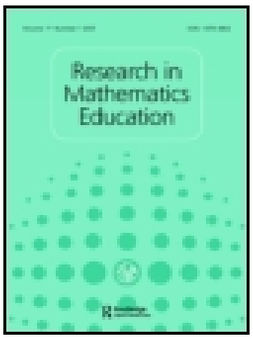

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Developing an analytical tool for radical socially-just teacher educator action research about language diverse mathematics classrooms

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ABSTRACT
In our action research projects as teacher educators, we focus on broadening preservice teachers' understandings about language diversity in mathematics classrooms, away from just improving students' fluency in the language of instruction. To undertake these projects, we developed a flexible analytical tool, for identifying those pedagogical practices which could be improved and so broaden pre-service teachers' understandings about language diversity. By documenting the development of the analytical tool, we provide insights into how action research projects can contribute to aims for social justice. Using data from a wider project, the analytical tool is discussed in regard to its possibilities for identifying practices that could be improved, its validity and reliability, and its potential for use in other action research projects.

KEYWORDS
Mathematics teacher education; language diverse classrooms; social justice action research

Introduction

To support mathematics teaching in multilingual classrooms, Essien (2010) suggested that teacher education should provide experiences and encourage practices that give insights into the complexities connected to language-diverse, mathematics classrooms. Without engaging with these complexities, preservice teachers (PTs) could gain limited understanding about supporting language diverse students (see for example, Rangnes & Eikset, 2019). Yet, raising these complexities is challenging for teacher educators (TEs) (Eikset & Meaney, 2018). In survey studies, Vomvoridi-Ivanović and McLeman (2015) and McLeman and Vomvoridi-Ivanović (2017) found that TEs' self-reported practices to resolve challenges in their teaching about equity did not match the nature of those challenges. This resulted in PTs being underprepared to utilise equitable mathematics pedagogies.

The complexity of issues includes understandings about language diversity as a right, as a problem and as a resource (Planas & Setati-Phakeng, 2014; Ruiz, 1984). For example, Thomassen and Munthe (2021) found that PTs, at several Norwegian teacher education institutions, lacked knowledge of multilingual students' legal rights and the regulations concerning teaching resources at schools. In a small study, Rangnes and Eikset (2019)

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identified in PTs' reflections on mathematics teaching on practicum that they lacked pedagogical knowledge about working with language diverse students, including utilising their backgrounds as a resource. In their reflections, the PTs referred to deficit perspectives that situated students' use of languages other than Norwegian as a problem. Research on how TEs support PTs to utilise students' languages as resources have tended to focus on what happens within the classroom (see for example, Wessel & Erath, 2018), with a few exceptions such as Aguirre and Zavala (2013).

After investigating teacher education in three countries, Essien et al. (2016) suggested that TEs rarely focused their PTs on practices for language diversity in systematic and structured ways, even though the TEs were aware of language diversity issues in mathematics education. Instead, TEs prioritised mathematical content and forming PTs' mathematics teacher identities. Eikset and Meaney (2018) concluded that, by prioritising mathematical content, the TE restricted opportunities to raise discussions about language diversity. This is a tension experienced by many TEs in balancing addressing issues of equity and social justice with mathematical content (e.g. Vomvoridi-Ivanović & McLeman, 2015). This suggests that even with a clear focus on social justice concerns in educational planning, it might be challenging to undertake actions, when these concerns are not shared among classroom members (Bartell, 2013).

To overcome these potential barriers, de Araujo et al. (2021) suggested that teacher education programmes need to disrupt notions of "effective teaching". Similarly, Parra and Trinick (2018) suggested that to "deviate" from purely mathematical content goals and proficiencies there is a need to attend to cultural issues, language practices and to problematise mathematics. Implementing such suggestions is likely to improve possibilities for providing PTs with a more socially-just mathematics education. However, to implement such changes TEs need to systematically investigate their own practices.

Gates and Jorgensen (2009) labelled the disruption to socially-unjust teaching practices as radical social justice in teacher education and considered it to be the hardest kind of social justice to achieve. Although they focused on class differences, language-diverse mathematics classrooms are also areas of marginalisation, through normalising the acceptance of language diversity as a problem (Planas & Setati-Phakeng, 2014). In our individual, wider action research projects, we focus on improving our practices to broaden PTs' understandings about language diversity as a form of radical social justice, requiring us to raise issues that look beyond what occurs in the classroom.

In this article, we describe the development of an analytical tool to support us to improve our practices in raising issues about mathematics and language diversity. In earlier research, it was challenging to identify those practices that needed improvement and determine ways to improve them (see for example, Eikset & Meaney, 2018; Meaney, 2013). Therefore, an analytical tool needed to be developed that could be adapted for our individual action research projects and for others to match their own focus.

Teacher educator action research and analytical tools

Before describing the process of developing the analytical tool, we begin by introducing the need of such tools in action research projects. Kemmis (2009) described action research as a meta-practice, or a "practice-changing practice", which aims at changing not only practitioners' practices, but also their understandings of these practices and

the conditions under which they are implemented. To develop an understanding of practices there is a need for self-reflection (Carr & Kemmis, 1986). Therefore, as part of our commitment to social justice, it was important to document the development of the tool so that our own biases become evident and could inform other action research projects.

Griffiths (2009) made a distinction between action research *as* and *for* social justice:

- Action research *as* social justice, where social justice issues are reflected into the research process, such as its epistemology and collaborative practices and actions.
- Action research *for* social justice, where the outcomes of the research are focused on improving social justice issues.
- Action research that includes both of these approaches, or moves from one to the other over time, when researchers are *mindful* of social justice concerns and the processes of their research.

In previous mathematics education research, TEs' action research has generally focused on developing PTs' pedagogical content knowledge (e.g. Erbilgin, 2019), and doing action research for social justice. For example, in an action research project by de Freitas (2008), the focus was on exploring the complexities connected to PTs' resistance to social justice issues in mathematics education, rather than on her own teacher education practices.

