A new method to determine the added mass of 2-dimensional objects in potential flow

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Abstract

The random walk source model is a displacement model where particles coming out of a source are displacing neighbor particles. The deformation of particle fronts moving towards objects carry the information about the fluid dynamical properties of the flow, as lift coefficient and drag coefficient. It is shown in numerical experiments, that the deformation of straight fronts pushing towards and passing an object also define an area behind the object where particles lack in comparison to an undeformed front. This aera (in the moment the separated fronts meet again), multiplicated with factor 2, is the added mass, assuming the particle represented by pixel, have a mass. This is a new way to determine the added mass in a random walk experiment.

Keywords: potential flow, drag, unsteady flow, added mass, random walk, displacement model, D'Alembert's paradox, induced drag.

1 Introduction

A body moving with constant velocity in an ideal fluid of constant density does not feel resultant forces, which is known as D'Alembert's paradox. The same body moving with unsteady motion in the same environment feels forces which are seen as the result of the acceleration of particles in the flow field, according to their position and movement in relation to the object movement. In 1888 Riecke [1] showed the physical effect, that for a sphere moving with constant velocity through an inviscid infinite fluid, the displaced particles do not return to their former positions. Sir Charles Darwin showed 1953, that the permanently displaced mass of the fluid between the initial and final position is the added mass, which is shown as a force when the object is accelerated [2,3]. For some objects the added mass can be calculated mathematically with the help of the potential flow theory or numerical modelling of potential flow with definition of sinks and sources distributed in the plane in parallel flow. We can find a number of tables defining mathematical and experimental data concerning the added mass of 2- dimensional and 3- dimensional objects

The random walk source model [4] is a model that generates particle fronts moving and generating an unsteady potential flow. This model shall be used to calculate the added mass of objects in 2 dimensions.

2 Methods

The random walk source model is a displacement model for ideal fluids, where particles coming out of a source are displacing neighbor particles. Particles close to the source must move more, far away particles less, according to the continuity hypothesis. The problem is that the exact movement of each particle bases on the shape of the geometrical environment and the contact of each particle with all other particles and their relation to the defined boundaries. It is a complex task to look on each particle in relation to all other particles and the geometrical conditions in the field. The solution is to statistically track the movement of each particle during flow development is the random walk source model. Each particle is exploring the field which results in a statistical picture of the geometrical conditions in the field during flow development. The results are particle fronts moving towards objects in an unsteady potential flow, see as an example figure 1.

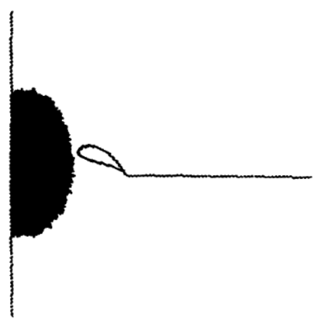

Figure 1: A particle cloud moving towards a wing profile in the random walk source model

The deformation of the fronts carries the fluid dynamical properties of the flow as lift coefficient and drag coefficient [5]. It is assumed, that the deformation of straight fronts passing an object also define an area behind the object which is connected to the added mass of this object. This shall be shown in numerical experiments.

3 Results

In the random walk source model, a vertical line source on the left side is releasing particles randomly on the start line. Each particle coming out of the source is randomly exploring the neighbor position in one horizontal or vertical step forward or backward. Let us imagine a clock- face. The outcoming particle is in the center. In the next step the particle choses by random to step to 3 o' clock, or 9 o'clock or 12 o' clock or 6 o'clock (Von Neumann neighborhood). In this position the particles askes "is a particle here" – if not, it settles down and a new particle starts on the source line. Is the position occupied, the particle checks another direction. A wall signalizes "you cannot settle down here, search in another direction". In figure 2 we see a vertical source line consisting of 100 start positions (100 pixel).

