CRITICAL MATHEMATICS EDUCATION AND POST-NORMAL SCIENCE: A LITERATURE OVERVIEW

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Abstract

This paper presents findings from a literature overview where the focus is to identify and critically reflect on concepts and perspectives highlighted as important in the literature of two fields, critical mathematics education and post-normal science. In addition, the combination of the two perspectives develops thinking about the concept of critical citizenship. The paper is written from a theoretical perspective of critical mathematics education. Critical mathematics education concerns social and political aspects of the learning of mathematics, how mathematics is used in society, and how the development of critical citizenship is carried out both inside and outside the classroom. Post-normal science concerns how complex science-related issues in society can be managed by involving critical citizens. Some researchers suggest that to develop pupils' critical mathematics perspective, real-world problems such as climate change, can act as a gateway for them to be(come) a critical citizen in an extended peer community.

Introduction

Citizens deals with mathematics in multiple ways in our society. From an educational perspective it is important to be aware of, and to learn how to deal with, these different intersections of mathematics and citizens in society. Hauge and Barwell (2017) argue that pupils can develop critical mathematics perspectives working with complex real-world problems. Such perspectives include students' knowledge and understanding of mathematics as means for self-empowerment to (re-)organize interpretations of social institutions and traditions, and for taking justified stands in social and political reform (Skovsmose, 1994). In order to further explore these views, this paper presents findings from a literature overview, where the focus is to *identify* and *critically reflect on* concepts and perspectives highlighted as important in the literature from two fields, critical mathematics education (CME) and post-normal science (PNS). CME concerns social and political aspects of the learning of mathematics, how mathematics is used in society, and how the development of critical citizenship is carried out both inside and outside the classroom (Skovsmose 2016). PNS concerns how complex science related problems are dealt with in society.

The extent of mathematics used in climate change and other socio-political issues, implies that mathematics education needs to be involved (Barwell 2013). Climate change is a complex issue, and involves uncertainty, conflicts, risks, and values. It is sometimes referred to as a wicked problem, characterized by no definitive problem-formulation, no trial and error for solutions, no given alternative solutions, and solutions not possible to define as right or wrong (Conklin 2006; Rittel & Webber, 1973). One might refer to climate change as a super wicked problem, because there are additional aspects like the time issue, no central authority, those causing it are those who seek to solve it, and even if aware of the impacts from inaction, one chooses at the very best, limited actions (Levin, Cashore, Bernstein, & Auld 2012). Questions to be asked could then be: if pupils are to develop a critical mathematics perspective, can working with wicked problems in the mathematics classroom be relevant? Also, is there something to learn from how these problems can be dealt with in real-life? CME and PNS can be a starting point for reflections at both a philosophical and an educational level. I will reflect on both levels and use the philosophical perspective to bring new perspectives into mathematics education.

Funtowicz and Ravetz (2003) argue that complex science-related issues need new conceptions of management. In the past, problems have been dealt with by including an expert-regime. The uncertainty and complexity of climate change makes it difficult for citizens, politicians, scientists, or other stakeholders to handle. They refer to issues where "facts are uncertain, values in dispute, stakes high and decisions urgent" as PNS (p. 1), and argue that PNS must involve a wide range of participants, not only experts, but also those who have to deal with the consequences. They refer to this as *extended peer community*. The extended peer community can contribute with extended facts, integrating both social and technological aspects, and critically evaluate materials provided by experts. The involvement of local non-expert citizens is important for decision-making, and can contribute with local knowledge and an ability to think outside the box. Hauge and Barwell (2017) argue that if students learn to manage uncertainty and to embrace a plurality of perspectives in the mathematics classroom, they can be better prepared and enabled as critical citizens in an extended peer-community.

There are similarities between CME and PNS, such as dealing with social and political aspects of society and citizens' role, critique of the existing, emphasis on uncertainty and complexity, and the emphasis that science and mathematics are neither objective and certain, nor neutral or value-free. However, where CME mainly revolves around educational perspectives, PNS is an insight which links governance to epistemology, and aims to enhance understanding for both research and action (Funtowicz & Ravetz, 1999 2003). CME is a philosophy with a strong

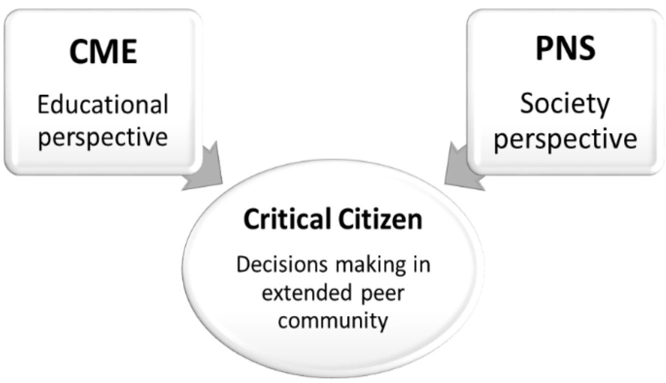

Figure 1: How can the perspectives from Critical Mathematics Education (CME) and Post-Normal Science (PNS) be combined for broadening our views on how to be(come) critical citizens?

