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SUSTAINABILITY AND MATHEMATICAL MODELLING IN 5TH GRADE

SOSTENIBILIDAD Y MODELADO MATEMÁTICO EN 5º GRADO

SUSTENTABILIDADE E MODELAGEM MATEMÁTICA NO 5º ANO

Lisa Steffensen
(Western Norway University of Applied Sciences, Norway)
lste@hvl.no

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ABSTRACT

This research investigates how pre-service teachers aim to bring awareness of environmental sustainability while learning mathematical modelling to their 5th-grade students. Theoretical perspectives from socio-critical modelling research can involve engaging students in societal issues, including focusing on action. Thematic coding of a mandatory task by a group of pre-service teachers, where they design, implement, and reflect on a modelling activity during practicum, is used to analyze how they combine environmental and mathematical aims. The findings show that the pre-service teachers emphasize the authenticity of tasks and data, engage students in actions in littering, reflect on students' critique of assumptions, and focus on the meaning-making of environmental numbers using students' everyday language. Implications to teacher education suggest that teachers can combine focusing on environmental sustainability through mathematics education. Further, teacher education can support pre-service teachers and students in actions through socio-critical modelling activities.

Keywords: mathematical modelling, socio-critical perspectives, sustainability, primary education, pre-service teachers.

RESUMEN

Esta investigación investiga cómo los maestros en formación pretenden generar conciencia sobre la sostenibilidad ambiental mientras aprenden modelos matemáticos a sus estudiantes de quinto grado. Las perspectivas teóricas de la investigación de modelos sociocríticos pueden implicar involucrar a los estudiantes en cuestiones sociales, incluido el enfoque en la acción. La codificación temática de una tarea obligatoria por parte de un grupo de futuros profesores, donde diseñan, implementan y reflexionan sobre una actividad de modelado durante la práctica, se utiliza para analizar cómo combinan los objetivos ambientales y matemáticos. Los hallazgos muestran que los futuros maestros enfatizan la autenticidad de las tareas y los datos, involucran a los estudiantes en acciones para tirar basura, reflexionan sobre la crítica de los supuestos de los estudiantes y se enfocan en la creación de significado de los números ambientales usando el lenguaje cotidiano de los estudiantes. Las implicaciones para la formación docente sugieren que los profesores pueden combinar el enfoque en la sostenibilidad ambiental a través de la educación matemática. Además, la formación docente puede apoyar a los futuros docentes y estudiantes en acciones a través de actividades de modelado sociocrítico.

Palabras modelado matemático, sostenibilidad, educación primaria, perspectivas sociocríticas, profesores en formación.

RESUMO

Esta pesquisa investiga como os futuros professores visam trazer a consciência da sustentabilidade ambiental enquanto aprendem modelagem matemática para seus alunos da 5ª série. As perspectivas teóricas da pesquisa de modelagem sociocrítica podem envolver o envolvimento dos alunos em questões sociais, incluindo o foco na ação. A codificação temática de uma tarefa obrigatória por um grupo de futuros professores, onde eles projetam, implementam e refletem sobre uma atividade de modelagem durante o estágio, é usada para analisar como eles combinam objetivos ambientais e matemáticos. Os resultados mostram que os professores iniciantes enfatizam a autenticidade das tarefas e dos dados, envolvem os alunos em ações de lixo, refletem sobre a crítica dos alunos às suposições e se concentram na criação de significado dos números ambientais usando a linguagem cotidiana dos alunos. As implicações para a formação de professores sugerem que os professores podem combinar o foco na sustentabilidade ambiental por meio da educação matemática. Além disso, a formação de professores pode apoiar professores e alunos em formação inicial em ações por meio de atividades de modelagem socio-crítica.

Palavras-chave: modelagem matemática, perspectivas socio-críticas, educação primária, sustentabilidade, professores em formação.

