optimisation of turbine blades and wind farms subject to mainly aerodynamic and structural constraints, as well as cost. In particular, the wake that shadows downstream turbines has a negative effect on the yield of a wind farm. First, the reduced wind speed in the wake leads to a lower power yield with a cubic relation. Further, rotor induced turbulence negatively influences the life span expectancy of the downstream turbine structures, and thus, increasing costs for repair and service. A general quantitative description of the power loss within a wind farm is difficult to get, as only few measurement data on wind speed and turbulence within a plant are available. In addition, performance data of turbines and the overall power plant are often held confidential. The Horns Rev 1 wind farm at the Danish North Sea coast and the Nysted wind farm in the Baltic Sea are two of the few exceptions; for these wind farms research data is available [1]. The most distinct power loss within a multi-row wind farm occurs between the first and the second turbine row. The power level of the following rows remains close to the level of the second row, which is about 60 % for both wind farms. In their investigations, Barthelmie et al. emphasise the importance of wake recovery and the lateral overlapping of the wakes for the described trend in yield. The interaction of the wake from a single turbine with the free stream flow is generally limited to the sides and the top of the wake due to the proximity to the ground. However, within the wind farm the wakes of several turbine rows overlap laterally. The flow-recovery and momentum balancing in this case largely takes place through interaction with the upstream wakes. As a result, the wind speed and turbulence between the turbines evens out [1]. Further, Barthelmie & Jensen identify both the axial spacing of the turbines and the ambient turbulence as the main reasons for the power drop [2]. The wake recovers increasingly with axial distance downstream of the rotor. Furthermore, larger ambient turbulence improves the flow recovery process through enhanced flow mixing between the wake region and the ambient flow. The wind farm power should therefore increase for larger turbine distances and ambient turbulence. The evaluation of measured data by Barthelmie & Jensen supports this hypothesis [2].

In this paper, an analytical, two dimensional steady state tool for modelling large offshore wind farms is presented. Since available measurement data is limited, the tool is used to investigate the relation between power yield and a wider variety of parameters and their combinations. First, an analytical wake model considering wake superposition methods as developed by Ishihara et al. is implemented and validated [3]. Then, the effects of turbine positions, ambient turbulence and yaw angle on the total power of a wind farm are investigated in a sensitivity study. Finally, the tool is tested by optimising the Horns Rev 1 wind farm considering the turbine layout and yaw angle at the dominating wind direction.

2. Methods

First, the methodology of analytical wake modelling and superposition is presented. Then, the structure and computational workflow of the developed tool is described, including information on simplifications and grid sensitivity.

2.1. Analytical wake model & superposition

The most used wake model in the wind industry is the Jensen model by Katic et al.. Therefore, it is integrated in most wind farm design tools such as WAsP [4]. However, the Jensen model does not include the effects of the rotor induced turbulence, which leads to a significant over prediction of power losses within a wind farm [3]. Over time, a number of other wake models have been presented, for example by Frandsen et al. [5], Bastankhah et al. [6] and Gao et al. [7]. Through empirical wake decay constants and extended equations considering the ambient turbulence intensity the authors increased the models accuracy. A more recent model was presented by Ishihara et al. [3]. They extended the equations to include rotor induced turbulence and wake deflection when applying a yaw angle, as displayed in Fig. 1. All mentioned models are two-dimensional, considering the velocities and turbulence in the horisontal plane at hub height. Qian & Ishihara introduced the third dimension along the tower axis, which takes the vertical profile of the flow into account. However, the inflow values for speed and turbulence of a turbine

(a) speed distribution at $\gamma \neq 0$ (b) turbulence distribution at $\gamma \neq 0$

Figure 1: Speed, u, and turbulence intensity, TI, in the wake region of a yawed turbine, based on a normal distribution. Inspired by [3].