educational perspective, aiming to bring interpretations, clarifications, and providing new perspectives into mathematics education (Skovsmose, 1994). PNS does not have an educational perspective, but has a political aspect by promoting decision-making in society in a dialogic and pluralistic process, as a response to problems involving uncertainties, values in dispute, and high stakes decisions. Although PNS is not explicitly a theory for educational purposes, there are several examples in the literature which combine PNS and education, mainly in science education. Also in mathematics education some researchers have used PNS; Hauge and Barwell (2017) argued that PNS and CME complement and enrich each other. By combining CME and PNS, I emphasize the role critical citizenship could take in society, and where CME looks at critical citizenship from the perspective of education, PNS emphasizes processes by which it might come about in society. By looking ahead at how students can contribute in a highly technological society, one can better prepare those students to be(come) critical citizens in the mathematics classroom. Barwell (2013, p. 9) suggests that “Mathematics education (together with science and environmental education) needs to consider the role of learners in the wider peer communities that post-normal science demands, and prepare them to participate as active, critical citizens”. When PNS urges decision and policy makers to involve multiple voices in controversies in society, CME does the same in education, by enabling and preparing students for an extended peer community.

Although there are relevant literature reviews within the two fields, none, so far, combine them and search for commonalities. Literature from these two academic fields are therefore investigated in this literature overview. As a literature overview, I will survey the literature, and describe some characteristics (Grant & Booth 2009). Furthermore, this paper has a critical aspect, and brings previous thoughts from the literature and combines these into further conceptual development.

Characteristics of the literature overview

The characteristics of this literature overview are presented according to focus, goal, perspective, coverage, audience and organization (Cooper 2016). **Focus:** the focus of this paper is on theoretical concepts and perspectives. **Goals:** the main goals are to identify and critically reflect on concepts and perspectives highlighted as important in the literature of CME and PNS. Essential in this work is to reflect on similarities, differences, or contradictions between concepts, and to synthesize and summarize. **Perspective:** in the literature chosen, there may be a preexisting bias towards being overly positive toward the two fields. Due to the goal of this literature overview, my biases have hopefully had little impact on which literature is selected, because my interest was in identifying concepts. My reflections in the paper may, though, be influenced. **Coverage:** I have used the term literature overview, due to the focus of the paper, and have made a purposive sample of the existing literature. Systematic approaches have been used to identify concepts highlighted as central in the literature. **Audience:** the overview perspective will give value for those who are new to CME or PNS, while the critical aspects of it can give new insights for those familiar to the fields. Researchers, mathematics educators, and mathematic teachers are considered the main audience. I will use this literature overview to inform, create a focus, and form a conceptual framework for my own PhD project (Maxwell 2006), where I am interested in how complex real-world situations like climate change can develop students' critical mathematics perspective. **Organization:** the text is structured (from Discussion of Findings, p.9) into four main areas, in accordance with identified concepts and perspectives: 1. Important characteristics of tasks; 2. Pupils' role in society; 3. Competences recognized as important; and, 4. Aspects of democracy.

Methods and methodology

The guiding principles for this literature overview are "to present results of similar studies, to relate the present study to the ongoing dialogue in the literature, and to provide a framework for comparing the results of a study with other studies" (Creswell, 1994, p. 37). The process has been in three main steps; defining research topics, selecting texts to include, and analyzing. In the first two steps, I initially had a broad approach to the field, using the search terms: "critical mathematics education" and "post-normal science". Multiple approaches have been carried out in the searching process, such as using scientific search databases, bibliography searches, scrutinizing reference lists, citation searching, author searching, consulting with experts, and identifying key search terms in iterative cycles. Doing so, I gained insight into the area, which helped me to narrow down the research topic and choose which texts to analyze. Important processes in my analysis were: identifying focus areas through key terms and systematization, data extraction, similarities, differences, conflicting or new perspectives, quality assessment, synthesizing, and pattern recognition. To obtain transparency and reproducibility, the search process is described in detail. Development of criteria for inclusion, exclusion, and quality assessment of the texts was an ongoing and iterative process, and these criteria are described below.

Table 1 presents some general criteria, whereas Table 2 and Figure 2 visualize the key terms in more detail. The methodology part ends with an excerpt of included items (Table 3). The text comments and elaborates on the tables and figure.

Criteria	**Included**	**Excluded**
Time frame	2006-2017	2005 and older
Publication	Book chapter, conference papers, journal article.	Books, report working groups, topical survey, monographs, master thesis, editorials, news, Audio/Videoclip, book reviews.
Language	English and Scandinavian language	Others
Level	Lower secondary school	Others (such as preservice teachers, kindergarten, after-school program, university)
Electronic search engines	ERIC, Science Direct, ProQuest, Web of Science, MathEduc/Scopus	Others
Others	Access to full-text articles, search terms, quality assessment	

Table 1: General criteria for inclusion and exclusion of articles

CME and PNS have been around roughly since the nineties, and the timespan was chosen to ensure diversity and an “up to date, but not forgotten” perspective. However, some items older than ten years were included due to their influential impact on their fields. Different research designs were included, but the final list consisted mostly of qualitative studies, with theoretical and empirical focus. Early research in mathematics education with a social perspective mainly used quantitative research methodologies (Atweh, Forgasz, & Nebres 2001). Although there is quantitative research that has a critical mathematics perspective (as exemplified by Skovsmose & Borba 2004, p. 209), I had a special interest in qualitative studies, due to own empirical research. I also had an interest in lower secondary school, and I do acknowledge that by excluding other levels, relevant items can be missed both in order to locate concepts and perspectives, and in recognizing their transferability. The search terms were set to all fields (title, abstract etc,). The quality assessment was based on several criteria, such as peer review, published in a recognized journal or publisher, transparency of methodology and theoretical grounding, quality of writing, bringing onboard new and different concepts and perspectives, and degree of reflexivity. Furthermore, thematic relevance was of great significance.