However, action researchers can encounter practical challenges related to developing research techniques and methods (Winter, 1989). Among these, Winter (1989) identified the need for research techniques which are specific enough to support small-scale studies by providing significant new insights, but not too simple to reduce validity, or too elaborate to be applied. For action research *as* and *for* social justice, it is particularly important that the analytical tool which is used provides valid and reliable results that can lead to improvement of practices and to understandings of social justice issues. Therefore, using an analytical tool as a basis for validity, is not a matter of how valid the findings are themselves, but of how the procedures undertaken to produce these findings are specific and rigorous (Winter, 1989). In addition, reliability of such tools in action research can be supported by ensuring that individual observations and interpretations are checked against observations and interpretations of another individual with similar perspectives and purposes (Lankshear & Knobel, 2004).

Developing the analytical tool – a focus on content

In considering appropriate tools for our action research projects, there was a need to identify relevant knowledge, in previous research, that should be made available to PTs and which could be a basis for the analytical tool as part of action research for social justice. Frameworks described in previous research about language diversity can be classified into two groups. The first group focuses on integrating language and language learning with mathematical content, but where critical perspectives are not discussed. The second group focuses on how aspects of culture, language and identity are more or less included under the umbrella of social justice for diversity.

The first group mostly includes frameworks developed for professional development programmes or design-research for in-service mathematics teachers, such as Prediger's

(2019) conceptual framework for content-specific teacher expertise and Wessel and Erath's (2018) tool for enhancing language learners' mathematics and language proficiency. These frameworks focus on the needs of teachers in supporting their language diverse students to develop mathematical and language competencies and so do not consider aspects beyond interactions in the classroom. Such frameworks in teacher education may lead to a limited focus on what happens inside the classroom which reinforces the gatekeeping role of mathematics. This could restrict PTs increasing their understandings about language diverse students' possibilities to explore and learn about the world with mathematics (Lange & Meaney, 2019).

The second group of studies broadens the focus from mathematics to the inclusion of social justice issues, often from a culturally-responsive approach. For example, Aguirre and Zavala (2013) designed a tool that enabled teachers to plan and analyse mathematics lessons from multiple dimensions of mathematical thinking, language, culture and social justice.

Both groups of frameworks provide insights about mathematics teacher education, however none of them had been used in action research projects. In the pilot study for this project, we chose to use the framework of Lange and Meaney (2019), called Learning About Teaching Argumentation for Critical Mathematics Education in multilingual classrooms (LATACME) (Lange & Meaney, 2019). The LATACME framework is theoretically similar to that of Aguirre and Zavala (2013) in that it identifies roles and responsibilities for teachers, but differs in that it focuses on language diversity and argumentation (see Table 1). The framework of Lange and Meaney (2019) was developed as part of a large project, also known as LATACME, in which our action research projects are part. Designed for TEs, this framework identifies understandings about language-diverse students in mathematics classrooms, incorporates how culture, both that of the students (Parra & Trinick, 2018) and the school systems, affects learning possibilities in language-diverse classrooms.

The first responsibility of mathematics teachers is to facilitate students' possibilities, including language diverse students, to explore and learn mathematics. As school mathematics acts as a gatekeeper for jobs and further studies, students need opportunities to become successful learners (Meaney, 2018). The second responsibility is supporting students to explore and learn about the world through mathematics, which relates to developing democratic competencies, a requirement in the Norwegian curriculum (Utdanningsdirektoratet, 2020).

The three roles are related to teaching in language-diverse, mathematics classrooms. The first one concerns how to teach so that students have the best possibilities to

Table 1. The LATACME framework (from Lange & Meaney, 2019, p. 2).

	Responsibilities	
Roles	Facilitating the exploring and learning of mathematics	Facilitating the exploring and learning about the world through mathematics
Teacher	Knowing how mathematical topics and mathematical argumentation can be developed where languages are considered a resource.	Knowing how to develop connections between critical mathematics education and argumentation.
Learner	Learning from multilingual students about their understanding of mathematical topics and argumentation.	Learning about critical mathematics education issues of interest and importance to multilingual students and their communities.
Advocate	Knowing how to provide input about mathematics education, including argumentation, to (multilingual) parents, school communities and government.	Knowing how to advocate that students need to use mathematical arguments in order to explore the world.

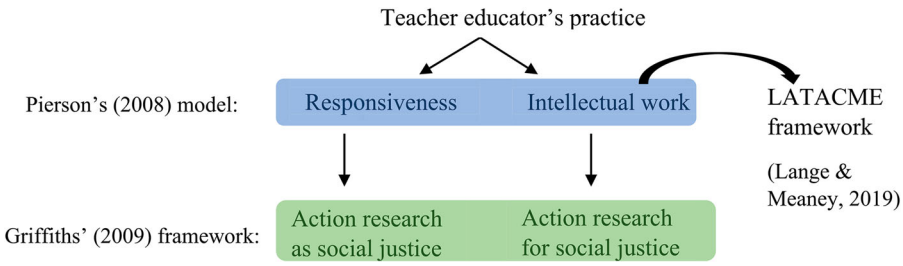

Figure 1. The integration of the frameworks into the analytical tool.

explore and learn mathematics and to explore and learn about the world through mathematics. The second role refers to the need for PTs to learn about the skills and knowledge their students bring to the classroom and their aspirations for engaging with mathematics. The third role is about advocating for students. The advocacy role, as Lange and Meaney (2019) noted, has rarely been discussed in mathematics teacher education. It comes from the assumption that some aspects of the wider society may need to be challenged if language diverse students are to gain appropriate learning opportunities. For example, PTs need to know about educational laws and students' rights in relation to language diversity (Thomassen & Munthe, 2021) in order to advocate for them.

