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Heuristics for shelf space allocation problem with vertical and horizontal product categorization

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Abstract

In this research, we proposed a practical shelf space allocation model with the vertical and horizontal categorization of products. Five groups of constraints, such as shelf, product, multi-shelves, orientation and band constraints, are implemented in the model. We proposed two heuristics to solve the retailer's profit maximization problem. The experiments were performed on small instances, and the solution was compared to the optimal solution found by the CPLEX solver. The experiments proved that heuristics could find the optimal solution in most cases (14/20 for heuristics H1 and 16/20 for heuristics H2) without checking the whole solutions space. The lowest solution quality is 97.68% which shows that the heuristics allow finding high-quality near-optimal solutions. Therefore the steps implemented in the proposed heuristics should be used in more complicated shelf space allocation problems for which it is impossible to find a solution by the solvers.

Keywords: Retailing, decision making/process, merchandising, shelf space allocation, planogram

1. Introduction

One of the most significant activities in retail management is the allocation of shelf space. This has a consequence on not only business performance and costs but also the level of customer relationship and customer happiness [1]. Furthermore, the method for allocating shelf space has a major impact on sales forecasting, delivery planning, procurement, financial flows, as well as other retail processes. Frequently, the shop might encourage customers to buy things that aren't really needed by them [2]. As a result, experienced retailers should carefully use

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assortment and packaging design parameters to draw attention and boost processing convenience [3]. The goal of the retailer is to easily implement category plans that optimize the product position on the shelf, performance, and layout of product categories to meet customer demand, boost sales, and maximize gained profit.

2. Literature review

2.1. Shelf space allocation

Planograms are an important communication tool in today's retail industry. Assortments must produce higher sales and ensure customer contentment. Therefore efficient planograms are critical to the retailers and manufacturers. Shelf space is an important but limited resource that must be effectively utilized and controlled in order to meet the primary goals of generating profits, lowering expenses, and improving customer delight. The product selection problem, in its most basic form, entails deciding which products should be displayed in the limited shelf space available, whereas the shelf space allocation problem entails designing the actual assembly of the chosen products on shelves and determining how many units of each product will be presented [4], [5].

A facing is one unit of the product that faces directly out to the customer. Borin et al. (1994) indicated that product assortment (choosing which products to show) and stockouts (products absent on shelves) are important factors to address in shelf space management [6]. Some works includes similar product grouping constraints in order to improve product substitution in case of out-of-stock [7], [8], [9].

2.2. Horizontal versus vertical shelf placements

Before buying the product, the customer takes it from the shelf, examines and evaluates it, i.e. the customers scan the items on the shelf. Although both horizontal and vertical scan patterns are used by customers in stores, the former is significantly more prevalent [10], [11]. Mowrey et al. (2019) estimated horizontal scan patterns because their problem included travel and eyesight in main aisles rather than product aisles (where vertical scanning would be more prevalent) [11]. Kahn (2017) investigated the patterns of concentration that consumers display when examining assortments thanks to recent developments in advanced eye-tracking facilities [3]. Hansen et al. (2010) found that vertical position is approximately two times more powerful than the horizontal location on store performance [12]. Chen et al. (2006) studied the connection between the positional distances of the products displayed and their sales, as well as the impact of shelf space vicinity on sales, in another study related to the location factor in retail [13]. A sophisticated method that relies on association rule mining was created for this purpose. Consistent with these findings, according to Drèze et al. (1994), shelf position has a significant impact on sales, whereas changes in assigned shelf space have a far smaller impact as long as a minimum stock level is maintained [14]. A practical retail shelf space allocation model which incorporates shelf division into horizontal segments of flexible size is presented in [7], [8], [9].