In the initial stage, the search words (CME and PNS) were chosen to ensure the inclusion of relevant literature. This generated some items with less relevance. Several other key terms were therefore added as inclusion and exclusion criteria. Some of these terms were chosen in line with the focus of my PhD project, such as critique, mathemacy, and climate change (Table 2). Others came from what was highlighted in the literature as important or relevant, such as social responsibility, extended peer-community and reflective knowing. In CME, the focus of the literature overview, identifying concepts and perspectives, was my guiding principle when selecting items. Items that had an overview-focus were therefore favored above items concerning more narrow parts of the field. When searching for literature in the field of PNS, I first favored items that concerned education, climate change, and extended peer community, then those that had an overview perspective.

The keywords listed in Table 2 (see below) give an indication of important search words. They are used in various forms (synonyms, etc), and with different Boolean logic. The keywords shown in the table give the reader an indication of the foci of the search. If the excluded keywords are the main focus of the paper, with less emphasis on the included keywords, papers have been excluded. Some of the excluded keywords, such as social justice and ethnomathematics, are closely related to CME. Teaching mathematics for social justice, as suggested by Gutstein (2012), does not explicitly use the label of CME, though e.g. Skovsmose (Alrø, Ravn, & Valero 2010, p. 4) considers it as an example of CME. Similar thoughts can be made in respect to ethnomathematics. It is not my intention to draw distinct lines between different areas of CME. Furthermore, some of the key terms are sometimes highly influenced by particular authors. A search by these concepts could therefore give a biased image of the literature. There are for instance several related concepts to mathematical literacy (see discussion below, main area 3, p.16) such as mathemacy, often associated with Skovsmose (1994), and matheracy, associated with D’Ambrosio (2010). For other key terms, such as reflective knowing (by Skovsmose, 1992), related concepts are not as easily found used in other authors (although they may write about reflective knowing without explicitly referring to this concept). Using this concept as a search word can therefore result in a skewed literature search result, favoring one particular author. I have listed all the different and related concepts in order to make it more transparent which search word was included.

	CME	PNS
Inclusion	Mathematical literacy (see text) Critique, Critical agency, Critical mathematics perspective, Critical mathematical thinking Democracy, Formatting power of mathematics, Critical citizen(ship) Reflective educational practice, Reflective knowing Social responsibility, Values, Ethics, Conflicts, Complexity, Uncertainty Real world problems (modelling, statistics)	Education, Transdisciplinary, Interdisciplinary, Multidisciplinary Extended peer-community, Democratization of science Social responsibility, Values, Ethics, Conflicts, Complexity, Uncertainty Climate change, Sustainability
Exclusion	Ethnomathematics, Social justice, Pedagogy of dialogue and conflict, Sociomathematics, Intercultural, Race, Gender and identity, Equity Curriculum, Education policies, Foci on specific topic (geometry, algebra etc.) Instructional practices, Assessment, Tests, Achievement gap, Disabilities Historic perspective, Epistemological and ontological foci	Policy and intervention, Economics, Production of Knowledge, Specific topics within specific fields (such as biotechnology, chemistry, fisheries etc.) Urban planning, Catastrophes Gender-issues, Historic perspective

Table 2: Key terms for inclusion and exclusion of items

Throughout the read and search process, I created a conceptual mapping (Figure 2, see below). When identifying concepts highlighted in the literature as important, they have been written onto a map to get an overview of the two fields. By visualizing the central concepts and themes this way, the interrelation between them appeared clearly. Later in the analyzing process, the interrelation of the identified concepts made possible the grouping into four main areas. The identified concepts within each area, have some common features, but also some overlapping properties. After identifying these main areas, my question became: if this is the answer, what is the question, resulting in four questions, one for each area.

In creating the mapping in Figure 2, some homogenizing of terms was undertaken. An example of this was when Sánchez and Blomhøj (2010) wrote about the important role mathematics plays in shaping our world, this was coded as "formatting power of mathematics", a termed used by Skovsmose (1994). Another example was when referring to mathematics and real life, there are multiple related terms: real-life education (Alrø & Johnsen-Høines 2010), realistic mathematics education (Jablonka 2015), real world problem solving (Jurdak 2016), real-world activities or real-world examples (Atweh 2012), mathematics of the real world (Brantlinger 2014), real situations and really real situations (D'Ambrosio 2015), real life applications and realistic problem (De Freitas 2008), real-life contexts (Goos, Geiger, & Dole 2014; Hauge & Barwell 2017; OECD 2003), real world projects (Gutstein 2006), and authentic applications of mathematics in socio-political applications (Sánchez & Blomhøj 2010). These were all coded under the term "real world problem", although they might each have a slightly different perspective.