Introduction

In 2017, a dead whale was found in Norway with its stomach full of plastic packing. As a result of the plastic whale, schools and local communities became engaged and increased their focus on littering (Lislevand, 2021). There was an increase in people who thought plastic was an environmental problem, from 2.5% in 2015 to 45% in 2018 (Sylte, 2018). It caused organisations, politicians and decision-makers to act toward marine littering. Initiatives on plastic pollution are still happening, and in September 2022, the "Clean Hardangerfjord" initiated what they refer to as the world's largest plastic clean-up, engaging schools, companies, and volunteers (Clean the Fjords, 2022). Similar clean-ups are seen globally, like The Ocean Clean-up (2022), founded by a Dutch teenager in 2013. In Norway, the quantities of waste are growing along with economic growth and rising consumption, and globally, waste production is predicted to increase by 70% from 2018 to 2050 (Sensoneo, 2019). Waste worsens climate change by releasing methane gas and carbon dioxide, and littering (incorrectly disposed waste) destroys wildlife and natural habitats. Within mathematics education research, Barwell (2018) argue that the prevalence of mathematics in environmental issues calls for an important role of mathematics education in understanding the challenges. Citizens need competencies to critically engage with the mathematical information in sustainability, for instance, by reflecting on their own behaviours. He problematises that teaching about, e.g. statistical literacy and mathematical modelling rarely go beyond students as consumers of statistical information or critical thinkers and highlights how it could also focus on incorporating values, social relations, or social or environmental effects of mathematical models. Mathematical modelling refers to the process of using mathematics to solve real-world problems.

Combining environmental sustainability issues and mathematics education can increase students' awareness of critical real-world problems while providing relevant context to mathematics education. Sustainability is one of three interdisciplinary topics in the Norwegian core curriculum (Ministry of Education and Research, 2017). Here, it is stated that students should develop competencies to "make responsible choices and to act ethically and with environmental awareness" (p. 15). In the mathematics curriculum (Ministry of Education and Research, 2019), it is highlighted that mathematical modelling should contribute to understanding relationships within society and nature. Combining sustainability and mathematical modelling can contribute to students' understanding of mathematics as a relevant subject while bringing awareness to important societal issues.

I investigate Pre-service Teachers' (PTs) modelling activities involving sustainability, and I particularly focus on *how PTs bring awareness to environmental issues while learning mathematical modelling to their 5th-grade students*. A mandatory task by one group of PTs attending a teacher education course is used to investigate this focus.

Combining sustainability issues and mathematical modelling

Several studies have combined environmental issues and mathematical modelling. For instance, English (2012, 2013a, 2013b) described how 1st-grade students engaged in recycling while selecting attributes, structuring and representing data, and making predictions. Also, English (2009) described how 5th-grade students constructed guidelines indicating the health of a creek. She highlighted that modelling promotes interdisciplinary learning, and the study included core concepts from mathematics, science, society and the environment and the interaction between these areas.

Solares-Rojas et al. (2022) described how 2nd-grade students investigated socio-ecological problems in their community. The nearby river is highly polluted by the region's factories and urban centres, and the students used tables and pictograms to present their classifications of local animals. The research involved collaborative work with school students, researchers, and the community. Aguirre et al. (2019) described a modelling task that combined modelling with social justice issues involving the context of polluted water. Teachers estimated the number of bottled water needed for the school children in the affected area. Aguirre et al. argued that after teachers engaged in such tasks, this led them to imagine other relevant topics to include when modelling in their own classroom, e.g. immigration issues, recycling, and food insecurity.

Gürbüz and Çalik (2021) researched 7th-grade students' investigations of waste management using plastic bottles instead of bricks when building houses. They found that students' dialogues involved discussions both of mathematical problem solving and exploring relevant environmental issues such as protecting the environment, waste saving, and informing society. They highlighted that the modelling activity facilitated students' inquiry, critical thinking, and acting as responsible citizens for the environmental issue under investigation. Another study exploring waste was done by Jung et al. (2021). They described a modelling activity where 7th-grade students investigated the Great Pacific garbage patch. The students designed a model to convey the size and density of the garbage patch, and they discussed various approaches. They engaged in, e.g. the origin of the trash, the effects on wildlife, and the extent and actions taken. Özdemir (2021) researched 5th-graders' integration of environmental and mathematics education and found that this combination brought awareness of their actions and roles and attention to the relevance of mathematics in solving real-world problems. They highlighted that the duality between environmental issues and mathematics could broaden students' perspectives, and both are needed to achieve meaningful reasoning. Johnsen-Høines (2020) described how 4th-grade students mathematically investigated issues rated to the plastic whale from Norway. They visited an exhibition about the plastic whale, had classroom talks, and worked on formulating their mathematical problems. Examples of the latter are "How much plastic is thrown into the ocean each day – each week – a year" and "How much garbage can we pick up in one year – two years – three years" (p. 200). When students presented their results, they focused on approaches and asked questions to peers to understand if they had suggestions on other approaches.