One of the key literature limitations is not taking into account merchandising rules based on simultaneous vertical and horizontal product categorization as the product could belong to the vertical category, and the shelf levels can also represent visible category divisions on the planogram. Only hierarchical or separate vertical categorization exists. Various merchandising rules for shelf space allocation with horizontal and vertical positions are deeply presented in [15]. Key characteristics of the proposed model are:

- Categorizing the products and placing them on different vertical shelf levels.
- Forming vertical and horizontal categories on a planogram using all product categories.
- Grouping similar products into clusters on one shelf ensure a substitution effect.
- Using more than one possible orientation of a product on the shelf.

3. Problem formulation

Nomenclature

Parameters and indices used in a model:

S total number of shelves;
P total number of products;
K total number of categories;
T total number of tags;
i shelf index, $i = 1,...,S$;
j product index, $j = 1,...,P$;
k category index, $k = 1,...,K$;
t tag index, $t = 1,...,T$;
r orientation index, $r \in \{0,1\}$; $r = \begin{cases} 0, \text{ for front orientation} \\ 1, \text{ for side orientation} \end{cases}$.

Shelf parameters:

s_i^l length of the shelf i;
s_i^d depth of the shelf i;
s_{ti}^g binary tag t of the shelf i; $s_{ti}^g = \begin{cases} 1, \text{ if shelf } i \text{ is tagged} \\ 0, \text{ otherwise} \end{cases}$.

Product parameters:

p_j^w width of the product j;
p_j^d depth of the product j;
p_j^u unit profit of the product j;
p_j^l cluster of the product j;
p_{tj}^t tag t of the product j;
p_j^k category of the product j;
p_j^s supply limit of the product j;
p_{jr}^w width/depth of the product j on orientation r; $p_{jr}^w = \begin{cases} p_{j0}^w, \text{ if } r = 0, \text{ width for front orientation} \\ p_{j1}^w, \text{ if } r = 1, \text{ depth for side orientation} \end{cases}$;
$p_j^{o_2}$ side orientation binary parameter of the product j; $p_j^{o_2} = \begin{cases} 1, \text{ if side orientation is available} \\ 0, \text{ otherwise} \end{cases}$;
$f_j^{\min} / f_j^{\max}$ minimum/maximum number of facings of the product j;
$s_j^{\min} / s_j^{\max}$ minimum/maximum number of shelves on which the product j can be allocated;

Category parameters:

c_k^m minimum category size as a percentage of the shelf length;
c_k^t category size tolerance between shelves in the category as a percentage of the shelf length.

Tag parameters:

b_t^n band name of the tag t, $b_t^n = \{H; H^+; V^+\}$;
b_{tij}^t product to shelf compatibility tag.

$b_{tij}^t = \begin{cases} 1, \text{ if } s_{ti}^t = p_{tj}^t \wedge b_t^n = \{H\} \\ 0, \text{ otherwise} \end{cases}, t = 1,...,T$ - for the horizontal bottle level shelves;

$b_{tij}^t = \begin{cases} \min(p_{tj}^t;1) \wedge b_t^n = \{V^+\} \\ 1, \text{ if } p_{tj}^t = 1 \wedge s_{ti}^t = p_{tj}^t \wedge b_t^n = \{H^+\} \\ 0, \text{ if } p_{tj}^t = 1 \wedge s_{ti}^t \neq p_{tj}^t \wedge b_t^n = \{H^+\} \\ 1, \text{ if } p_{tj}^t = 0 \wedge b_t^n = \{H^+\} \end{cases}, t = 1,...,T$ - for the horizontal and vertical jar shelves.

Decision variables:

$x_{ijr} = \begin{cases} 1, \text{ if product } j \text{ is placed on shelf } i \text{ on orientation } r \\ 0, \text{ otherwise} \end{cases}$ - product placement binary variable, for all $i = 1,...,S$, $j = 1,...,P$, $r \in \{0,1\}$: $x_{ijr} \in \{0,1\}$

f_{ijr} the number of facings of the product j on the shelf i on orientation r;

$y_j = \begin{cases} 0, \text{ if product } j \text{ is on front orientation} \\ 1, \text{ if product } j \text{ is on side orientation} \end{cases}$ - orientation of the product j, for all $j = 1,...,P$: $y_j \in \{0,1\}$.