The number of items included was initially 362 (202 CME, 160 PNS), and after the inclusion-exclusion and quality assessment processes the number was reduced to 57 (30 CME 27 PNS). In the initial search in CME, Skovsmose was the most frequently represented author (31 items). His book, *Towards a Philosophy of Critical Mathematics Education*, brought about seminal ideas to the field, and several of his publications met the inclusion criteria. In order to avoid an over-representation of Skovsmose, a representative publication with an overview focus was selected. This approach was also carried out on other authors, such as Frankenstein, D'Ambrosio, and Ernest. This method could give an incorrect image of which authors have contributed the most to the field. On the other hand, it allows authors who have not written the same amount as Skovsmose, to gain a voice. Similar issues were observed in PNS, where Funtovitz & Ravetz are the idea-holders. Items passing the inclusion criteria could also be excluded if there was a strong resemblance between different items and overlapping concepts and perspectives.

During the inclusion and exclusion process, there were several borderline cases, where one could argue for inclusion as well as exclusion, and judgment calls were therefore made. A shared decision-making with supervisor and colleagues strengthened this process. Notes were taken during the reading and searching process. An excerpt of selected items is presented in Table 3 to indicate this process (the full document contains more text).

Author	**Type**	**Content**	**CME**	**PNS**	**Focus**
Skovsmose (2016)	Journal article	Notions, the future	Y	N	1,2,3,4
Hauge and Barwell (2017)	Journal article	Education, critical citizens	Y	Y	1,2,3,4
(Funtowicz & Ravetz 2003)	Journal article	Uncertainty, complexity	N	Y	1,2,3,4