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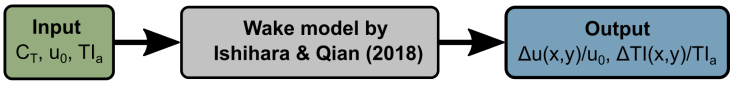


Figure 2: Input and output parameters of the wake model by Ishihara & Qian [3].

are still averaged to a scalar value [8]. Polster et al. found this model to have a good agreement with wind tunnel measurements [9]. In the context of the present study, only two dimensions of Ishihara's model are implemented. With this simplification, a reliable optimisation tool with time saving computational capacities can be built. The in- and output variables of the wake model are shown in Fig. 2. When applied, the model provides the ratios of the velocity deficit as well as the turbulence increase with respect to the inflow velocity and ambient turbulence of the turbine in the rotor plane. The output variables are calculated for each downstream point at an axial distance x or at a lateral distance y with the rotors centre being the zero point. At this point only one single turbine wake is considered. For the superposition of several flows, different approaches are investigated for the analytical modelling [8]. With respect to Qian & Ishihara, the rotor-based linear summation of the velocity deficit is used here [8]

$$u(x,y) = u_0 - \sum_{i=1}^{n} \Delta u_i \quad \text{with} \quad \Delta u_i = u_0(1 - \frac{u_i}{u_0}) \ . \tag{1}$$

The changes in velocity, $\Delta\, u$ resulting from each individual turbine i are linearly added, where n is the number of overlapping wakes. The superposition of turbulence has not been sufficiently investigated yet. Ishihara & Qian therefore propose their own superposition rule for the turbulence intensity at a point $\mathrm{P}(x,\, y)$ within several overlapping wakes, introducing the rotor-based quadratic summation

$$TI(x,y)^2 = TI_a^2 + \sum_{i=1}^{n} \Delta TI_i(x,y)^2 \ . \tag{2}$$

However, they point out that a higher weighting of the turbulence contribution of the turbine closest upstream to the point calculated may lead to more realistic results [8]. Since this form of superposition has not been sufficiently investigated, it remains unclear whether it provides a significant improvement of the modelling results. Therefore, the approach as presented in Eq. (2) has been implemented in the presented tool. Different weighting of the turbines' effects would mean a disproportionate implementation effort in addition to significantly higher computation time. Both, the superposition of wind speed and turbulence are calculated rotor-based. This means that the calculated values for Δu_i and ΔTI_i of each turbine are related to the actual inflow conditions of the corresponding rotor [8]. For several interacting turbines, a wake induced by one rotor can influence the inflow conditions of a downstream turbine. Therefore, it is necessary with an iterative calculation to take the effects of all into account.

2.2. Steady state wind park modelling tool

The flow chart for the developed program is illustrated in Fig. 3. For modelling a wind farm, input parameters are required. These include the turbine data (such as performance maps, rotor diameter etc.), boundary conditions of the wind farm (number and position of turbines) as well as the ambient conditions (ambient turbulence TI_a, magnitude u_0 and direction α_{wind} of the free stream inflow). Next, the wake of each turbine is calculated separately in a local coordinate system. In the first iteration, the inflow conditions for all turbines equal the ambient wind speed and turbulence intensity. After the initial calculation, the results are inserted into the global field. To rule out the dependence of the results on node resolution, a grid independence study is performed. With a global resolution of 0.22 D and a local resolution of 0.1 D, an acceptable maximum error of 0.5 % with respect to the converging value is obtained.