Heuristics parameters:

x_{ij} sequence of shelf allocations; f_{ij} sequence of product allocations.

The Shelf Space Allocation Problem (SSAP) in this research consists of a planogram that is visually divided into vertical categories. Each shelf of an investigated planogram is tagged horizontally, a meaningful tag is applied to the shelves. The shelves are also tagged vertically, i.e. a tag is applied for each category and includes all shelves. The problem can be formulated as follows. There are given number of products P that must be placed on S shelves of a planogram. The products are divided into K vertical categories. The retailer allocates each vertical category on a planogram, assigning the minimal size of the category that is visible enough on a planogram. The purpose is to define the proper shelf space for each category placed on a planogram defining the number of facings of each product in order to maximize the total profit.

The types or classifications of products are used to categorize them. Each product category has a vertical layout. The products and shelves are also tagged horizontally. Each product could have multiple tags p_{tj}^t at the same time. Several tags s_{ti}^t might be assigned on each shelf at the same time. As an example of a shelf: (1) A shelf is for a particular product packaging (jar, pot, can, bottle); (2) a shelf is for promotional items; (3) a shelf is at eye level; (4) a shelf is at touch level. As an example, consider the following: (1) a product is sold in a jar, and it must be placed on the jar's shelf; (2) a product is sold in a bottle, and it must be placed on the bottle's shelf at eye level; (3) a product is a bottle, and it must be placed on the bottle shelf at touch level.

In this research, we create 3 possible tags, $b_t^n = \{H; H^+; V^+\}$. The shelves and products may be tagged by T tags, but each shelf or single product could be tagged by one or more tags:

- H - the shelf in the horizontal layout is dedicated only for products of the defined packages (such as jars, pots, cans, bottles).
- H^+ - the shelf in the horizontal layout is dedicated only to products of different types (such as a lower-priced shelf, eye-level shelf). Therefore jars, pots, cans, as well as bottles could be placed on lower-priced or eye-level shelves.
- V^+ - the shelf in the vertical layout is dedicated to the specific product category. For the vertical product category, all shelves could be used to place products by colour or flavour (such as still water, soda, flavoured water).

This is a common practice in real-world retail establishments. The specificity of the vertical and horizontal bands on a planogram in the researched situation is shown in Fig. 1. There is an example case for the merchandiser how to allocate red and yellow categories on a planogram. The bottom shelf is reserved for bottles. Other shelves are reserved for jars or pots.

For some of the products, the tags are specified:

- Cranberry juice: red colour (V^+), package (H) is a bottle, must be placed on the shelf for bottles (H).
- Cherry jam: red colour (V^+), package (H) is a jar, must be placed on the shelf for jars or pots (H).
- Tomato jam: red colour (V^+), package (H) is a jar, must be placed on the shelf for jars or pots (H).
- Strawberry jam: red colour (V^+), package (H) is a jar, must be placed on the shelf for jars or pots (H).
- Lemonade: yellow colour (V^+), package (H) is a bottle, must be placed on the shelf for bottles (H).
- Orange juice: yellow colour (V^+), package (H) is a bottle, must be placed on the shelf for bottles (H).
- Honey: yellow colour (V^+), package (H) is a pot, must be placed on the shelf for jars or pots (H).
- Orange jam: yellow colour (V^+), package (H) is a jar, must be placed on the shelf for jars or pots (H) at eye-level (H^+).

In Fig. 1 (a), red and yellow colours depict the category assignment. Orange jam has specific shelf requirements - at eye level. Other jars or pots could be placed on any shelf (cherry, tomato, strawberry, honey). In Fig. 1 (b), the above-mentioned products in bottles (cranberry juice, lemonade, orange juice) are coloured with darker colours, jars or pots are coloured with lighter colours.