Table 3: Excerpt - Overview of selected items

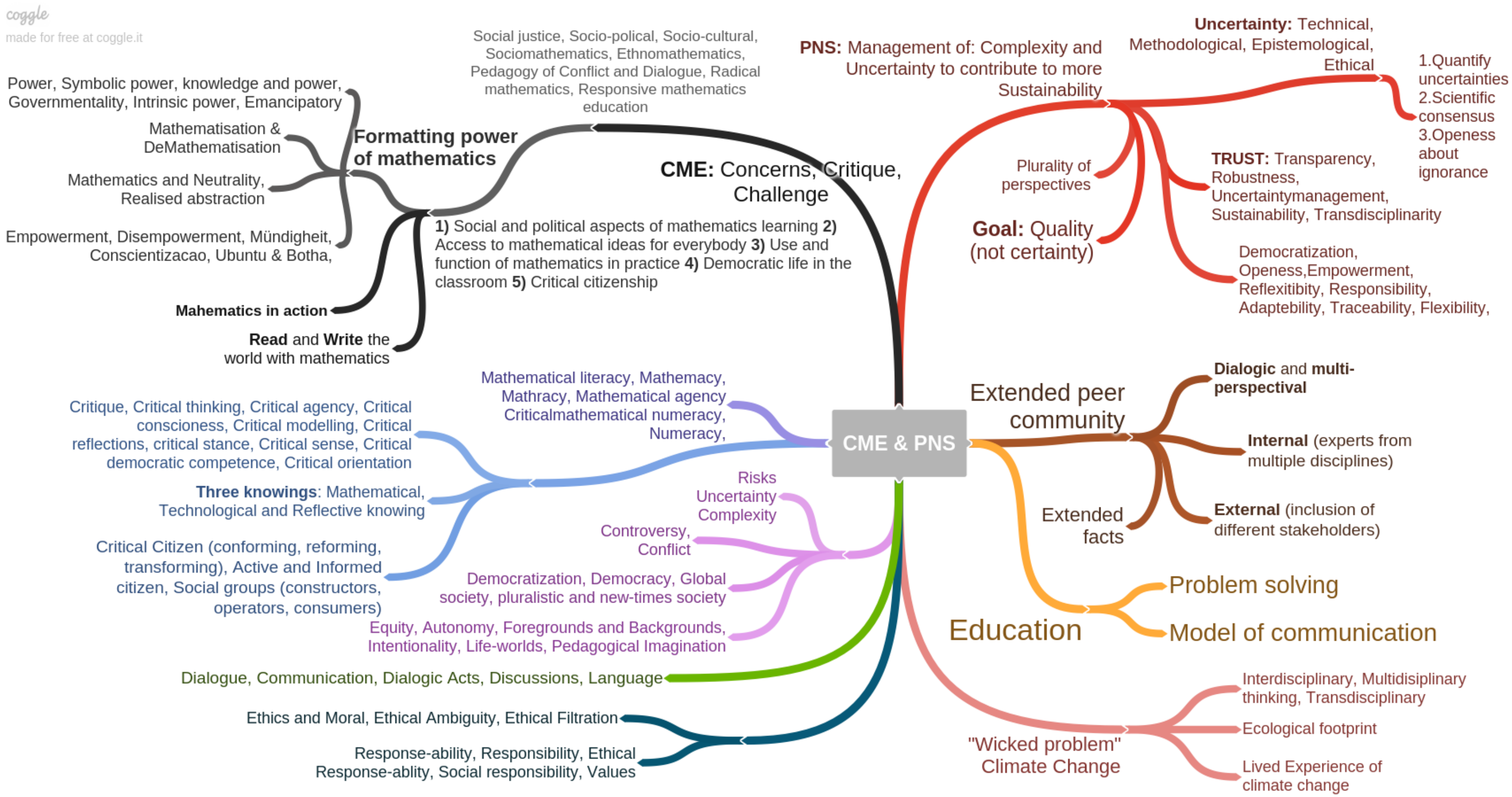

Figure 2: Conceptual mapping of identified concepts and perspectiv

Discussion of findings – the concepts and perspectives

The findings are structured, and discussed, in four main areas: Important characteristics of tasks (1); Pupils' role in society (2); Competences recognized as important (3); and, Aspects of democracy (4). Although the results are structured into different areas, each containing different concepts, there is an overlap between the different areas. Table 4 gives an overview, with accompanying questions, and the findings as keywords.

Main area	Questions	Findings
1. Important characteristics of tasks	What characterizes complex real-world problems?	Wicked problems, Uncertainty, Complexity, Controversy, Risks Multi/Inter/Transdisciplinarity
2. Pupils' role in society	Why should pupils (be prepared to) contribute in complex problems in society?	Critical Citizen, Extended peer community
3. Competences recognized as important	What kind of competences do pupils need when reflecting on complex real-world problems (in the mathematics classroom)?	Mathematical literacy, Reflective knowing, Critical agency, Critique and Dialogue
4. Aspects of democracy	Why is it important to bring real-world problems into mathematics education from a democracy perspective?	Power, Formatting Power, Responsibility, Ethics, Value Democratization and Global Society

Table 4: Overview of four main areas

Important characteristics of tasks (1)

Mathematics classrooms educate students for today's society. An important part in this can be to invite students into solving real-world problems (Jurdak 2016; Skovsmose, 1994). A key question is then: *what characterizes these real-world problems?* The identified concepts emphasized in the included literature regarding these characteristics are **wicked problem, uncertainty, complexity, controversy, risks** and **multi- inter- and transdisciplinary thinking**.

Wicked problems: in wicked problems, there are many levels and perspectives of the problem. In climate change, there are debates on whether climate change is in fact anthropogenic, and if recognized as such, multiple questions and issues surfaces in the wake of this (e.g. which factors contribute the most, the best approach towards energy supply, transport solutions). Hulme (2010, p. 303) argues that climate change has transformed from a physical phenomenon being studied, to an idea being contested. Several mathematics researchers and educators argue for dealing with real-world problems, and put emphasis especially on unmasking their often hidden and non-neutral aspects, e.g.,: Frankenstein (2010); Gutiérrez (2013); Gutstein (2012); Jurdak (2016). Barwell and Suurtamm (2011) argue for the importance of making the invisible visible in modelling climate change in order to make information accessible to citizens. D'Ambrosio (2015, p. 27) emphasizes that we do not need to try to contextualize mathematics education, the real situation is already here, "…waiting to be recognized and dealt with …". Jurdak (2016, p. 110) connects real-world problems to Freire's emancipatory problem-posing pedagogies (in contrast to banking education), and emphasizes students being "…critical co-investigators in dialogue with the teacher". Furthermore, he identifies challenges implementing CME in practice, such as teachers' insecurity, students' attitudes and dispositions, and reluctance from schools to provide room for CME. Also, curricula can impede goals such as developing "… critical consciousness of the world for changing it…", as seen in the work of Vithal and Gutstein (pp. 113-114). Atweh (2012) emphasized that real-world problems should do more than engage with the mathematics, they should also engage in real-world situations. From a perspective of PNS, Funtowicz and Ravetz (2003) suggest that when facing complex, uncertain, and value-loaded problems, one needs different ways to deal with these issues, including different actors. Wildschut (2017) reflects on how our society is changing towards a peer-to-peer society, with new roles for both science and citizens. What these roles might be is still to be seen, but some examples are observed in several areas, such as in environmental issues.

Wicked problems have earlier been left to scientists to deal with, but within this rationalist, science-led approach, claims can be made to include lived experiences as part of policy-making. Lived experience can be defined: as our individual and collective agency to reflect and engage; as our experiences and responses to events and potential impacts; and, as structures and power relations that define our capacities for actions (Abbott & Wilson 2014, p. 3). By implementing lived experience in policy-making, unintentional harm can be avoided, you can forward ideas that already work from a bottom-up-perspective, and enhance public acceptance and legitimacy (p. 6). But as Yasukawa (2007, p. 12) states, when reflecting on a mathematics education *about* and *for* sustainable development, in the case of the latter, we are expecting a great deal from the learners: understanding and skills; actions based on moral stands; and, perhaps in situations where individuals traditionally have little influence and power. Furthermore, the context is seen as a vehicle for learning mathematics more than "… factors that shape the purpose of mathematics teaching and learning" (2007, p. 18).