Socio-critical modelling perspectives

Socio-critical modelling perspectives emphasise the role of mathematics education in addressing the nature and role of mathematical models in society (Barbosa, 2009). By bringing students awareness of how mathematics shapes their community, students can engage in societal issues. Skovsmose (2023) argued that mathematics education could empower students to take action in society. Schools could educate students to believe their actions can make a difference in society, and idealistically, educational processes can turn into (collectively) actions outside the classroom. For example, the focus on the plastic

whale in the media and schools resulted in substantial public actions where citizens volunteered to pick up litter. Related, Rosa and Orey (2015) argued that a socio-critical modelling dimension should engage students in taking action to solve problems in their reality, and Gutstein (2006) described how students could act by reading and writing their world toward a more just society. He highlighted that teachers can provide students opportunities to develop a sense of social agency by engaging in and taking action on issues like disparities in wealth, racism, and other forms of inequality in their mathematics classroom. Combining sustainability and mathematics can provide opportunities for students and can be a way of engaging them in action.

When teaching societal issues, social, political and economic dimensions are typically avoided and deemphasised (Bhattacharya et al., 2021). Therefore, teachers and teacher educators should reflect on their responsibility to include a focus on these dimensions when teaching mathematics to students. Ernest (2019) argued that mathematics teachers have ethical obligations to use mathematics to raise ethical issues in the classroom, such as care for environmental issues. He described that modelling real-world examples can help develop skills, concepts and strategies and motivate problem-solving. He problematised that educators focus on instrumental numeracy and teaching to the tests instead of empowering students to contribute to a democratic society concerned with social justice and environmental problems. As an example of the latter, a study from Villarreal et al. (2015) described a situation when PTs engaged in socio-critical modelling tasks. They were encouraged to choose their topic of interest, investigate, report, and present their work. Their findings show that the PTs included such as socio-economic issues, ecological conscience (e.g. trash and recycling), and problems affecting them personally. The PTs engaging in trash and recycling issues expressed that when designing their modelling activity, their aim was not "obtain a super formula"; instead, it was to "modelling to raise awareness".

Quantifying and comprehending ecological entities can be challenging for various reasons, potentially resulting in a lack of action toward the problem. When imagining large numbers, Renert and Davis (2012) described that humans struggle with our quantity sense, the "ability to comprehend magnitude and size" (p. 99). For instance, we have difficulties conceiving the amount of 29 trillion ($2.9x10^{13}$) kilograms of carbon emissions emitted to the atmosphere. This struggle is problematic when it comes to dealing with environmental issues. When working with environmental issues in the classroom, teachers must be aware of the challenges in comprehending large numbers, particularly for young students. Johnsen-Høines (2020) described how students' perceptions from their everyday life are part of forming how they perceive and interpret concepts. While their conceptual content is thoughts and meanings of one's surroundings, conceptual expressions are the oral expressions, signs, and body language used to express your thoughts and meanings. Johnsen-Høines (2020) used the terms 1st and 2nd order language to refer to students' language (verbally, hand signs, drawings, etc.). Students understand words and signs from their 1st-order language and can think and learn through it. Their 2nd order language needs to be translated. An example of a student's 1st-order language is a visual image of three fishes or showing three fingers, while "|||" or "3" is part of their 2nd-order language. While 1st order language is strongly connected to their perception, meanings, and thoughts of a concept, 2nd order language has weak links to students' conceptual content. In Johnsen-Høines's (2020) study with 4th-grade students, they try to visualise the amount of plastic waste from the statement: "15 tonnes of plastic is thrown every minute" (p. 199). The teacher facilitated students grappling with large numbers by focusing on familiar units. For instance, when imagining 15 tonnes, they started by imagining 1 kilo of sugar and physically measured the time by counting 60 seconds. When modelling real-life experiences such as the plastic whale with young students, it is imperative to consider the student's language to comprehend environmental numbers.

Vos (2011) emphasised that providing authentic contexts is essential for students' modelling experiences. She described how authentic resources (e.g., a basket) can be used in inauthentic activities (e.g., finding the basket's circumferences) or task variations can facilitate different forms of authenticity. Also, some parts of a modelling task can be authentic; for instance, students can work with authentic data or use authentic modelling software. When working with authentic or personally relevant contexts,

Suh et al. (2021) found authenticity was a criterion for successful practice and engagement for elementary students' participation in modelling activities. Young people's engagement in initiatives like "Clean Hargangerfjord" (Clean the Fjords, 2022) shows that environmental issues are important to their lives. Suh et al. (2021) described that teachers emphasize criteria for success, for instance, connections to students' lived experiences, how the task connected to students' interests and emotions, or degree of civic and social awareness.