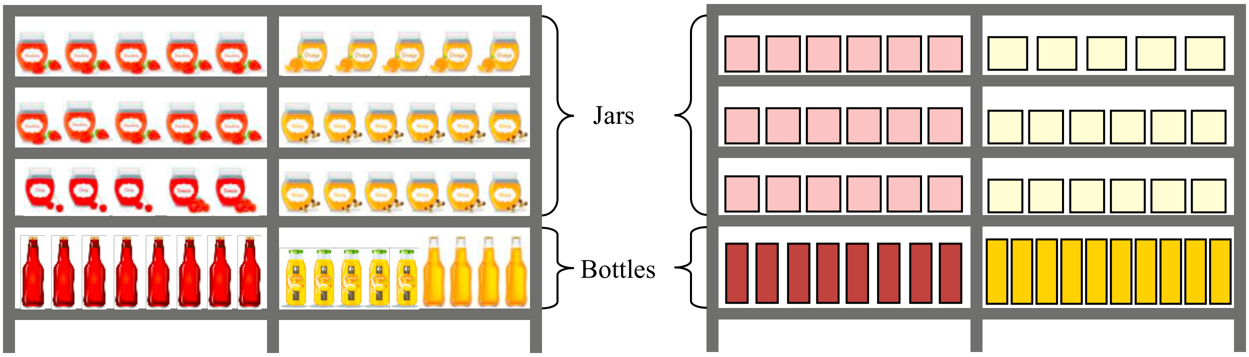

Fig. 1. Planogram with vertical and horizontal category bands:
(a) vertical - red, yellow, horizontal - jars, bottles; (b) vertical - red, yellow, horizontal - light, dark

Several shelves are devoted to a single product category. Each product could also be placed on several shelves. The merchandiser determines the minimum and the maximum number of shelves for the product and the minimum and the maximum number of facings for each product on the shelf in order to make it visible enough to customers. If the product is placed on several shelves, the maximum availability of it is defined by its supply limit p_j^s.

If the product is packed in a box, it can be placed on the shelf in two different orientations: front and side. This is also true for jars and pots if they are labelled on both sides and have different widths and depths. Obviously, for bottles, the orientation is not applied because the width and depth is equal, and rotation of the bottle doesn't reduce the total width occupied by products on the shelf. By default, all products come with a front orientation. So, based on the package and label visibility printed on the package, the orientation binary parameter $p_j^{o_2}$ determines whether the product can be placed on its side. Some products may be grouped into clusters based on the cluster parameter p_j^l in order to place them on the same shelf. Cluster products are not depicted in the figures above.

Only the front visible facings row is investigated in this study. The number of vertical facings and the number of depth facings are not taken into account. Because the bottom shelves of a planogram are typically deeper, the shelf depth varies, but the product depth and shelf depth are only considered for the front facings row. If the product's depth on the shelf is exceeded, but both front and side orientations are accessible, the product could be rotated on this shelf or placed on the deeper shelf in this scenario.

To solve the problem, the retailer must decide whether the product should be placed on the shelf, next define the number of facings each product on each shelf, finally determine whether it should be placed on the front or side orientation, and consider a set of constraints that are classified on four categories: shelf constraints, product constraints, orientation constraints, and bands constraints. The purpose of placing products on a planogram is to maximize total profit.

For the provided SSAP definition, we formulate the problem with the following decision variables: x_{ijr} - if the product j is placed on the shelf i on orientation r; f_{ijr} - the number of facings of the product j on the shelf i on orientation r; y_j - orientation of the product j.

The problem can be formulated as follows:

$$\max \sum_{j=1}^{P} \sum_{i=1}^{S} \sum_{r=0}^{1} p_j^u f_{ijr} \tag{1}$$

subject to:

1. Shelf constraints:

- shelf length limit $$\forall(i)[\sum_{j=1}^{P} \sum_{r=0}^{1} p_{jr}^w f_{ijr} \le s_i^w] \tag{2}$$
- shelf depth if a product is placed on front orientation $$\forall(i, j, p_{j1}^w > s_i^d)[f_{ij0} = 0] \tag{3}$$
- shelf depth if a product is placed on side orientation $$\forall(i, j, p_{j0}^w > s_i^d)[f_{ij1} = 0] \tag{4}$$