Uncertainty: Skovsmose and Greer (2012b, p. 376) highlight mathematics education as a field where certainty is pursued, but in the face of current crisis (e.g. environmental), it is not always possible to achieve. Skovsmose (2016, p. 12) underlines that "…the basic epistemic condition for a critical activity is uncertainty", and that the constant changes in society, such as environmental problems, require us to always redefine what has been. In science education, Christensen (2009, p. 207) emphasizes two dimensions of uncertainty, connected to the construction and application of scientific knowledge: complexity; and, science-in-the-making. In the first dimension, Christensen claims that even if the problem is extremely complex, there seems to be a focus on demonstrating direct relationships between cause and effect in a decontextualized setting. In the second dimension, there are often found examples of science-in-the-making in media (such as climate change, genetically modified crops and food), but less seen in classrooms. Christensen suggests that "acknowledging the uncertainty of scientific knowledge is not to decrease its value. It is not to deny the reliability of much scientific knowledge, the wealth and wonder of the vast number of scientific 'discoveries' or the crucial role that science plays in problem solving" (p. 208). Similarly, Turnpenny (2012, pp. 403-404) argues responses to uncertainty in wicked problems may not be working towards more certainty, but to recognize that problems are more than being reduced to numbers, statistics, and scientific facts, and recognizing that "…'facts' and 'values' are locked together …", and therefore need to be treated accordingly.

Hauge and Barwell (2017, p. 8) highlight three different sorts of uncertainty from the literature of PNS: technical; methodological; and, epistemic uncertainty. Technical uncertainty can be inexactness in methods or techniques. Methodological uncertainty can occur when connecting different system components where there are difficulties accurately quantifying them. Giampietro and Saltelli (2014) argue that 'ecological footprint' involves methodological uncertainty (e.g., different scales or quantitative variables belonging to different descriptive domains), and if used in policymaking this could conceal important uncertainty, and lead to paradoxes. Epistemic uncertainty can arise from lack of knowledge or lack of awareness of some features of situations. They argue that wicked problems (which they refer to as post-normal situations) are characterized by epistemic uncertainty, and should be dealt with PNS and by an extended peer community. They furthermore link these three types of uncertainty to Skovsmose's three types of knowing (see later in the text, and Figure 4).

Three different policy strategies dealing with uncertainties are identified: quantify uncertainty; build scientific consensus; and, openness about ignorance (Van Der Sluijs 2012; Van der Sluijs, Van Est, & Riphagen 2010). The first strategy is a technocratic view where uncertainty is seen as temporary "shortcoming of knowledge", resulting in more research and more complex models as "answers". Calculations presented may appear more certain than they are, and give the impression of unified certainty. The second strategy, scientific consensus, attempts to reflect the best of our knowledge from a multidisciplinary expert panel. The Intergovernmental Panel on Climate Change (IPPC) serves as an example. The third strategy, openness about ignorance, is a deliberative model suggested to include and appreciate a plurality of perspectives (2010, p. 410). It recognizes uncertainty as something unavoidable and permanent, and involving reflection and argued choice (p. 413). The reflexivity in the latter strategy is also highlighted by Spangenberg (2011, p. 279), who defines reflexivity as a "…capacity of an individual agent to act against influences of socialization and social structure, based on critical self assessment". Furthermore, reflexivity accepts different kinds of knowledge, and acknowledges different roles and perspectives. Funtowicz and Ravetz (2003, p. 4) emphasize that uncertainty and inconclusive arguments sometimes cause paralysis in policymaking, and suggest an awareness, in management, of uncertainties. This awareness can be accomplished by a quest for quality rather than for truth, and through an open dialogue between participants (p. 6).

Complexity: by recognizing systems as not fixed, not static and not in equilibrium, complexity is acknowledged (Beilin & Bender 2011, p. 159). Furthermore, complex systems involve interrelated subsystems, involving both natural and social systems (Funtowicz & Ravetz 2003). Each subsystem is interwoven and embedded in its societal and natural context. Funtowicz and Ravetz (2003, p. 2) argue therefore "there can be no single privileged point of view for measurement, analysis and evaluation", and that complex phenomena cannot be reduced to individual components and dealt with accordingly, but need a holistic approach. In mathematics education, a majority of problems tackled in classrooms are such isolated tasks, and Funtowicz and Ravetz (2003) argue that facts taught from textbooks no longer are sufficient, because in practice there are several plausible answers, or no well-defined answer at all. Ravetz (2007) claims that complex systems thinking needs to consider multiple perspectives on several relations, such as quality, knowledge, facts versus truths and evidence, ethical judgments, and ramifications of effects. An illustration of the latter is when making an ethical “good” judgment, but the effect is unintentionally harmful (law of unintended consequences, or Murphy’s Law), which often is seen in environmental problems (p. 280). Colucci-Gray (2009, p. 196) suggests an “…awareness of the different representations of the world is a step toward building awareness of complexity and a way of knowing which is more respectful of other people and other living beings”. She challenged students through an educational setting to handle complexity, and identified competences both on an individual, collective and an ecological level (pp. 199-200).