By focusing on the LATACME framework, as the content we wanted to highlight in our teacher education (action research for social justice), we hoped to develop PTs' understandings about language diversity by challenging their notions of effective mathematics instruction (de Araujo et al., 2021). As part of our approach, we also needed to ensure that the requirements for action research as social justice (Griffiths, 2009) were also met. To do this we adapted Pierson's (2008) constructs of responsiveness and intellectual work as an analytical tool for radical socially-just teacher educator action research about language diverse mathematics classrooms. Figure 1 outlines how three different frameworks (Griffiths, Pierson, and LATACME) are integrated before describing the analytical tool in more detail in the next section.

Developing the analytical tool – a focus on process of identification

Pierson's (2008) model was originally developed to improve teachers' professional skills, such as active listening, setting goals, and facilitating interactions in classroom dialogue to improve students' mathematical understandings. It had two constructs, responsiveness and intellectual work. Responsiveness was about the extent to which the teachers took up students' ideas during mathematics classroom interactions, focusing on student thinking or their own thinking (see Figure 2). Intellectual work referred to the cognitive activity that the teachers wanted students to engage with during those interactions (see Figure 3). In a recent study of the same author, Bishop (2021) found that the level of intellectual work that the teacher requested from students was associated with the level of students' responses, highlighting the relationship between the two constructs.

From our perspective, the constructs could be modified to focus of broadening PT's understandings about language diversity in mathematics classrooms and fulfil Winter's

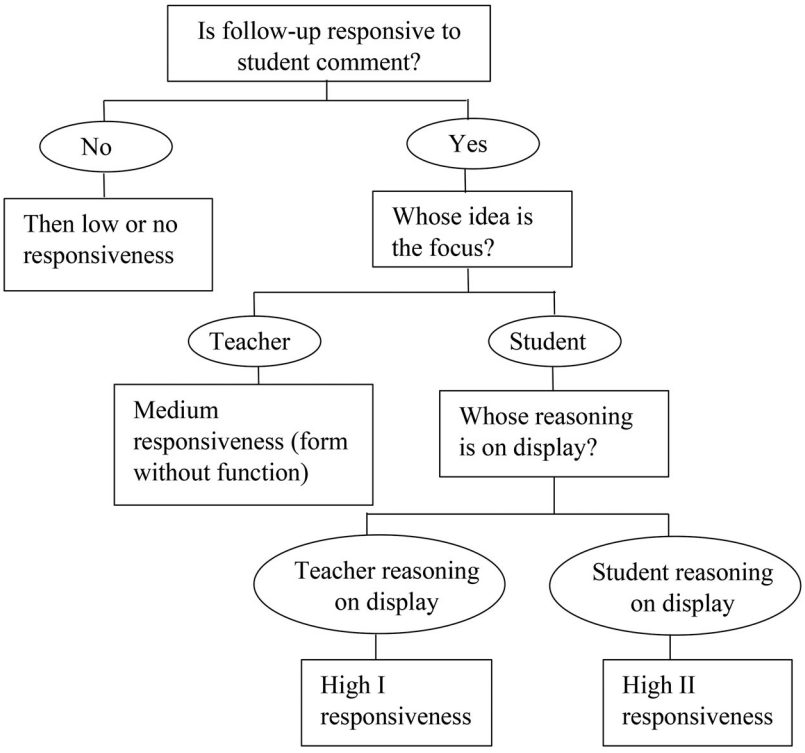

Figure 2. Flowchart for responsiveness – short version (Pierson, 2008, p. 69).

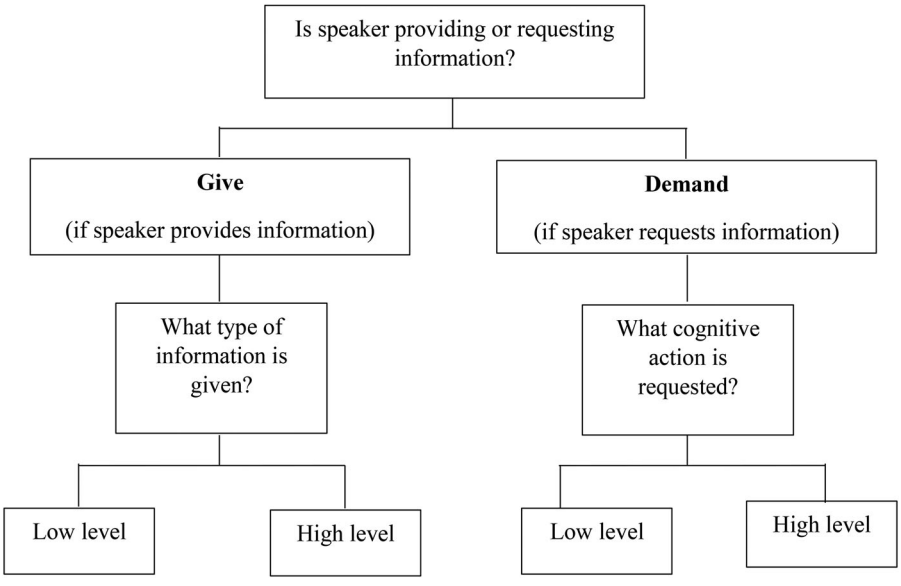

Figure 3. Flowchart for intellectual work – short version (Pierson, 2008, p. 81).

(1989) requirement for an efficient analytical tool for action research projects. As can be seen in the flowcharts in Figures 2 and 3, the questions to determine the levels of responsiveness and intellectual work, provided an efficient way to determine practices that could be improved. Although Aguirre and Zavala (2013) also had a series of questions for reflecting on teacher practices in their suggested tool, theirs were more complex and required more time, making them difficult to use in an action research project and be in alignment with the indications of Winter (1989) mentioned previously.