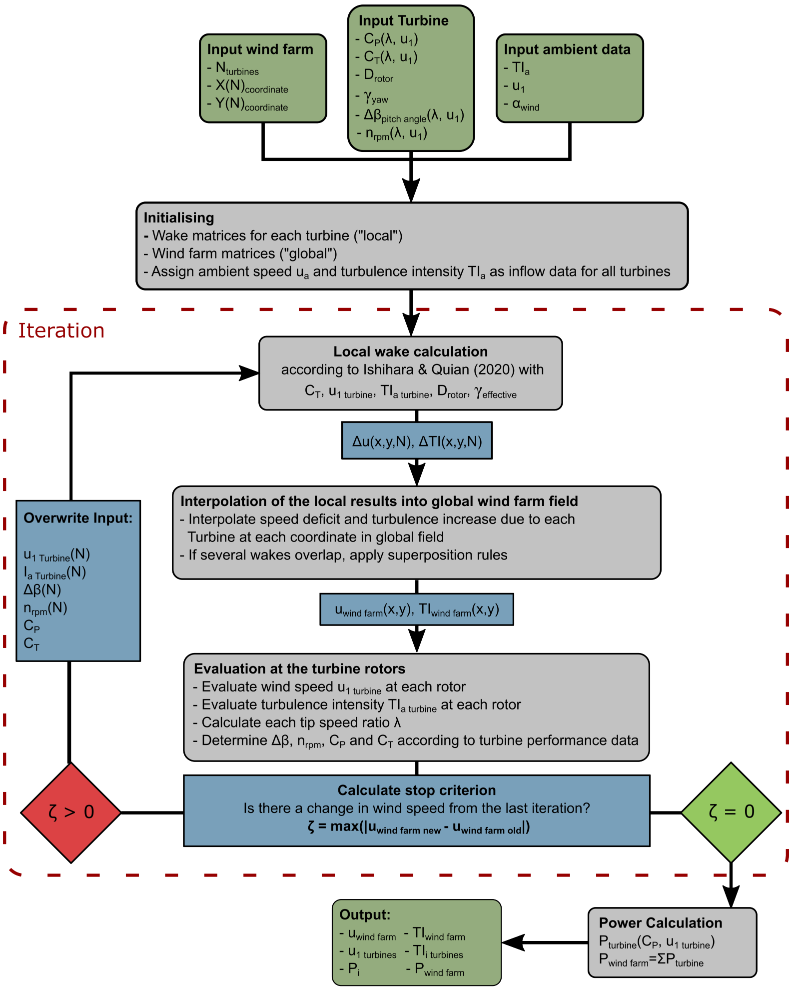


Figure 3: Structure and algorithm of the calculation tool.

Fig. 4 schematically shows the wake area of three turbines in the global field. The nodes that are unaffected by wakes, only contain the ambient free stream values. The nodes within the light green areas are only influenced by one wake. Here, the interpolated value is used without applying any superposition rules. In the darker areas, there is an overlapping of several wakes. For this case, the superposition rules according to Eq. (1)-(2) are applied. Once the interpolation and superposition computation is performed, the velocity and the turbulence intensity in the rotor plane of each turbine are evaluated. Qian & Ishihara recommend an average of 100 points on each rotor plane [8].

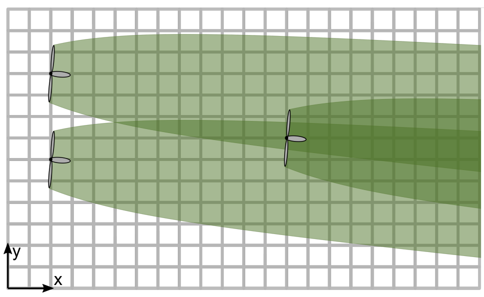


Figure 4: Three turbines with wake regions on schematic global calculation grid. Darker areas indicate increased number of overlapping wakes.

After the inflow parameters of all turbines have been computed, the values are stored as new input parameters and used as inflow condition for the turbines in the next iteration. The next iteration is therefore conducted with altered inflow conditions for each turbine. After each iteration, the wind speed in the global field is calculated. Here, the parameter ζ is used to terminate the calculations, according to

$$\zeta = \max(|u_{\text{farm,new}} - u_{\text{farm,old}}|) \; . \tag{3}$$

Hence, if $\zeta < 10^{-9}$, no change in the flow field has been registered, and the calculations of the wind speed and turbulence are converged. Finally, the power of all turbines is computed based on the inflow wind speeds determined during the last iteration and the performance maps of the turbines. For yawed conditions, the reduced power is determined as

$$P_\gamma = P \cdot \cos(\gamma)^\kappa \; , \tag{4}$$

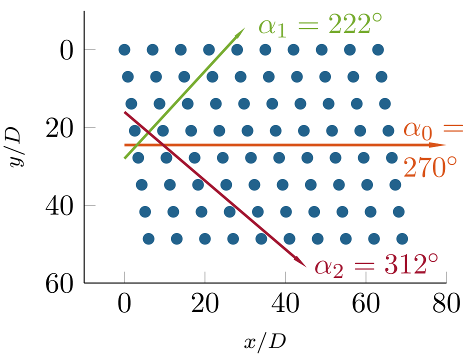
where κ is the power yaw loss coefficient and γ is the yaw angle with a positive yaw corresponding to a clockwise rotation. According to Liew et al., different models and measurement publications indicate that κ should be between 1.88 and 5.14 [10]. However, a definitive statement on the correctness of the parameter is not possible. In this work, $\kappa = 2$ is chosen for all calculations.