Controversy: in many real-world problems, there is controversy and conflicting interest. Atweh (2012, p. 337) calls for a “…willingness to deal with controversial topics … debate and difference of opinion…”. Also Gray and Bryce (2006) stress the need for education to include controversies in order to involve reflections on science, social and ethical dimensions, uncertainty, and complexity. They claim that students should be allowed to engage in critical reasoning, discussions, value judgement, and the messiness of decision-making (p. 176). This is exemplified by Colucci-Gray (2009), where students deal with controversies on prawn farming through role-play. There are cycles of controversies about environmental impact, such as trees being cut down to facilitate economic growth, which leads to loss of mechanical protection from the sea, which again causes reduction in biodiversity. Other aspects of controversy are: how one selects information; how value judgements are performed; and, how certain power positions can be persuasive (p. 192). Suggestions for education are to facilitate a language for accessing pluralism, and a connectedness to reality in order to get to “deeper and fluid engagements between the observer and reality” (p. 193). Controversies do not always represent interest between two equal parties, one group can have less legitimization than others (such as local inhabitants versus scientists). Another element is how different parties use facts. The same numbers and statistics are used by counterparties in debates about climate change, and therefore serve as an example that science and mathematics are not a “monolithic truth tablet” (Turnpenny 2012, p. 400).

Risk: risk can be considered as the possible loss of something valuable. Christensen (2009) elaborates on several conceptions of risk, and traditionally it involves measurement of some sort (e.g., quantification of physical harm). Social and cultural context can play a role in how risk is perceived, and environmental risk is influenced by traditional views on risk, but also people’s perceptions of risk from a cultural perspective (p. 213). Furthermore, the precautionary principle is highly relevant in climate change, where irreversible damage can occur, and the uncertainty involved should not lead to postponing preventative action (Christensen 2009). Risk management involves making “right” decisions, where different concerns such as safety, costs, reputation, are to be balanced adequately (Aven 2013, p. 270). Focus is on more than economics and fatalities, and towards issues such as possible reversibility, and possible violation of equity (who benefits versus who bears the costs) (p. 274). Lidskog (2008) investigates three different proposals for public engagement in risk regulation: new production of knowledge; PNS; and, scientific citizenship. All concern how science can be democratized, and the relation between experts and citizens (p. 71). He mentions examples of political declaration, highlighting the importance of involving citizens in risk management, decision-making, and moving towards deliberative negotiation. He does not see public inclusion as the total cure, but welcomes this cautiously. In science education, there is growing awareness of how knowledge about risk connects to “value free” scientific knowledge.

Students can engage in considering risk and uncertainty, both on a personal and on a societal level. Three significant challenges are then relevant: how to deal with uncertainty; how to deal with the "power and limitations of science in social context"; and, how these two challenges can be handled in educational contexts (Christensen 2009, p. 207). Furthermore, Christensen highlights argumentation, collective argumentation, small-group discussion, and team teaching as important.

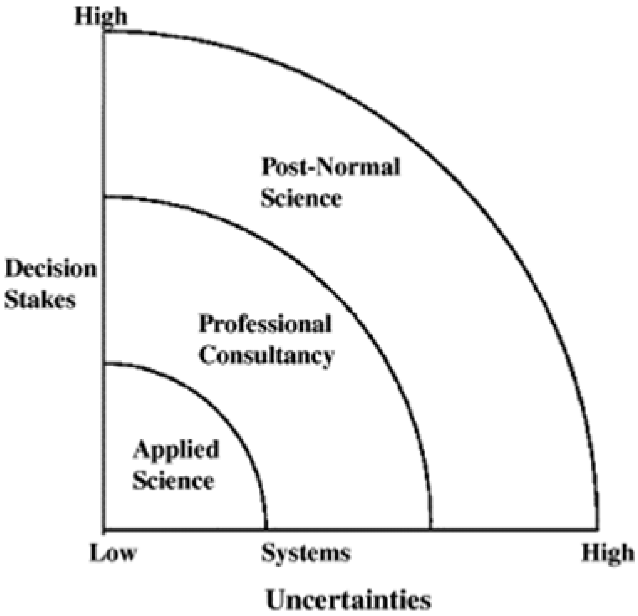

Figure 3: PNS – Uncertainty-Risk model from Funtowicz and Ravetz (2003).

Funtowicz and Ravetz (2003) have developed an uncertainty-risk model (Figure 3), with one value dimension (decision stakes), and one knowledge dimension (system uncertainties). It has three categories of risk problem-solving strategies: applied sciences; professional consultancy; and, PNS. When both the uncertainty and decision stakes are low, applied sciences are efficient. When one or both are medium the involvement of professional consultancy (e.g. senior engineer, or surgeon) may be effective; and, when one or both are high (e.g., risks cannot be quantified or possible damage irreversible) we need to engage in post-normal science.

Inter- Multi- Transdisciplinary thinking: educational systems tend to be disciplinary based, although several mathematics researchers suggest a more holistic approach to learning. Wicked problems require competences from several subject disciplines at once. In science education, Spangenberg (2011) provides an overview of sustainability science, and distinguishes between a traditional disciplinary-based science *for* sustainability and a transdisciplinary science *of* sustainability. He characterizes the former as being more of a basic science with descriptive and analytical properties, while the latter is more reflexive and on a meta level. Funtowicz and Ravetz (2008) argue that transdisciplinary research and PNS are a complementary pair, where a variety of perspectives are taken into consideration. Colucci-Gray, Perazzone, Dodman, and Camino (2013) highlight that a progression from multi disciplinarity to inter- and transdisciplinarity in the pursuit of theoretical pluralism. In contrast to multi- and interdisciplinarity, where the main focus is on the disciplinary thinking, transdisciplinarity can bring focus on dialogue and different forms of knowledge and by this, an ontology of multiplicity and differences (Colucci-Gray et al. 2013). Transdisciplinarity "combines epistemological reasons associated with incompleteness and contingency of knowledge with political and ethical arguments" (p. 136). Renert (2011, p. 25) emphasis transdisciplinarity as a decentralized network of specialists working together towards common goals. Beilin and Bender (2011, p. 159) included PNS in the curriculum in an environment degree and found that educators moved from a multidisciplinary approach, where each had their own methods of handling particular aspects of problems, towards an interdisciplinary approach with focus on how these different views could enhance a better understanding and a holistic picture. This is supported by Colucci-Gray et al. (2013, p. 145), who also found, in such an approach, the importance of meta-reflective competences and inter- and transdisciplinary knowledge (they suggest e.g. unchaining concepts from the rigid framework of a discipline, and recompose them into a complex whole).