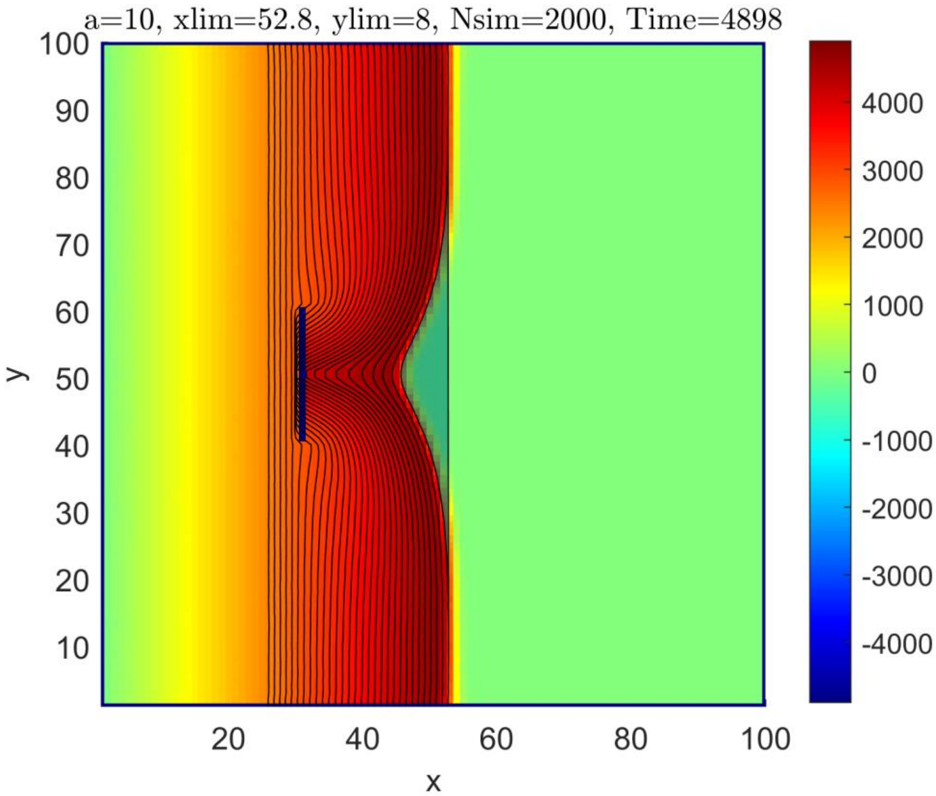

Figure 2: A vertical particle front moves from left to right towards the flat plate, generating an area of lacking particles.

A vertical particle front moves from left to right towards the object, a flat plate (length 20 pixel, thickness 1 pixel), which is in the position 35 pixel in x direction from the source line, shown in figure 2. The vertical equipotential lines in the figure are showing the deformation of the front of particles while passing the plate. The prior vertical straight line is separated into two lines curved around the corner of the plate. The area between the deformed fronts when meeting again on the backside of the plate and a vertical line representing the undeformed position of these fronts multiplicated with the factor 2 represents the added mass of the plate.