3. Results

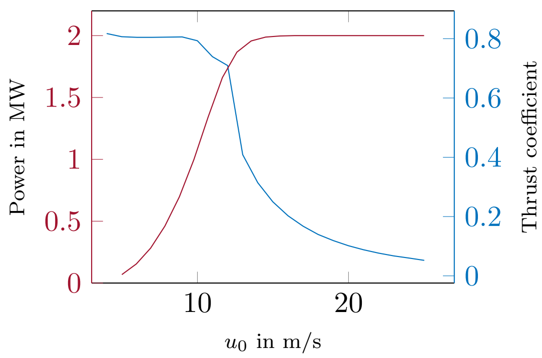
First, the validation of the tool is presented. Then, the influence of several wind farm and ambient parameters on the power yield is investigated. Finally, the optimisation results are shown.

3.1. Validation

To validate the developed tool, the Horns Rev 1 wind farm is modelled. This offshore wind farm consists of 80 V80-2MW wind turbines from Vestas. The layout of the Horns Rev 1 wind farm, as well as the power and thrust for the turbines, are given in Fig. 5. Using the measurements of Barthelmie et al. [11], the power loss within one row of the farm for three wind directions α (cf. Fig. 5a) is investigated. In addition, the predicted drop in wind speed and increase in turbulence intensity is investigated in detail in Fig. 6. To validate the flow conditions, the results of a Large Eddy Simulation (LES), carried out by WU & Porté-Agel, are used [12]. In addition, data according to Qian & Ishihara are evaluated, which contain analytical results of their in-house code as well as results obtained using the WAsP software [8]. The validation is performed for a wind speed of 8 m/s and an ambient turbulence of about 8 %. The measurement uncertainties

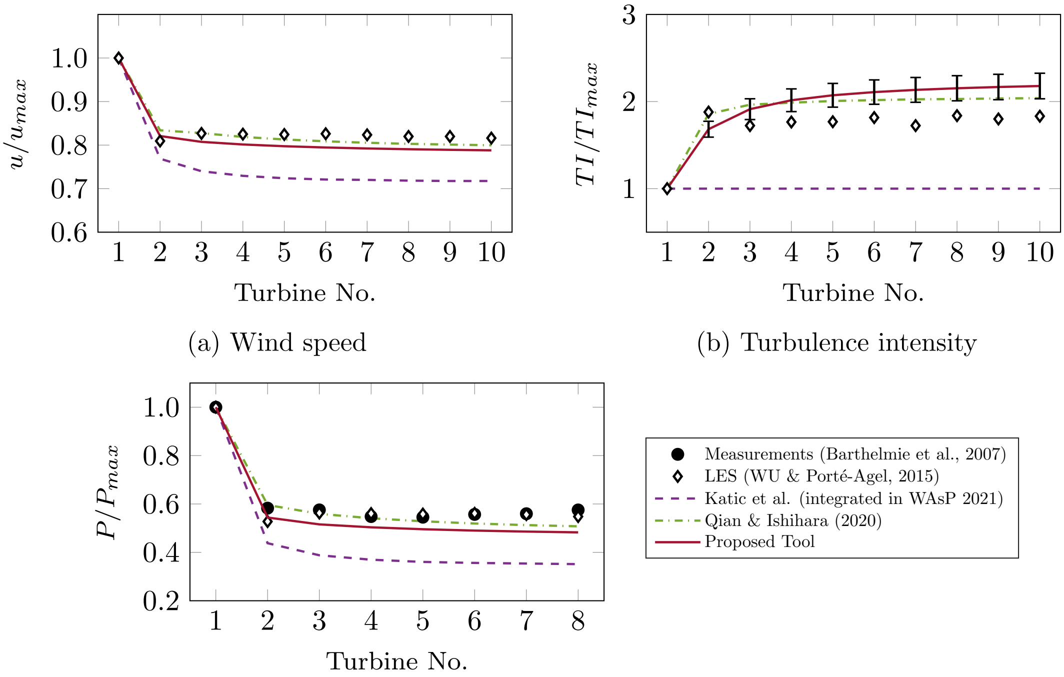


(a) Layout with three main wind directions



(b) Power- and thrust coefficient curve

Figure 5: Layout of Horns Rev 1 and performance map of Vestas V80 2MW-offshore [8].