Pierson (2008) analysed responsiveness by whether the teachers responded to the student and how they built on students' contributions. When the teachers merely evaluated or acknowledged a student idea, their responsiveness was categorised as low (see Figure 2). When the teachers built on students' contributions, but brought up their own ideas or reasoning, then the responsiveness was considered high-I level. Teacher responses which invited students to clarify, expand or make sense of each other's idea were categorised as high-II responsiveness level (Pierson, 2008).

Intellectual work focused on the cognitive demands made on students. Pierson (2008) distinguished between "give" moves and "demand" moves (see Figure 3). Give moves include information that is provided to students in form of statements to which they are not expected to respond. By explaining, evaluating or connecting ideas and giving examples, teachers' give moves support students to make sense of previous moves in the interaction. Demand moves request information from students, where they are responsible for making connections, judgements or justifications (Pierson, 2008).

Both give and demand moves can require lower or higher levels of student activity related to the content (see Figure 3). Low level moves are those where the teacher describes a mathematical procedure (low giving), or requests students to recall a mathematical formula or definition (low demand). Higher levels include the teacher explaining and justifying procedures (high giving) or requesting students to explain or justify their reasoning (high demand) (Pierson, 2008). Levels of intellectual work provide different opportunities for learning and understanding the lesson content.

We considered that Pierson's (2008) model could be modified for teacher education because the two constructs could be related to Griffiths (2009) distinction between action research *as* and *for* social justice (see Figure 1). Action research as social justice is about ensuring that social justice is enacted within the research process which includes the collaborative actions between participants. As responsiveness includes responding to and utilising PTs' own ideas about language diverse mathematics education, it can be connected to action research as social justice. Intellectual work can be related to action research for social justice as it focuses on content, in our case the roles and responsibilities identified in the LATACME framework (Lange & Meaney, 2019) that we wanted PTs to become aware of.

Modifying the model in alignment with TE data

As Pierson's (2008) model was originally developed for studying mathematics teachers' work in secondary schools, some adaptions were needed. Consequently, we used data from a teacher education workshop, collected as part of the wider LATACME project, to determine how to adapt Pierson's (2008) model for focusing on language diversity in mathematics teacher education. Although the data came from the second author's

work as a TE, the intention was for the modified model to be used in action research projects by both authors and potentially by others. The second author had previously analysed aspects of her mathematics teacher education practices to do with language diversity (Eikset & Meaney, 2018; Meaney & Rangnes, 2020) but identified the need for an analytical tool that could be used quickly and systematically.

The data came from the TE's workshop in an international mathematics education course, which ran in the final year of a Master of education in our institution. The PTs were focused on teaching grades 1–7 or 5–10 and had previously completed at least two bachelor's level courses in mathematics education. The workshop was about the development of mathematical argumentation in a second language and focused on aspects of the LATACME framework (Lange & Meaney, 2019).

Most participants in the workshop were native speakers of Norwegian, but there were also some international participants, who did not speak Norwegian. Therefore, communication was primarily in English. This created opportunities for discussions about mathematics in a second language, which would not necessarily be the case in a regular course. The second author is a native English speaker, with developing Norwegian language skills.

The video-recorded workshop consisted of approximately two 40-minute teaching blocks. The TE provided input through lecturing and using three short videos, which acted as springboards into small-group and whole-class discussions. PTs' voices were captured through a recorder worn by the TE. PTs' faces were not visible in the video recordings, which had been agreed in the consent between the TE and PTs. The transcript of the workshop was the primary source for analysis, with the video being used to check unclear or incomplete utterances.

Developing the analytical tool – adapting with data

Keeping in mind Pierson's (2008) model, we wanted to identify similar sets of questions to those in Figures 2 and 3. To do this, we identified interactions in which the TE made a response directly after a PT expressed an idea or question. To determine a level of responsiveness, we considered how the TE made use of the PT's contribution and how it could contribute to collaboration. For intellectual work, we identified how the TE provided or requested information about the roles and responsibilities described in the LATACME framework (Lange & Meaney, 2019). As in Pierson's (2008) model, in our data some TE follow-up moves showed both responsiveness and intellectual work. Identifying when responsiveness and intellectual work were present in the same move enabled us to see how action research *as* and *for* social justice were related.

We began by using the questions in Pierson's (2008) original flowcharts to classify the interactions, initially to determine if the interactions were relevant for our focus. We then compared relevant interactions with the examples that Pierson (2008) provided in her analytical tables, and classified them as being of a low, medium or high level. From these comparisons, we produced adapted versions of the tables for responsiveness, and for giving and demanding intellectual work.

Table 2 provides an example of the adapted table for high-I responsiveness. The common feature between Pierson's (2008) description of this level and our description of the level was that in both cases learners' (students' or PTs') contributions were the

Table 2. Adapted description and example of responsiveness high-I level.

Pierson's (2008) description	Our adapted description	Transcript
Follow-up that is responsive to S(tudent) idea, question, or perceived misconception. The teacher's thinking is on display, but in response to the student's idea. This includes answering student questions and responding to student misunderstandings. T can expand on S comment but takes over S's idea and puts his/her thinking as focus (*p*. 77)	Genuine attempt to respond to PTs' ideas or questions – clarifying directions/ instructions, where the TE's thinking is on display.	*Extract 1* PT: How would you go about it if you're the only one speaking your language? Would that be possible? TE: No, if you've got someone in your class, where there's no one else to talk about it, then that is where it's absolutely essential that you have a language teacher, a mother tongue language teacher alongside. That's where your role as a teacher advocate for the student has to come into play.

basis for the educator's (teacher's or TE's) response but where the educator's reasoning became the focus. In the example connected to a task about learning fractions in Welsh, the TE responded to a PT's question about possibilities of supporting language diverse students in a mathematics classroom who did not share a first language, by describing the need to advocate, given the restraints in the system.