2. Product constraints:

- minimum and maximum number of facings $\forall(i,j)[f_j^{\min} x_{ijr} \le \sum_{r=0}^{1} f_{ijr} \le f_j^{\max} x_{ijr}]$ (5)
- supply limit if the product is placed on multiple shelves $\forall(j)[\sum_{i=1}^{S}\sum_{r=0}^{1} f_{ijr} \le p_j^s]$ (6)
- product is placed on the shelf $\forall(i,j,r)[f_{ijr} \le x_{ijr} f_j^{\max}]$ (7)
- if products are grouped into clusters, they are placed on the same shelf

$$\forall(i)\forall(a,b: p_a^l = p_b^l,\ a,b = 1,...,P)[\sum_{r=0}^{1} x_{iar} = \sum_{r=0}^{1} x_{ibr}] \quad (8)$$

3. Multi-shelves constraints:

- minimum and maximum number of shelves $\forall(j)[s_j^{\min} \le \sum_{i=1}^{S}\sum_{r=0}^{1} x_{ijr} \le s_j^{\max}]$ (9)
- if the product is placed on multiple shelves, the next shelf only is available

$$\forall(j)\forall(a,b: |a-b| \ne 1 \wedge a < b,\ a,b = 1,...,S)[\sum_{r=0}^{1} x_{ajr} + \sum_{r=0}^{1} x_{bjr} \le 1] \quad (10)$$

4. Orientation constraints:

- side orientation is possible for the product $\forall(i,j)[y_j \le p_j^{o_2}]$ (11)
- only one orientation (front or side) is available $\forall(i,j)[\sum_{r=0}^{1} x_{ijr} \le 1]$ (12)

5. Bands constraints:

- tags compatibility for the shelves and products $\forall(i,j)[\prod_{t=1}^{T} b_{tij}^t \ge \sum_{r=0}^{1} x_{ijr}]$ (13)
- minimum category size if the category exists on the shelf

$$\forall(i,k)[(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} p_{jr}^w f_{ijr} \ge \left[s_i^l \cdot c_k^m\right]) \vee (\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} f_{ijr} = 0)] \quad (14)$$

- category size tolerance $\forall(k)[\max_{i=1,...,S}(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} p_{jr}^w f_{ijr}) - \min_{i=1,...,S}(\sum_{\substack{j=1,\\ p_j^k=k}}^{P}\sum_{r=0}^{1} p_{jr}^w f_{ijr}) \le \left[\max_{i=1,...,S}(s_i^l) \cdot c_k^t\right]]$ (15)

6. Relationships constraints:

- facings relationships $\forall(i,j,r)[f_{ijr} \ge x_{ijr}]$ (16)
- facings and orientation relationships $\forall(i,j)[f_{ij0} \le (1-y_j) f_j^{\max}]$ (17)
- facings and orientation relationships $\forall(i,j)[f_{ij1} \le y_j f_j^{\max}]$ (18)

7. Decision variables:

- the product placed is on the shelf $\forall(i,j,r)[x_{ijr} \in \{0,1\}]$ (19)
- the number of facings $\forall(i,j,r)[f_{ijr} = \{f_j^{\min} ... f_j^{\max}\}]$ (20)
- orientation $\forall(j)[y_j \in \{0,1\}]$ (21)

4. Heuristics development for the SSAP

In this research, we develop two heuristics that allow us to solve the proposed SSAP with vertical and horizontal product categories. The proposed approach can be represented along these lines. Let x_{ij} is a sequence of shelf allocations, $x_{ij} = \sum_{r=0}^{1} x_{ijr}$, which indicates if the product is placed on the shelf. Let f_{ij} is a sequence of product allocations, $f_{ij} = \sum_{r=0}^{1} f_{ijr}$, which represent the number of facings on the shelf. Variable y_j is used to indicate the front or side orientation of the product.