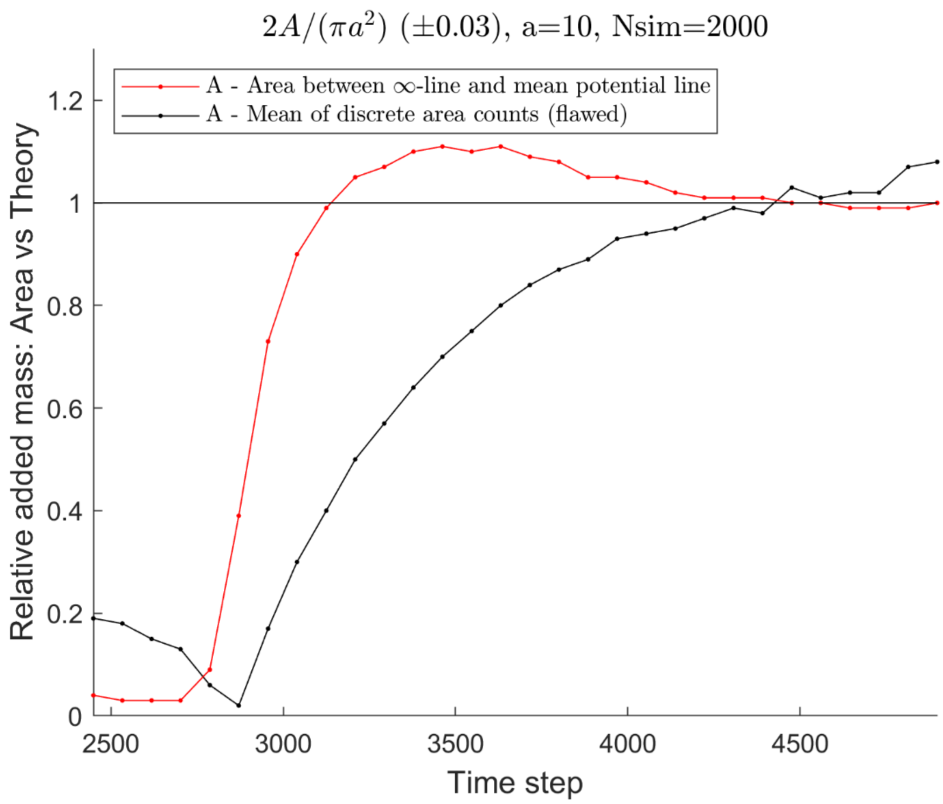

Figure 3: The vertical axis defines the relation between the area of lacking particles A multiplicated by 2 (2 A) divided by the published theoretically result for the added mass of a plate which is π a^2. The horizontal axis is showing the time steps representing the time development of the front.

In figure 3 we are looking on the magnitude of a deformation area A lacking particles, which is continuously deforming in contact with a flat plate. The vertical axis defines the relation between the deformation area A multiplicated by 2 (2 A) divided by the published theoretically result for the added mass of a plate which is π a^2. The horizontal axis is showing the time steps representing the time development of the front. The red curve is, as assumed, approaching the value 1 when the separated fronts meet again behind the obstacle, here in timestep 4500.