In analysing the data, it became clear that Pierson's (2008) two levels, low and high, for giving and demanding intellectual work (see Figure 3) were insufficient. In her later work, she included a "potentially high" level between low and high intellectual work, for interactions where the provided or requested information was not justified (Bishop, 2021). However, we chose to add a medium level of intellectual work. We described medium level of giving as "Use a PT response to highlight a different point (use it as a springboard to somewhere else)", and a medium level of demanding as "The demand to have PTs rethink their ideas is implicit in the points that the TE makes".

Although the addition of these medium levels arose from the need to classify interactions from one workshop, it seemed reasonable to anticipate that a similar need would arise for action research projects in teacher education. This is because the relationship between adults (TEs and PTs) is different to that between and an adult and children (teacher and students). In Eikset and Meaney (2018), it was noted that the same TE had difficulty demanding PTs to engage with confrontational and complex issues about language diversity. Therefore, to understand how teacher education practices could be improved, it seemed important to focus on how issues were raised and not just if they were raised.

Once the descriptions of the levels were consistent across the interactions, the tables were then used to create the flowcharts in Figures 4 (responsiveness) and 5 (intellectual work). Along with the tables, these provided a consistent approach to determining the levels of responsiveness and intellectual work. We then re-analysed the interactions in the transcript to check that both the flowcharts and the tables were sufficient for identifying practices that could be improved in the next round of action research. As a result of this analysis, it became clear that we needed to specifically identify which aspects of the LATACME framework were evident in the giving and demanding of intellectual work, and not just focus on whether intellectual work was given or demanded.

In this workshop, the aim of the intellectual work was to understand which aspects of the LATACME framework (Lange & Meaney, 2019) were evident. Given that the workshop only

covered 40 min, not all aspects could be raised. In other action research projects, it may be that other frameworks are more appropriate as the basis for the intellectual work, such as Prediger (2019) and Wessel and Erath (2018). Nevertheless, whatever framework is the basis for the intellectual work, it seemed important to be explicit about the aspects of language diversity were raised with PTs. Eikset and Meaney (2018) found some issues to do with language diversity were easier for the same TE to raise than others and, therefore, there is a need to be attentive to identify which issues are raised over time.

Consequently, we expanded the tables with a column for identifying which, if any, of the three roles (Teacher/Learner/Advocate) and the two responsibilities (Facilitating the exploring and learning of mathematics; and Facilitating the exploring and learning about the world through mathematics) from the LATACME framework could be identified in the interaction. Table 3 provides two examples from one utterance of high demand intellectual work, identifying the connections to the LATACME framework.

These examples were classified as high demand intellectual work, as the PTs were asked to reflect on issues that would affect students learning mathematics. In the first example, the TE required the PTs to reflect on their learning of mathematics education in English, to challenge views about language diverse students' first language being a problem. Considering the interaction against the LATACME framework, we classified that the PTs were being asked to take on the role of learner by reflecting their own experiences of learning in a second language as a way to understand their future students' experiences. In the second example, the PTs were asked to think about why they may need to be advocates for language diverse students and in what circumstances. This was done by challenging whether it was likely that children could tell a teacher that their schooling was not appropriate. Both interactions were seen as being connected to the first responsibility, exploring and learning about mathematics.

The identification of aspects of the LATACME framework (Lange & Meaney, 2019) was complex. Some examples were identified as implicitly related to the roles and responsibilities, while some were identified as related to more than one role or responsibility. Other interactions were not considered to be related to the framework at all, but to classroom management or issues to do with mathematics learning, such as the learning of fractions or algebra. This latter kind of interactions might be relevant if the intellectual work was based on another framework. However, the process of identifying if and how specific content as intellectual work was raised provided insights into what needed attention in the next action-research cycle. Thus, being clear about the intellectual work was important, regardless of the framework which was in focus.

Below, we elaborate on the levels of responsiveness and intellectual work in Figure 3 and 4, using examples from the workshop. Our intention is to focus on how the levels indicate the connections between action research and social justice.

Low responsiveness

In the workshop, three types of interactions were classified as low responsiveness, but only one was directly related to the LATACME framework. This suggests that low responsiveness interactions may not overlap with intellectual work or only with low levels of giving and demanding. This is perhaps not surprising, as Bishop (2021) identified such relationships between the two constructs.