1. Create an initial set of shelf allocation sequences for each of the product orientations (front or side) with regard to side orientation (11)-(12) and cluster (8) constraints.
2. Next, using this set, create a set of shelf allocation sequences that allows allocating the products on the shelves of defined tag $b_t^n = \{H; H^+; V^+\}$, i.e. apply the initial shelf allocation sequences for each shelf and exclude inappropriate allocations. From the given set, exclude allocations that do not satisfy shelves tags compatibility (13), minimum and maximum number of shelves (9) and next shelf (10) constraints.
3. Create a set of product allocation sequences for each shelf sequence with regard to allocation product on the shelf (7), minimum and maximum numbers of facing (5), shelf length (2), shelf depth (3)-(4), supply limit (6), constraints.
4. From the achieved set of product allocation sequences, exclude the ones that do not satisfy facings allocation on multiple shelves, i.e. minimum category size (14), category size tolerance (15) constraints.
5. Relationships constraints in this algorithm will be satisfied, therefore they are not referenced within these steps.

After performing these preparation steps, the number of shelf and product sequences can be estimated. This is needed in order to predict how many of them we can check in a reasonable time, so we generate only a part of these sequences, which will give a good enough solution. Moreover, based on the minimum and maximum numbers of facings parameters, the profit of a product allocation sequence could be estimated. Therefore in order to execute this algorithm, some steering parameters could be set.

Steering parameters:

- number of generated product allocations for each category to be checked;
- input profit for each category to be checked, the product allocations with the profit below it are not considered.
- total profit for a solution on all shelves, the allocations with the profit below it are not considered.

All shelf allocations which satisfy the SSAP constraints will be examined, but the number of product allocations will be reduced by steering parameters.

6. Next, according to rules specified by each of the heuristics, reduce the number of product allocations and analyze only a part of them in order to find the best solution from the examined part of product allocations.

- H1 – sort the product sequences in not ascending order of profit, next, sort the parts inside sorted ones in not ascending order of profit ratio.
- H2 – sort the product sequences in not ascending order of profit ratio, next sort the parts inside sorted ones in not ascending order of profit.

In this context, profit ratio is the ratio of the total profit of the products allocated on a planogram divided by occupied space, free space is not taken into calculations.

5. Computational experiments

The goal of the experiments was to check the solution quality achieved by the heuristics and compare it to the optimal solution found by CPLEX solver. Obviously, not all problems can be modelled as linear; therefore, the optimal solution could not be found. In this research, we checked if the proposed principles inside the heuristics give satisfactory results in order to apply these heuristics in more complicated for SSAPs later.

Experiments were performed on 10-product sets which must be allocated on a 4-shelf planogram with the shelf length 250 cm, 375 cm, 500 cm, 625 cm, 750 cm. There were 2 vertical product categories. 4 product sets per 10 products with different product parameters (width, depth, profit, etc.) were generated.

Table 1 presents the performance of the developed heuristics compared to the CPLEX solution. For each heuristics, we calculated the profit ratio, which is the profit found by heuristics divided by the optimal profit found by CPLEX solver. It could be observed that both heuristics found the optimal solution for the 1st instance. For the 2nd instance, heuristics H2 was better than H1 and found optimal solutions on 4 cases from 5 ones, while heuristics H1 found optimal solutions only in 2 cases. But it is not bad because the rest of the feasible solutions have a high enough profit ratio, which varies from 97% to 99% approximately. For the 3rd instance, heuristics H2 was also better than H1 and H3 because H1 and H3 didn't find a solution for 375 cm shelves in the appropriate time. This means that in this case, there were so many allocations to be checked that the time was very long. Reducing the number of allocations to be checked results that there was no solution that satisfied all of the constraints. This means that algorithm H1 is inappropriate for this case. For the last 4th instance, both heuristics performed the same way and found optimal solutions on 3 cases from 5 ones. The solution time of CPLEX was from 1 to 4 seconds. It could be noticed in Table 1 that most of the heuristics found the solution in less than a minute. The longest time was approximately 6 minutes for the 3rd instance on a 500 cm planogram. But even for this instance, the optimal solution was found by both heuristics. This is the reason why we did not reduce the number of product allocations to be checked, which definitely reduced the computational time.