(a) Wind speed (b) Turbulence intensity (c) Power loss

Figure 6: Normalised wind speed, turbulence intensity and power within one row for $\alpha_0 = 270°$.

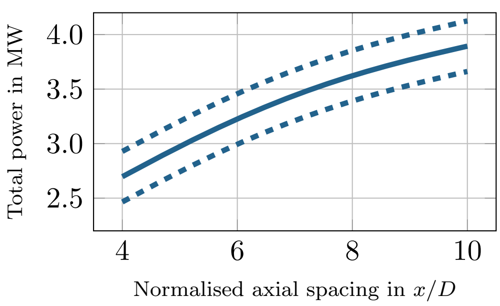
for the wind direction and the speed are given by Barthelmie to be $\pm 1°$ and ± 0.5 m/s [11], while the turbulence uncertainty of $\pm 2\,\%$ is taken from a study by Hansen et al. [13]. Both uncertainties have been included in the model to obtain estimates for the impact on the final results. For the power and wind speed losses, the computational deviations due to the given parameter uncertainties are $< 1\,\%$. An exception is the turbulence, where the error bars are shown in Fig. 6b. In general, it can be seen that the presented tool follows both the qualitative trend of the measured values and the results of the model given by Qian & Ishihara. The WAsP software clearly overestimates the loss in wind speed and turbine power. This deviation occurs because WAsP does not model the rotor induced turbulence within the wind farm (cf. 6b). Compared to the results given by Qian & Ishihara, the presented tool slightly overestimates the power losses. It is difficult to make a clear statement as to what causes this deviation, since not all key data are available in the literature. One reason could be that the presented tool neglects the third dimension and thus the shear profile of the flow. In contrast to the other variables, it can also be seen in Fig. 6b, that the turbulence has the largest deviation from the LES and reference model results. It can be assumed that the higher weighting of the turbine, as implemented by Qian & Ishihara, leads to this difference. The validation suggests that the superposition as proposed by Qian & Ishihara could give better alignment to both measurement and LES. A summary of the relative deviation of the model results are shown in Table 1.

Table 1: Max. relative error of power drop w.r.t. model data by [8] and measured data in [11].

Model	$\alpha_0 = 270°$	$\alpha_1 = 222°$	$\alpha_2 = 312°$
Katic et al. (as integrated in WAsP 2021)	38.6 %	24.6 %	24, 8 %
Qian & Ishihara (2020)	11.4 %	4.4 %	11.3 %
Presented tool	16.1 %	4.8 %	8.0 %

3.2. Sensitivity study of wind farm parameters

To investigate the interactions of turbine positioning, yaw angle, and ambient turbulence intensity on the power yield of a wind farm, a numerical experiment is conducted. To keep the computational time reasonable, a wind farm with fewer turbines is used for this study. This wind farm has 9 evenly spaced turbines of the type NREL offshore 5-MW as developed by [14]. The ambient wind speed is kept constant at $u_0 = 11.4\,\mathrm{m/s}$, for all parameter combinations. This corresponds to the nominal wind speed of the 5-MW turbine. All calculations are performed for a wind direction of $\alpha = 270°$. The investigated parameters are the axial $\Delta\,x$ and the lateral turbine spacing $\Delta\,y$ as well as the ambient turbulence TI_a. Additionally, the impact of yawing only a first-row turbine on the wind farm is investigated. A total of 15 000 different parameter combinations are computed using latin hypercube sampling. The axial spacing is analysed for a range of 4 to 10 x/D, while the lateral spacing is varied from 2 to 7 y/D. The parameter ranges are chosen based on the actual spacing for different wind directions at Horns Rev 1, thus guaranteeing the same safety distance. In addition, the ranges ensure that the downstream turbines are not placed in the near wake at distances $< 3\,D$. According to the results used for the validation, the turbulence intensity within the wind farm Horns Rev 1 increases to nearly 20 %. Therefore, TI_a is examined for a range of 4 to 20 %. In order to keep the structural loads and losses of the single turbine relational, the yaw angle is varied between $-20°$ and $+20°$. For the lateral turbine spacing and the yaw angle, no positive influence on the overall wind farm performance could be achieved and no results are shown in this work. The insensitivity to the yaw angle can be explained by the fact that the wakes continue to overlap despite the induced deflection. The axial distance as well as the ambient turbulence were found to be the most significant input variables (Fig. 7). This agrees with the results obtained by Barthelmie & Jensen [2]. In order to improve the power yield by altering the wind farm layout, the results suggest that the axial distance should be increased, which leads to the authors schematic layout proposal visualised in Fig. 8. While the original layout allows two turbines into the free stream, the optimised version shows the opportunity for having 4 turbines under undisturbed inflow conditions. At the same time, the axial distance for the downstream turbines increased significantly. Therefore, since the baseline layout is close to the optimisation reference as shown in Fig. 5a, the optimised layout proposal is expected to be close to this proposed, diagonally shaped pattern. For the yaw angle optimisation, the sensitivity study suggests that a performance increase due to yaw control is not possible when wakes are fully overlapping (as displayed in the left part of Fig. 8).