Dealing with environmental issues can lead to hopelessness due to their severity and complexity. So, introducing the burdens of environmental issues to young students could be accompanied by providing hope through action (Ryan & Steffensen, 2021). Freire et al. (2014) describe how hope has its foundation in believing that we can organize ourselves to different possibilities for the future. For instance, believing we are able to reduce the amount of litter in the future and mobilize ourselves accordingly.

Methodology

This study is part of a larger design-based research, where researchers and teacher educators aim to improve mathematics teaching and learning, see, e.g. Hansen (2021). Here, PTs were introduced to mathematical modelling in their second year of teacher education and asked to design a modelling activity. To investigate how PTs bring awareness of environmental issues while teaching mathematical modelling, I use one task from one group of PTs as a case study. This group was chosen because their modelling activity involved littering. They implemented the modelling activity during their practicum, which they described in a mandatory written assignment.

The practicum school asked the PTs to introduce the topic of sustainability and take an interdisciplinary approach. The PTs designed a modelling activity consisting of natural science, social studies, Norwegian, arts and crafts, and mathematics. They focused on littering in nature. Their students were two 5th-grade classes, with 21 and 20 students. The students worked in groups of about ten. In the first part of the activity, the students were outside and asked to collect as much litter as they could in one hour. The second part took place inside the classroom, and the PTs used their litter measurements to design four modelling tasks for their students, each containing subtasks (a total of 11 tasks). In their assignment, the PTs expressed mathematical objectives such as mathematical modelling, standardized and unstandardized measuring, whole numbers, and decimals, and the four basic arithmetic operations. They also expressed that they wanted to use the subject of mathematics to shed light on littering issues and assist sustainability. The assignment was thematically coded in NVivo and categorized into main themes. Due to page limitation, three main themes are included in this paper, chosen because they provide illustrating examples to reflect on the research question. The above theoretical perspectives are used to reflect on the emerging themes.

Combining the subject of mathematics with awareness of environmental issues

Three main themes are used to discuss how PTs bring awareness to environmental issues while teaching mathematical modelling to their 5th-grade students: (1) Authenticity, problem-posing, and critique, (2) Mathematical modelling focusing on action, and (3) Meaning-making using students' everyday language.

The authenticity, problem-posing, and critique in mathematical modelling

The PTs start the modelling activity by letting their students pick litter in their local community. They ask students to "pick as much as you can" and "the result will be measured" and tell them that both classes will do the same activity. The PTs' focus is linked to community engagement on littering issues, and they aim to make the data relevant to the students by using their own litter measurements. Using authentic data aligns with what Vos (2011) and Suh et al. (2021) highlighted as essential to engage

students in modelling tasks and make the modelling activity more authentic. The PTs display the data in a table, see Table 1:

	Class 5A	Class 5B
Number of students	20	21
Litter in kilos	15.3	6.4

Table 1: Measurements of litter collected by the two classes.

The PTs use the authentic data and pose questions such as: (1) Both classes used one hour. How much litter could class 5A pick up in two hours? (2) How much litter could both classes pick up in one week if they use one hour daily? (3) Each student picked about 0.5 kilos in one hour. How much litter could you and your family pick up in one hour? (4) Make your own task about littering. The PTs pose most of the questions (10 out of 11) but include one where the students should pose a question themselves (Q4). There are strong arguments for letting students pose their own questions based on their interests when modelling, as Villarreal et al. (2015) described. However, the PTs have limited time for the activity, and both students and PTs are unfamiliar with modelling.

Some of these modelling problems resemble regular world problems and could trigger students to solve them as routine textbook problems. For instance, answering (1) by merely doubling what class 5A picked in one hour and not giving it more thought. However, the PTs express that students ask questions such as "Where did class A go since they managed to pick that much?" and "Was it more litter there?". These utterances imply that the students compare their results and consider that location impacts these. Thus, one could argue that students use their knowledge about the data to be critical to the generalizability of the collected data. As one student said, "If we pick litter for two hours, it is not sure that we find as much. Or perhaps we find more". Their reflection on the data would perhaps not have happened if they saw this as a genuine word problem in a textbook. The authenticity of data (Suh et al., 2021; Vos, 2011), their experience, together with the discrepancy between the two classes' results (15.3 kilos compared to 6.4 kilos), might have contributed to shedding light on the real situation and bringing this knowledge into play when modelling and predicting results.