Table 1. Performance of the developed heuristics.

Instance	Shelf width	Profit ratio of H1	Profit ratio of H2	Time of H1 [min]	Time of H2 [min]
1	250	100.00%	100.00%	0.02	0.02
	375	100.00%	100.00%	0.14	0.14
	500	100.00%	100.00%	0.18	0.18
	625	100.00%	100.00%	0.36	0.36
	750	100.00%	100.00%	1.01	1.02
2	250	100.00%	100.00%	0.01	0.01
	375	100.00%	100.00%	0.06	0.06
	500	99.78%	99.78%	0.17	0.18
	625	98.07%	100.00%	0.58	0.60
	750	97.68%	100.00%	0.97	0.96
3	250	100.00%	100.00%	0.32	0.32
	375		99.59%		0.89
	500	100.00%	100.00%	1.32	6.23
	625	100.00%	100.00%	0.45	0.45
	750	100.00%	100.00%	4.89	4.89
4	250	99.02%	99.02%	0.07	0.07
	375	99.84%	99.84%	0.07	0.08
	500	100.00%	100.00%	0.70	0.70
	625	100.00%	100.00%	0.70	0.71
	750	100.00%	100.00%	0.40	0.40

Table 2 presents the steering parameters of the developed heuristics. Values are repeated that for most of the test cases, the input values were the same. We changed input values for the heuristics only if the input parameters which were good for another heuristics was inappropriate for the tested heuristics.

A number of generated allocations to be checked shows the quantity of product allocations that were achieved after excluding the product allocations with a profit less than the defined steering one. In most cases, we excluded the allocations with a profit less than the average profit calculated by all possible product allocations. Profit input ratio columns mean the input profit divided by the average profit of all product allocations. Values of 100% mean that we excluded allocations with profit less than average profit. Values less than 100% mean that we reduced the input profit because in some cases, excluding product allocations below the average profit values gave a worse solution which should be improved. There are no values more than 100%, but if they exist, they mean that the increase of this input profit value does not worsen the solution but helps to reduce computational time. This happened if a large number of product allocations could be generated, which later might be checked.

Table 2 shows that for both heuristics, we checked all of the product allocations for the 1st category. Otherwise, the number of product allocations of the 2nd category was large enough that we reduced this number and checked only a part of them, i.e. for H1 and H2 for 3 test cases. The largest number of allocations to be checked was for the 3rd instance on 375 cm.

The number of shelf allocations in the general case is $(r+1)^{PS}$ which equals $1.22 \cdot 10^{19}$. The number of product allocations in the general case is $\prod_{j=1}^{P}(f_j^{\max} - f_j^{\min} + 1)^S$ which equals $1.1 \cdot 10^{52}$.

In the proposed heuristics, the number of generated product allocations to be checked is significantly smaller. It varies from 7.8 thousand up to 46 million for heuristics H1. The number of generated product allocations to be checked is significantly greater than in H1 and varies from 7.8 thousand up to 375.5 million because it takes into account also the 3rd instance on 375 cm for which heuristics H1 could not find the solution. It could be summarized that the developed heuristics could find even the optimal solution checking only a very small part of the product allocations ($1.1 \cdot 10^{52}$ the total number of product allocations).

Table 2. Steering parameters of the developed heuristics.