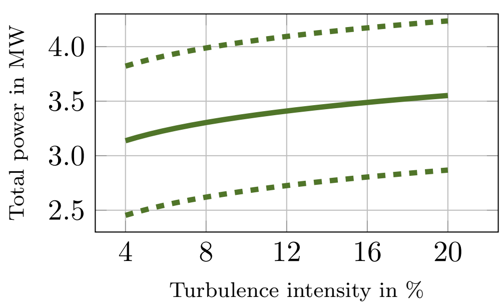


Figure 7: Total power of small scale 3x3 wind farm as a function of axial turbine spacing Δx (left) and ambient turbulence intensity TI_a (right) with 95 % confidence intervals (dashed line).

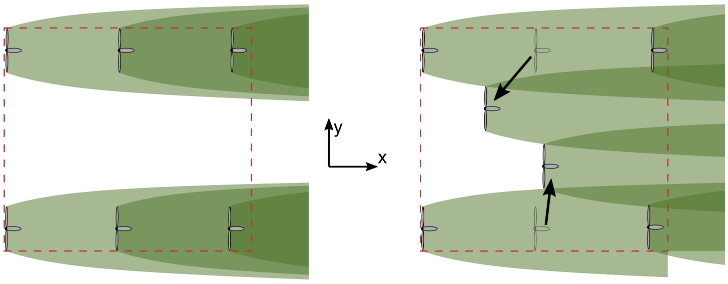


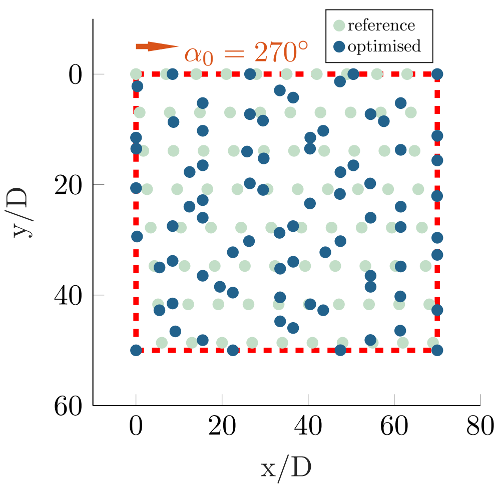
Figure 8: Schematic representation of baseline (left) and possible optimised layout (right).

3.3. Wind farm layout design using gradient optimisation

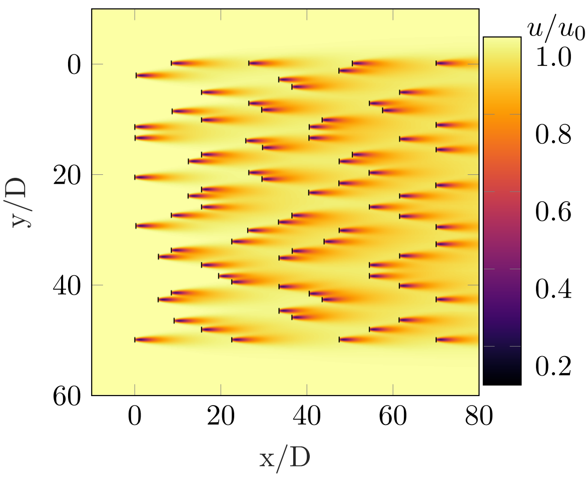
Two different optimisation approaches will be investigated in this section. First, the optimal turbine layout for one given wind direction is presented. Then, a yaw angle optimisation for an off main wind direction case is shown.