The PTs acknowledged the students' implicit critique and argumentation about the generalizability. They write: "After all, all their thoughts were correct and reasonable and highlighted that it is not always as simple as just calculating, and then the answer is given". Critically engaging with mathematical information is essential for critical citizens (Barwell, 2018). To acknowledge and provide opportunities to dwell on critical questions is essential for teachers and students when engaging in numbers and data. Although the PTs express the student's critical approach to the data as valuable, they also express worries that students became "more engaged in investigating whether the numbers were representative or fair than working with the problems". It may seem like the PTs perceive these critical questions as students being "off-track". Teachers raising concerns about what is "on track" when it comes to modelling conversations were also described by Aguirre et al. (2019). Due to the nature of modelling tasks and their strong connections to the real world, it is relevant to include discussions that perhaps seem non-mathematical at first glance. In the PT's reflections, they write that as an improvement of their actions in the classroom, they could have taken students' critical questions more seriously, for instance, by dwelling on why class 5A had managed to collect more litter than class 5B. Although the problem-posing tasks designed by the PTs inhabit traits from ordinary mathematics tasks, the students perceive them as authentic and pose relevant and critical questions about how to deal with the data and their generalizability. Providing more space for students' reflections could have provided opportunities for students to further elaborate and reflect on the nature of models, the assumptions, and the consequences of the selected criteria in line with Barbosa (2009) ideas of socio-critical modelling.

Action in mathematical modelling

The PTs research question in their mandatory task is: "How can we use the subject of mathematics to shed light on littering issues and assist sustainability in primary school?" Their research question and their activities with the students involved at least two forms of action. First, there is an action toward using mathematics education to "assist sustainability". Their focus on using mathematical modelling as an approach to promote sustainability is in line with Ernest's (2019) and Barwell's (2018) call for mathematics education to empower students' regarding environmental issues. They actively address how mathematics education can be part of empowering primary students with environmental issues through the subject of mathematics. Also, using mathematical modelling to raise ethical issues in the classroom is in line with what Ernest (2019) highlighted as the ethical obligations of teachers. The PTs invite the students to contribute ideas on how they could connect mathematics to littering. They write that their students "initially struggles to answer, 'How can we use mathematics in picking litter?'". By asking the students how mathematics is relevant to littering, the PTs bring awareness to how mathematics is part of real-world issues. The PTs further write: "After some guidance and time to think, several contributed with ideas, and we saw a clear engagement when discovering this [how to use mathematics when picking litter]". Allowing the students time to reflect on the duality between mathematics and littering is related to the findings of Özdemir (2021); working with environmental issues in the mathematics lesson engaged the students. And it also provides meaningful reasoning on how mathematics has relevance to real-world problems and engagement with environmental issues, and vice versa.

Secondly, there is an action toward hands-on activities by students. When the PTs engage their students in picking litter and dealing with the numbers involved, they focus on action in line with what Gutstein (2006) and Rosa and Orey (2015) proposed. It is contrary to teachers' typical approach toward a descriptive approach (Bhattacharya et al., 2021). The PTs state that one aim of picking litter was because they wanted "to visualise how small individual contribution can contribute to making a difference for the environment". They focus on the fact that individual efforts matter. Visualising litter can help bring forward important messages. For instance, seeing the plastic bags found inside the "plastic whale" was a wake-up call for many people, resulting in individual action towards littering (Sylte, 2018). Also, the emphasis that individual action matters can provide hope rather than a feeling of hopelessness caused by the magnitude and the severity of this problem, in line with what Ryan and Steffensen (2021) describe. The PTs further write: "We wanted to give the students the experience that they contributed to a larger community. That's why we cleaned up nature, which led to the students gaining a strong sense of belonging". They emphasise that they want their students to see that their contribution count in a larger picture. By giving their students the opportunity to contribute, they provide hope for a better future along with the ideas of Freire (2014). The focus on individual contributions and the responsibility one has as a citizen could have been balanced with a focus on the responsibility of large companies, decision-makers, and government. Norway's largest contributors to marine plastic waste are fisheries, aquaculture, shipping, tourism, and consumers. When teachers choose to focus on individual contributions, they should emphasise that littering is not an individual responsibility alone, and systemic changes in.