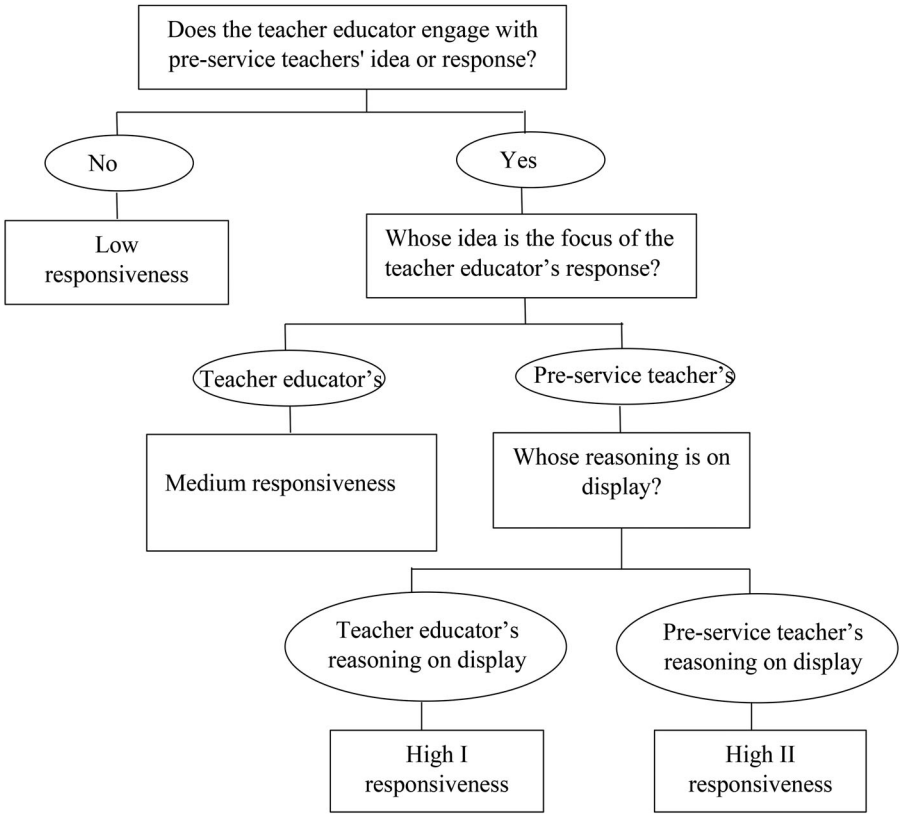

Figure 4. Adapted flowchart for responsiveness.

An example of an interaction identified as low responsiveness came late in the workshop, following a video of a culturally-responsive lesson, connected to algebra. In this interaction, the TE asked a question about how the written mode was used by the teacher in the video. A PT responded about how the teacher allowed students to discuss in language they felt comfortable with. The TE, however, did not respond to the PT's point, but elaborated on her earlier question about written language.

In this example, the TE was asking the PTs to develop their understandings of the teacher role for facilitating the exploring and learning of mathematics, by reflecting on the modes of language and their use in the videoed lesson. The TE's response to the PT's idea, as explained in the previous paragraph, was categorised as low responsiveness, indicating that collaboration with PTs was not realised. There is a need to identify such instances so that the TE can consider how to better utilise PTs' own contributions and to develop understandings from this, as these would potentially contribute to action research as social justice.

Medium responsiveness

In the interactions classified as medium responsiveness, the TE used the PTs' ideas to focus on the intellectual work the TE considered important. As such, this was very similar to Pierson's (2008) original description of medium responsiveness.

An example of medium responsiveness came from a task in a video about solving fraction problems on a digital app using oral and written Welsh, a language that neither the TE nor the PTs knew. The PTs reported that they recognised the task as being about equivalent fractions, by using their knowledge of fractions to interpret the diagrams and symbols the Welsh speaker showed. The TE then asked PTs if they were able to continue the discussion about fractions in Welsh. When they indicated that they could not, she used these responses to raise understandings about how students with language diverse backgrounds in monolingual classrooms might feel when asked to learn mathematics in a language in which they were not yet fluent. In relationship to the LATACME framework (Lange & Meaney, 2019), this example focused on the role of Teacher with the responsibility of facilitating the exploring and learning of mathematics. The TE raised the issue about how the PTs, as teachers, could reduce the mathematical learning opportunities of language diverse students by adopting a teaching practice of only using the language of instruction in their classrooms.

As with interactions classified as indicating low level of responsiveness, medium level interactions cannot be considered as being sufficient for action research as social justice. Nevertheless, identifying the prevalence of these levels of interactions provided input about what could or should be changed in the next cycle of action research to achieve a high level of responsiveness.

High responsiveness

High responsiveness was separated into high-I and high-II. In high-I, the PTs' idea was the focus of TE's response, but the TE's reasoning was on display, while in high-II PTs' reasoning was the focus. High responsiveness required the TE to listen to PTs' points and respond to them, not always by agreeing but respectfully listening to what they had said.

One example of high-I responsiveness came when a PT shared a story about a student, who did not have language support in the mathematics classroom. In the transcript in Table 2, the TE challenged PTs to be advocates for such students. The interaction continued between the PT and the TE as follows:

Extract 4

PT: But then again, it's a matter of economics. I've had a student who has been one year in Norway, and she had like five hours a week with a teacher and (unclear) to the Norwegian language. […] And so, in every other subject she was on her own. And it does not matter how much we wanted to advocate for her

TE: I know, but she has a right. You know? Under the law she has a right.

PT: Yes, but it doesn't make sense, very much, over a few hours.

TE: No, but this is the issue of where do you stop? Do you just give up on the student? And as a professional you can't do that. So, yes of course it's economics and, no, you can't solve all of the problems.

The TE's responses to the PT continued to highlight the need to advocate for multilingual students in mathematics classrooms, even with the constraints expressed by the PT, in connection with ensuring students' possibilities to explore and learn mathematics. Such interactions were identified as high-I responsiveness as they require PTs' points

to be in focus while the TE offers alternative views showing their own reasoning about these points.

On the other hand, high-II responsiveness, related to the intellectual work of the LATACME framework, occurred when the TE supported a PT to discuss their understandings about an issue and in so doing increased their understanding of that issue. The following extract also came from the task using the video of culturally-responsive teaching of algebra. The TE began this interaction by asking, "What were some of the things that she (the teacher in the video) did with the oral language to ensure that they actually opened up communication and didn't shut it down?" The interaction continued as follows:

Extract 5

PT5: She had to repeat what the other students just said?

TE: Yes. Because by focusing in on that mode, she (the teacher) couldn't be sure that everyone had got it. So she needed to check on a couple of those girls in particular. … to ensure that there was meaning being conveyed. The choice of the oral language meant that she had to do some double checking. Yeah?

PT6: But their language didn't seem to be a problem in this group. So we discussed the language with, how much learning is there to repeat if I was to repeat what she (the teacher) just said? Even if I did not understand it probably it doesn't make me understand anything more.

TE: And I think this is an interesting point and there are two things which go with that. It is that you can have very good conversational language, but the academic language can get a bit lost. But here is a case where she's presuming that there will be some misunderstandings, but maybe there's not.