Instance	Shelf width	Num. of generated alloc. to be checked H1	Num. of solutions H1	Checked alloc. for cat. 1 H1	Checked alloc. for cat. 2 H1	Profit input ratio H1	Num. of generated alloc. to be checked H2	Num. of solutions H2	Checked alloc. for cat. 1 H2	Checked alloc. for cat. 2 H2	Profit input ratio H2
1	250	358 470	15	100%	100%	100%	358 470	15	100%	100%	100%
	375	16 898 112	8	100%	100%	100%	16 898 112	8	100%	100%	100%
	500	503 552	128	100%	100%	100%	503 552	128	100%	100%	100%
	625	3 439 680	754	100%	100%	100%	3 439 680	754	100%	100%	100%
	750	472 444	385 854	100%	100%	100%	472 444	385 854	100%	100%	100%
2	250	7 840	647	100%	100%	100%	7 840	647	100%	100%	100%
	375	2 686 772	2 371	100%	100%	100%	2 686 772	2 371	100%	100%	100%
	500	1 587 612	12 847	100%	100%	90%	1 587 612	12 847	100%	100%	90%
	625	2 347 020	59	100%	83%	91%	2 347 020	13 566	100%	83%	91%
	750	1 525 810	48	100%	36%	93%	1 525 810	5 517	100%	36%	93%
3	250	33 467 486	65 464	100%	100%	88%	33 467 486	65 464	100%	100%	88%
	375						375 531 768	861	100%	100%	100%
	500	46 337 964	183 384	100%	59%	100%	46 337 964	2 337 969	100%	59%	100%
	625	25 500	24 260	100%	100%	100%	25 500	24 260	100%	100%	100%
	750	2 022 300	2 022 300	100%	100%	83%	2 022 300	2 022 300	100%	100%	83%
4	250	5 039 320	19 974	100%	100%	100%	5 039 320	19 974	100%	100%	100%
	375	1 479 200	1 864	100%	100%	100%	1 479 200	1 864	100%	100%	100%
	500	7 869 164	226 085	100%	100%	100%	7 869 164	226 085	100%	100%	100%
	625	486 336	184 967	100%	100%	100%	486 336	184 967	100%	100%	100%
	750	37 720	7 488	100%	100%	100%	37 720	7 488	100%	100%	100%

6. Conclusions and future research

In this research, the SSAP with horizontal and vertical categories is focused on maximizing the retailer's profit. Retailers are interested in expert guidance or solutions on how to allocate products on shelves which results in avoiding lost income and an increase in boosting total profit.

In this research, we enriched the basic SSAP with simultaneous vertical and horizontal category bands. We developed heuristics H1 and H2 demonstrate that they can solve the proposed SSAP optimally with vertical categories on small problem instances. Their quality was compared to the solution found by CPLEX solved. Obviously, in large instances, the optimal solution for non-linear models, even with the CPLEX solver, could not be found. That's why heuristics must be developed. Some of the principles in the heuristics could be applied for solving other SSAP problems. The developed heuristics allows the retailer to create, maintain, optimize, and distribute in-store planograms rapidly and profitably.

Heuristics H1 found the optimal solution in 14 from 20 test cases, while heuristics H2 was slightly better and found the optimal solution in 16 from 20 test cases. Both heuristics execute without checking the whole solutions space; nevertheless, they could find an optimal or near-optimal solution. The lowest solution quality of near-optimal is 97.68%.

The maximum numbers of product allocations checked by heuristics H1 and H2 were 46 million and 375.5 million, consequently. At the same time, the total number of product allocations is $1.1 \cdot 10^{52}$. This shows that the proposed heuristics allow reducing solution space without worsening the result. Therefore solution time was very fast, varying from a couple of seconds to 4.89 for heuristics H1 and to 6.23 for heuristics H1.

The computational experiments reveal the strong and weak sides of heuristics. Therefore in order to apply them in solving large instances, some of the steps should be improved. Future heuristics improvement should consider:

- a method of reducing the number of shelf allocations to be generated and to be checked because in this research, all shelf allocations were investigated;
- another method of reducing the number of product allocations to be generated and to be checked because the given method was inappropriate for one instance of heuristics H1;
- a better method of estimating input profit below which the solutions should not be generated because, for one instance, we received some millions of product allocations;
- a better method of adjusting steering parameters which allows to generate a satisfactory number of allocations and not too much increase the computational time because, for some instances, the solution time was longer than 1 minute.

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