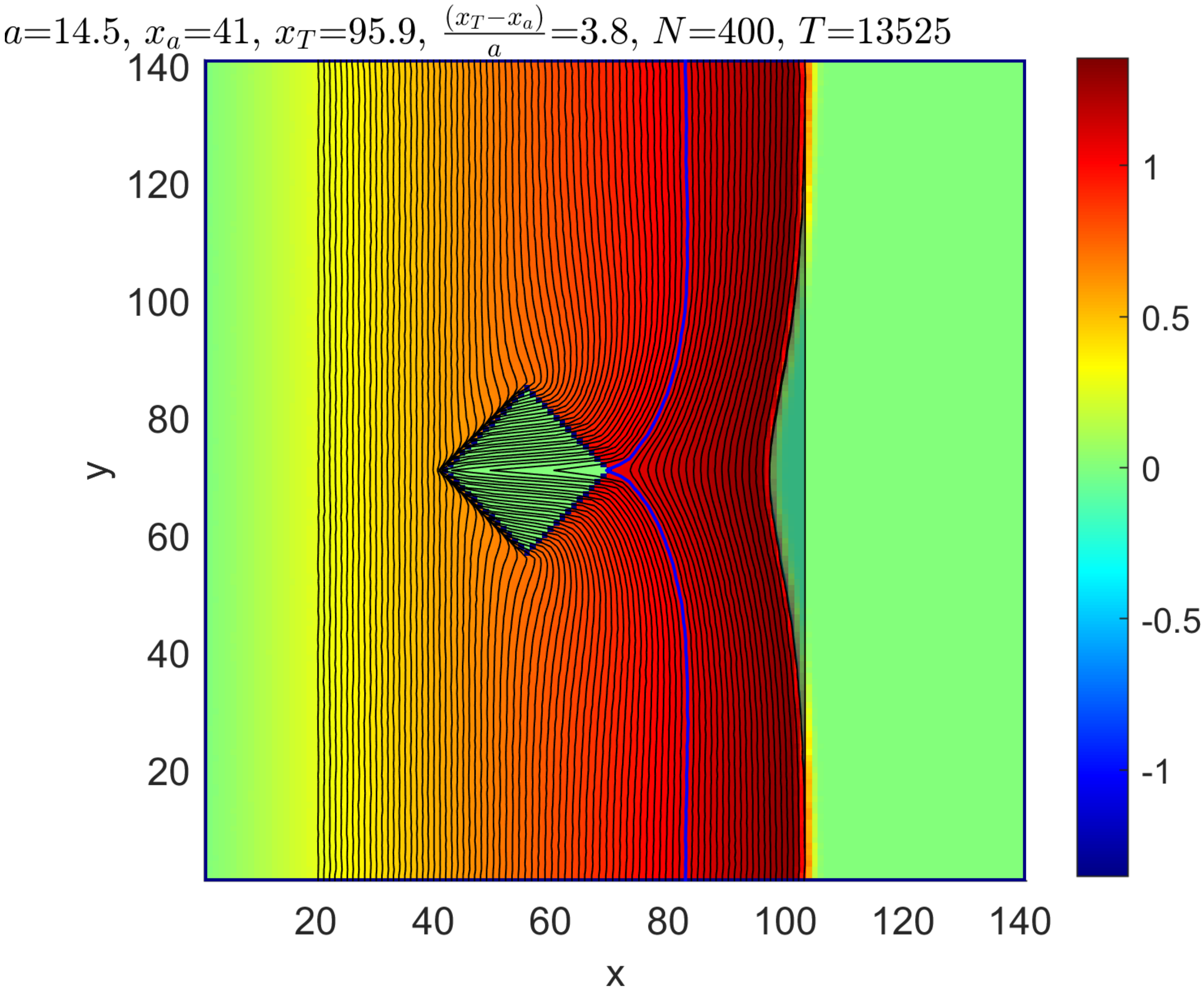

Figure 4: A vertical particle front moves from left to right towards a diamond shape, generating an area of lacking particles.

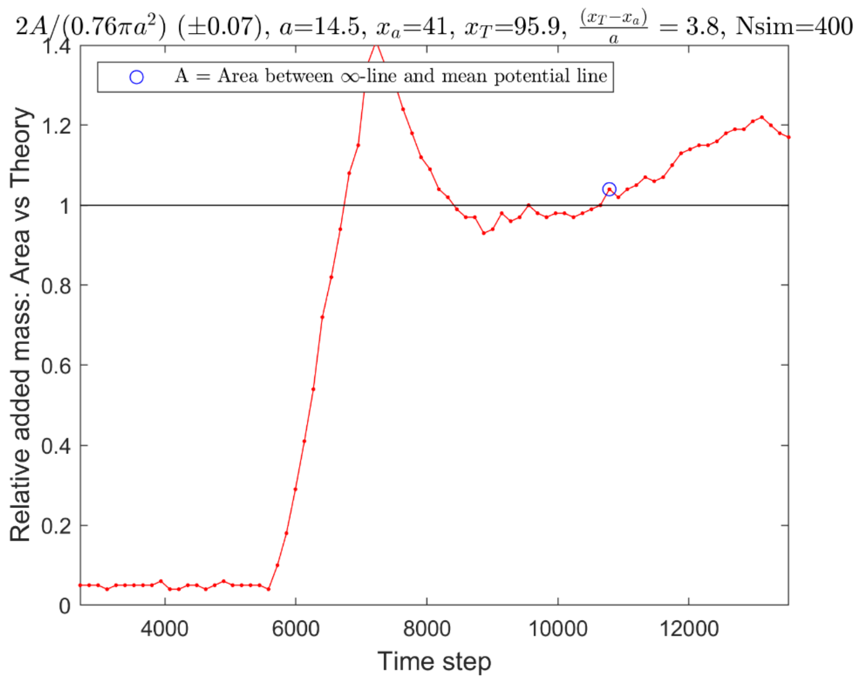

Figure 5: The vertical axis defines the relation between the area of lacking particles A multiplicated by 2 (2 A) divided by the published theoretically result for the added mass of a diamond shape which is 0.76 π a^2. The horizontal axis is showing the time steps representing the time development of the front.

Figures 4 and 5 show the same experimental results in connection with a diamond shaped body.

4 Conclusions and Contributions

When Riecke in 1888 showed the effect, that for a sphere moving with constant velocity through an inviscid infinite fluid, the displaced particles do not return to their former positions but establish in new positions, it was later possible to show that the permanently displaced mass of the fluid between the initial and final position is the added mass.

The random walk source model is a displacement model where particles coming out of a source are displacing neighbor particles. The deformation of fronts moving towards objects carry the information about the fluid dynamical properties of the flow, as lift coefficient and drag coefficient. We have shown in numerical experiments, that the deformation of straight fronts passing an object also define an area behind the object where particles lack in comparison to an undeformed front. This area (in the moment the separated fronts meet again), multiplicated with factor 2, is the added mass, assuming the particle represented by pixel, have a mass. This is a new way to determine the added mass numerically. We assume, that it will be possible to also calculate the added mass in potential flow in 3- dimensions, looking on the volume of lacking particles behind an object when connecting again behind the 3- dimensional object. The front defines here a deformed membrane surface.

Acknowledgements

The authors would like to thank Sveinung Erland for his impressive work writing the software and generating the figures and our colleague Zhenhui Liu for fruitful discussions.

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