3.3.1. Layout optimisation

The previous sections show that there is great potential for optimising the overall wind farm performance. Increasing the axial spacing Δx between the turbines strongly impacts the power yield. A power-increasing layout change by varying the turbine coordinates x_i and y_i should therefore lead to better aerodynamic yield. The optimal layout is searched using *fmincon*, a non-linear, gradient-based extreme value search function implemented in MATLAB. The total wind farm power P_{total} is defined as the target value. The position vectors of the 80 turbines, x_i and y_i serve as design variables, defining this problem with 160 degrees of freedom. The optimisation of the positions is based on the already presented reference layout of the wind farm Horns Rev 1 (cf. 5a). The turbine used is the NREL offshore 5-MW turbine [14]. The flow condition for all calculations have a wind speed of 10 m/s for all calculations, which corresponds to an ambient turbulence of 6.4 % [13]. The layout optimisation is performed for non yawed conditions and a yaw angle of $\gamma_i = 0°$ is applied to all turbines. The resulting optimal layout is shown in Fig. 9. The seemingly unstructured arrangement shows similarities to research results of Charhouni et al. [15]. By looking at the corresponding velocity distribution (cf. Fig. 9b), a clustered diagonal arrangement of the turbines normal to the wind direction can be observed. Hence, by positioning



(a) Optimised vs. reference layout



(b) Wind speed distribution

Figure 9: Optimised layout for the wind direction $\alpha_0 = 270°$.

the turbines in the free stream, the wind farm performance is increased. For this wind farm, the number of turbines not affected by wakes is increased to 24 for the optimised solution, compared to the reference layout, which has 8 turbines located in the free stream. A power loss of about 40 % can therefore be prevented for $\Delta N = 16$ turbines for this wind direction. Comparing the total power of the optimised wind farm $P_{\text{optimised}}$ to the reference layout $P_{\text{reference}}$, a potential aerodynamic power increase of 57.17 % is obtained (cf. Table 2). Despite the fact that only one wind direction is investigated in this optimisation, the new layout also achieves a significant increase in power production for the other two main wind directions. Nevertheless, the results indicate how important the wind direction is for the layout optimisation of a wind farm. Future optimisation studies should include further constraints such as different wind speeds, directions and their corresponding probability distributions, multiple weights for superpositioning wake influences as well as structural mechanics and costs for maintenance, logistics and cabling.

Table 2: Optimised power yield for all main wind directions.

Optimised for main wind direction $\alpha_0 = 270°$			
α	$P_{\text{reference}}$ in MW	$P_{\text{optimised}}$ in MW	ΔP_{total}
270°	162.5	255.4	+57.17 %
222°	206.7	245.0	+18.53 %
312°	216.3	238.2	+10.12 %

3.3.2. Yaw angle optimisation

The sensitivity study performed, suggests that little gain in power production can be obtained from yaw steering of the wakes. However, Qian & Ishihara proposed in [8], that successful wake steering may be possible for some wind directions. Fig. 10 schematically shows such a case. Steering the wake out of the downstream turbine is expected to be possible for non main wind direction cases. Therefore, an optimisation will be performed using the baseline layout of Horns Rev 1 for a wind direction deviating by $\Delta\,\alpha = 5°$ from the main wind direction $\alpha_0 = 270°$. All other inflow conditions are equal to the layout optimisation problem. In analogy to the sensitivity study, the maximum yaw angle of the rotor with respect to the inflow wind direction is set to $\alpha_{max} = \pm 20°$. Here, the optimisation problem is compared using the baseline layout of Horns Rev 1 with NREL 5-MW offshore reference turbines. The resulting wind speed distribution with given optimal yaw angles is visualised in Fig. 11. The last turbine within the row has a yaw angle of 0° with respect to the inflow direction. At the same time, the first turbine shows a significant yaw angle of 19° close to the parameters boundary condition. It becomes clear, that optimising yaw angles becomes significantly more interesting for non main inflow angles $\Delta\alpha > 5°$. The resulting power gain with respect to the current common operating state of not yawing the turbines is shown in Table 3. The results highlight the aerodynamic potential of a yaw angle control. Even though the potential yield increase is significantly lower compared to the layout optimisation, the great advantage of the yaw angle control is that it could be used in existing wind farms. In order to install the yaw angle control, the built in yaw mechanisms of the turbines can be used. In addition, there is no need to change the position of the turbines in