This interaction was related to the intellectual work about the LATACME framework, as it involved taking a critical perspective on how children can be situated as struggling, even if they were not. This, therefore, focused on PTs being learners about their students as part of their responsibility for facilitating the exploring and learning of mathematics, by questioning some of their assumptions about how to check on language diverse students' understanding. The interaction was collaborative in that the TE reflected on what the PTs said and without presenting a particular perspective. High responsiveness II, therefore, provides information about the kinds of interaction that fulfils the requirements for action research as social justice, and an aim for future cycles of action research.

Giving intellectual work – medium

Although examples of low level of giving intellectual work were identified, these were related to classroom management or had unclear references to mathematics education. Information classified at the medium level was implicit, indicating that PTs might miss the information being given.

For example, in an interaction about the Welsh video, the focus was on the role of the teacher and the responsibility for facilitating the exploring and learning of mathematics. The TE took a PT's response about how the symbolic representation provided meaning, to raise a new point about how some languages have better ways to discuss mathematical

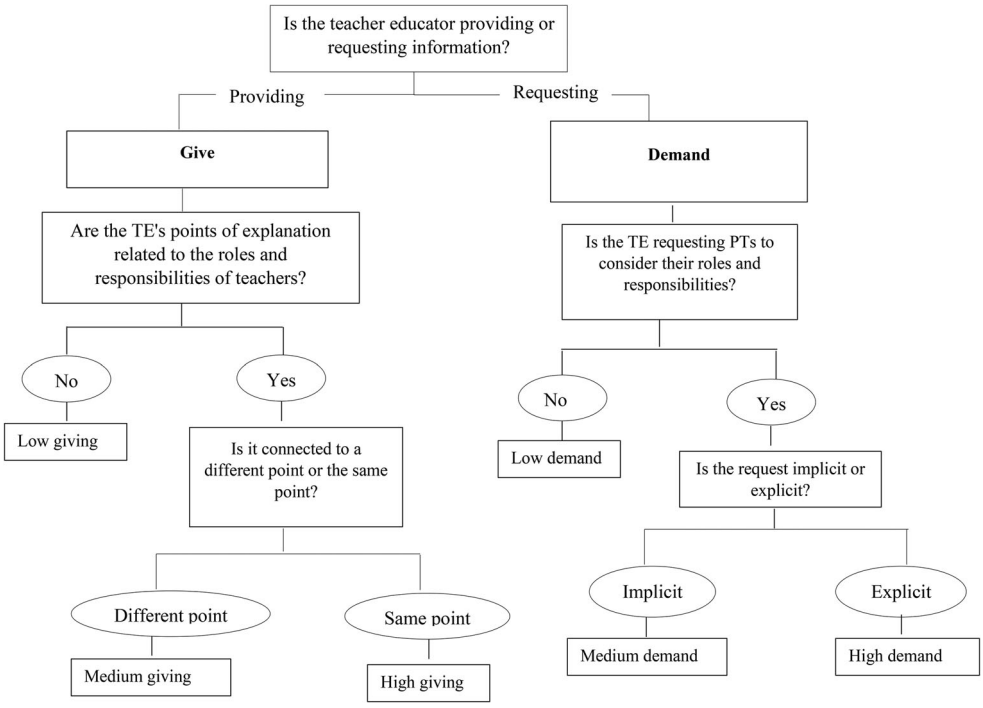

Figure 5. Adapted flowchart for intellectual work.

Table 3. Adapted and extended description and examples of high demand intellectual work.

Description	Examples	Transcript	LATACME framework
Request to evaluate what PTs experienced in the tasks they were asked to do, and to connect it to what they knew about themselves as a teacher or learner and to ask questions from that evaluation	(1) PTs are asked to reflect on their own language use in learning about mathematics education in English.	*Extract 2* TE: And while you're looking at it, think about how learning about mathematics education research in English. Does your first language mean that you have a problem doing that? Or do you think of it as a resource?	(1) Role: Learner Responsibility: Exploring and learning of mathematics
	(2) Questions about the teacher's role of being an advocate for multilingual students.	*Extract 3* TE: So, one of the interesting things with this video is that it says there's a role for advocacy. There's a role for parents to have discussions with teachers, but also for children in classes to say, "you are not giving me enough interesting work to do". Now I wonder, just how well received a child would be if they told their teacher that their schooling was not appropriate?	(2) Role: Advocate Responsibility: Exploring and learning of mathematics

ideas (see Extract 6). This was categorised as medium giving of intellectual work because it was an implicit attempt to bring up cross-linguistic awareness.

Extract 6

PT: I think it was about what the numbers represent, so how the numerator and the denominator and line between, which represents how it's divided.

TE: Did you all notice that there appeared to be the word for the numerator (in the video), then another word, and then the word for the denominator? And that other word seemed to indicate fraction. We don't have that in English. So, when we say, "two fifths", we have to always emphasise the "ths", because mostly the children can't hear it. … So, within Welsh, you can hear there is an advantage already, because they've got that extra word which makes a distinction, so you know you are talking about a fraction, yeah?

Identifying where the TE provided information implicitly gave insights into action research for social justice as it suggested practices that could be improved and how they could be improved in the next cycle.

Giving intellectual work – high

The high level of giving intellectual work differed from the medium level in that it was more likely to extend the PTs' reasoning. One example was when the TE asked PTs to consider how the different modes used in the video about learning algebra with cultural artefacts, affected the communication. One PT raised that, because the teacher and the students had the same cultural heritage, they could recognise the algebraic pattern in the cultural artefact. The TE used this point to problematise how a lack of cultural background could restrict students' possibilities to engage with mathematical problems. She then asked follow-up questions about the likely impact on the students' learning from using a known cultural artefact. The TE explicitly highlighted the impact of cultural contexts in mathematics education and so was related to the LATACME framework role of learners about language diverse students' cultural backgrounds. The responsibility was about exploring and learning the world through mathematics, as it raised issues with the types of problems that could be solved when cultural contexts were unknown. The PTs were much more able to engage with the topic because the teacher education was explicit about what she wanted them to consider.