Meaning-making using students' everyday language

When designing the modelling activity, the PTs and students decide to weigh all the litter. The PTs ask students to make assumptions about the weight of their group's litter bag while collecting. They get suggestions like "it is heavier than the school bag" and "it weighs about the same as my little sister". The PTs express that they want their students to use their everyday experiences and that "it is important to create a dialogue with the students about the non-standardised measurements […] and we tried to create a translation between 1st and 2nd order language". The PTs lean on the mathematics curricula, where it is stated that students should use non-standardised units (Ministry of Education and Research, 2019). Further, they use the concepts from their mathematics curricula literature, students use of 1st and 2nd order language, to describe students' oral language, hand signs, drawings, etc. As Johnsen-Høines

(2020) highlighted, because of the weaker links between conceptual content and conceptual expression in students' 2nd order language, the PTs emphasise it is important to translate between students' 1st and 2nd order language. Thus, they aim to build stronger links between students' perceptions, meanings, and thoughts of a concept in line with Johnsen-Høines' (2020) ideas. When students express that the litter bag is heavier than their school bag, it is meaningful to them; they have first-hand experience and, thus, probably part of students' 1st order language. The PTs write that they "let the students lead the conversation with their language and experience, and gradually build and introduce concepts that we want to be part of the students' 1st order language". When the PTs express awareness of using students' 1st order language while simultaneously using standardised measuring units, it could help students translate their everyday language toward standardised units.

Comprehending the problems of littering involves an understanding of the amounts involved. The weight is one way of quantifying the amount. The PTs write that "the students kept running over to us so we 'could feel how much the small pieces of litter weighted, too'". The students share their tactile experiences with the PTs. And the PTs emphasise students' engagement in actually feeling the weight of litter. As highlighted by Renert and Davis (2012) and Johnsen-Høines (2020), visualising environmental numbers that may be difficult for students to grasp can be one way of understanding mathematical-based sustainability issues.

Concluding comments

The findings from how PTs' can bring awareness to environmental issues while learning mathematical modelling to 5th-grade students are summarised in **Table 2**.

Themes	Combining mathematical modelling and environmental issues
Authenticity, problem-posing, and critique	• Students collect their own authentic data in their community. • PTs pose problems based on students' authentic data. • PTs value students' critique regarding the generalisability of the data but worry that students are "off-track" mathematically.
Action and modelling	• Students engage in hands-on action towards littering. • Students provide ideas on combining mathematics and littering. • PTs emphasise that individual action also contributes to society. • PTs have an action on using the subject of mathematics to bring awareness of and promote sustainability.
Meaning-making in an everyday language	• Students lead conversations and use their everyday language. • PTs help translates between students' 1st and 2nd order language. • PTs support students' visualisations and comprehension of environmental numbers.

Table 2: Overview of findings.

This research supports researchers like Ernest (2019), Barwell (2018), and Renert and Davis (2012) call for mathematics education to engage in environmental sustainability. It adds to empirical research on environmental and sustainability issues in mathematics education. In particular, it adds to research on socio-critical modelling perspectives, as expressed by, amongst others, Aguirre et al. (2019), Rosa and Orey (2015) and Barbosa (2009); modelling activities with primary students focusing on interdisciplinary learning, as described, for instance, by English (2009, 2012, 2013a, 2013b); and

modelling with an environmental focus (Gürbüz & Çalik, 2021; Johnsen-Høines, 2020; Jung et al., 2021; Solares-Rojas et al., 2022).

The challenge presented by global environmental threats implies that teacher education, schools, and teachers have a role in supporting students' engagement in environmental issues through mathematics. The findings from this research imply implications for teacher education, for example, to encourage PTs to combine mathematics education with environmental sustainability. When combining these two fields, teacher educators can emphasise issues such as the authenticity of tasks and data, engaging students in action on an individual level as well as at a systemic level, enabling meaning-making of environmental numbers using students' everyday language, critique models and their use in society, and how teachers can act as socio-critical agents on serious societal problems.

Limitations of the research involve representativeness and generalisability due to being a case study. Further, the mandatory task is the PTs' reflections about what happened in the classroom and does not display the full picture of the classroom activity. For instance, the data does not show whether the PTs or the students reflected on behavioural changes, e.g. reducing individual (and collective) waste. Nevertheless, the research provides insight into how PTs use mathematical modelling to contribute to environmental awareness regarding littering.

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