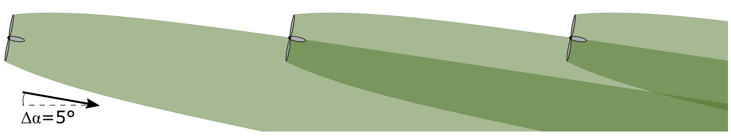


Figure 10: Schematic representation of possible yaw-optimisable inflow case.

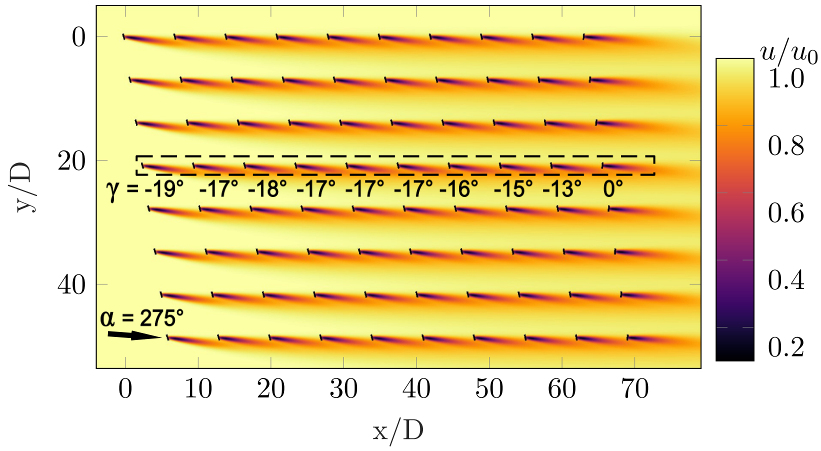


Figure 11: Optimised yaw angles for wind direction $\alpha = 275°$.

the farm. However, before giving a final evaluation, a more in-depth investigation of the optimal yaw angles at different wind speeds, directions and their distributions for a given location must be carried out, taking structural analysis and maintenance costs into account.

Table 3: Achieved performance increase with turbine yaw.

Optimised for wind direction $\alpha = 275°$		
$P_{\text{reference}}$	$P_{\text{optimised}}$	ΔP_{total}
196.317 MW	209.500 MW	+6.29 %

4. Conclusion and future studies

This work highlightsthe importance of wake effects and the resulting power losses within a wind farm. The most influencing parameters, the axial turbine spacing and the ambient turbulence, were identified and presented graphically. An effective tool for the investigation, prediction and performance-oriented optimisation of the turbine positioning and yaw angles is presented. The developed wind turbine farm simulation tool shows good agreement with measured values and LES simulations of the wind farm Horns Rev 1. Based on available data for Horns Rev 1, the power yield optimal layout for one main wind direction has been proposed, which could possibly lead to an increase in yield of 57,17 % in the main as well as 18,53 % in the non-optimised wind direction. Not shown results of layout optimisations for the non main wind directions 222° and 312° show possible yield gain of 34,34 % and 39,20 %, respectively. An optimisation of the yaw angles has been carried out for a 5° off-main wind direction with an estimated 6,29 % yield increase. Extended optimisation studies with increased inclusion of the wind direction and velocity distribution are necessary for more significant optimisation results. It should also be noted that other aspects such as structural mechanics and turbine maintenance work were neglected in the course of this study.

Future work will include an extension of the tool by reducing simplifications. In order to improve the accuracy and reliability of the results, adaptations and practical validations of the calculation tool must be included in future work. First the third dimension can be added, and secondly, a more detailed model of turbulence induction and superposition should be investigated. A wake-focused rotor blade optimisation with more powerful optimisation algorithms will be added. For that purpose it will be necessary to improve the existing wake model by radial distributed inflow parameters to close the gap between small scale studies on rotor blades and BEM theory and the large scale wind farm modelling. Finally the influence of wind direction and it's distribution will be added as additional constraint to the optimisation problem.

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