Interactions that involve high level of giving intellectual work can contribute to action research for social justice because they explicitly raise points, particularly relevant for specific moments in the teacher education classroom, which may not be raised otherwise. As well, imagining how high giving tasks could become high demanding tasks provided insights into potential practices that could be changed in the next cycle of action research.

Demanding intellectual work – medium

Interactions classified as medium demanding occurred when the TE implicitly challenged the PTs' ideas. In the interaction about advocating for a student who was the only one in the mathematics class who spoke a particular language (see Extracts 1 and 4), the TE assumed, perhaps from how the PT had initially raised the issue, that PTs would not necessarily see being an advocate for language diverse students as part of

their professional role or if they did, perhaps they did not know how to do it. This example was classified as medium demand because the main part of the TE's response in Extract 4 was indirectly requesting PTs to rethink their role in teaching language diverse students by implying having knowledge about the educational law ("You know? Under the law she has a right") and showing sensitivity to students ("but this is the issue of where do you stop? Do you just give up on the student? And as a professional you can't do that"). The TE, however, rather than checking her assumptions with the PTs, insisted that they rethink these views and adopt new practices regardless. This raised the issue of whether the TE could discuss with PTs, rather than imply, how they could act to advocate in this circumstance, which would potentially raise the level of intellectual work.

Demanding intellectual work – high

Interactions classified as high level of demanding were different to the medium level in that the TE's request for intellectual work was explicit. The example about high-II responsiveness in Extract 5 was also classified as an interaction illustrating high demand, in that the PTs were asked to consider how different modes of communication in a video of culturally responsive teaching of algebra contributed to the communication. This resulted in a PT sharing a concern about the practice of asking students to repeat words. The TE then used that to ask the PTs to consider the balance between improving language fluency and situating students as struggling. Examples such as this, where high responsiveness is connected to high demand, suggest that it was possible to approach both action research as social justice, in that the TE collaborated with the PTs, and action research for social justice, in that PTs' ideas could be used to discuss the complexity of language diversity and mathematics education. Identifying these examples provided potential practices that could be achieved in TE that were most likely to affect PTs' future mathematics teaching in language-diverse classrooms.

Reliability and validity of the analytical tool

As action research requires an efficient analytical tool (Winter, 1989), it was important to consider the reliability and validity of the tool for identifying areas for improvement in teacher education about language-diverse, mathematics classrooms that could be the basis for subsequent action research cycles. As well, Kemmis (2009) stated that action research should contribute to changing a practice more generally (practice-changing practice), than just for individual researchers' personal and professional growth. Therefore, the analytical tool needed to support our individual action research projects and contribute to wider understandings about teacher education practices.

Reliability in action research requires that individual observations and interpretations can be checked with those of others who have similar perspectives and purposes (Lankshear & Knobel, 2004). We had shared interests in challenging PTs' understandings about language-diverse, mathematics classrooms. Because of our shared interest, we then checked our individual interpretations against each other when adapting the analytical tool and categorising the data from the workshop accordingly. This allowed us to confront each other's ideas about challenging PTs' understandings about language-diverse, mathematics classrooms. For instance, when encountering data that

were difficult to categorise, we revisited similar examples we had coded previously and compared them to our interpretations. By doing this, we were able to produce the final version of the analytical tool. Figures 4 and 5, alongside the exemplary tables, allow us (and others) to replicate our data categorisation.

The validity of the tool has to do with ensuring that the analytical process is specific and rigorous, so that significant new insights can be made for the action research, without the process being too simple or too complicated (Winter, 1989). In the analysis of the data, we used the analytical tool to identify practices, such as the TE giving her own opinion rather than checking with PTs and building on their responses, that were similar to practices found in Eikset and Meaney (2018), where a different analysis was undertaken. This indicates that the analytical tool we adapted from Pierson (2008), provided similar results, although more quickly and thoroughly.

Our collaboration in the research process is also in alignment with Griffiths (2009) argument about collaboration in action research mindful of social justice. Therefore, the social justice approach in our action research is strengthening the validity and reliability of the analytical tool. We consider that the consistent use of this analytical tool to identify practices that need changing and in what ways can contribute to a radical socially-just teacher education (Gates & Jorgensen, 2009). Results for its use also have the possibility to provide a research basis for discussions about how and in what ways the teacher education profession could improve its practices around raising the complexity of issues to do with language diversity in mathematics education.

Conclusion

Research on how TEs challenge PTs to rethink deficit perspectives connected to language diverse mathematics classrooms has often shown disappointing results (see for example, Eikset & Meaney, 2018; McLeman & Vomvoridi-Ivanović, 2017; Vomvoridi-Ivanović & McLeman, 2015). Yet, such changes are necessary for achieving aims for radical social justice (Gates & Jorgensen, 2009). Action research projects provide ways to determine appropriate alternative teacher education practices, by identifying: what practices need to change; how they need to change; and why they need to change. In action research projects, this identification is achieved by having an efficient analytical tool (Winter, 1989).

In this article we describe how we developed an analytical tool for our action research projects. Identifying responsiveness in interactions with PTs provides insights into action research as social justice (Griffiths, 2009), because it highlights the possibilities for collaboration with the PTs. Identifying intellectual work connected to the LATACME framework (Lange & Meaney, 2019) provided insights into practices that could change and in what ways so that the complexity of issues connected to language-diverse mathematics classrooms could be raised more appropriately with PTs and support action research for social justice. The reliability and validity of the analytical tool indicates that it has the potential to provide other TEs, as well as ourselves, with possibilities to reflect on and improve practices related to language diversity in mathematics education. We anticipate that the knowledge that the tool generates can contribute, not just to better understandings of our own work as TEs, but also to the profession as a whole, so that PTs could work in more socially-just ways in their future school classrooms.

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