Real-world problems are not necessarily wicked problems, but perhaps it could be important to include elements of wicked problems, giving students opportunities to meet problems where there is no right or wrong, and no obvious solutions. The characteristics of complex real-world problems entail uncertainty, complexity, controversy, conflicting interest, risk and multidisciplinary thinking. Funtowicz and Ravetz (2003) suggest transparency of processes in dealing with these matters: how one communicates; how given policy problems are structured; which assumptions and ethical positions are taken; which scientific perspective one holds; and, which interests and values are considered. If education can embrace and implement these dimensions of real-world tasks in the mathematics classroom, students can reflect on these issues in a multi-perspective environment that the classroom can represent.

Pupils' role in society (2)

A legitimate question is to ask why students should deal with real-world problems in mathematics classrooms, or rephrased: *why should students (be prepared to) contribute in complex problems in society*? In this area, I emphasize in particular two identified concepts, namely **critical citizen** and **extended peer community**. The first originates from CME, the latter from PNS.

Critical citizen: citizenship can be defined as "the knowledge, skills, values and dispositions of citizens" (Cogan 2013, p. 4). A critical citizenship involves responsibility, collaboration, concern for social justice, and a motivation to change society (Dejaeghere & Tudball 2007). A critical citizenship relates to young people's ability to critically reflect and engage in their own lives. Many authors from CME emphasize critical citizenship, and Skovsmose (1994, p. 59) asks if mathematical competences could "provide students with a competence fundamental to a critical citizenship?". He suggests mathemacy as a precondition for this social and cultural emancipation. Mathematics education dealing with problems important for students' life can turn into something more than a subject taught at school. Furthermore, Skovsmose emphasizes three aspects of reflection, important for a critical citizenship: reflection *on*, *with* and *through* mathematics. Similar thoughts are seen in Ernest (2010), who suggests moving away from a traditional mathematics curriculum focus on content, and towards a perspective that empowers students. He relates critical citizenship to "social empowerments through mathematics", and calls for a mathematics education enabling students "to function as numerate critical citizens, able to use their knowledge in social and political realms of activity, for the betterment of both themselves and for democratic society as a whole" (p. 23). Hauge et al. (2015) highlights critical reflections with students on societal issues (e.g., climate change) as a possible way to learn to critique and to be(come) critical citizens. They emphasize reflective knowing and controversies. By using Skovsmose's steps of reflection as a framework for analysis, they suggest that reflective thinking about *the mathematics* is necessary for reflective knowing, and that a key capability for critical citizenship is to question the data, model assumptions, and the perspectives involved (pp. 1581-1582). From the PNS perspective, Funtowicz and Ravetz (2008, p. 364) mention that "increasingly, 'consumers' become critical citizens". From an environmental perspective, this can have different impacts, such as demands for different products that do not emit CO_2 throughout their life cycle (cars, energy supply, etc.).

Extended peer community: an extended peer community (as argued in the introduction), is an important element in PNS, and Funtowicz and Ravetz (1993, p. 747) suggest that an extended peer community should "…include all with a stake in the product, the process, and its implications both local and global". This can also include students, who can ask critical questions, and open-ended questions that start with *what if*, and *what about*, and could, through participation, develop ownership to the problem (Funtowicz & Ravetz 2008, p. 365). They further emphasize that mutual respect and a range of perspectives permit development of a genuine and effective democratic element in science (1993, pp. 741-742). This implies that students' voices in wicked problems have a say, and are considered as valued contributions. Different extended peer communities already exist, and are called a variety of names. Examples are citizen juries, consensus conferences, focus groups, consultations and planning cells, and these can consist of both experts and lay people (Levinson 2010, p. 87). They can involve knowledge from experts (e.g. advisory boards, hearings, internet), or lay knowledge (e.g. steering committees, advisory board, consultation processes) Spangenberg (2011, p. 284). Furthermore, they can have different forms and powers, but they all assess the quality of (scientific based) policy proposals (Funtowicz & Ravetz 2003, p. 7). Though they may already exist, they are not always taken seriously, and sometimes are seen as shallow political correctness (Ravetz 2012). Extended peer communities can be internal or external: the former "involves expert elicitations where multiple disciplines work together on the assessment of quality and uncertainty", while the latter "is the inclusion of representatives from all relevant stakeholders in the processes of problem framing, choices of indicators, and quality assurance" (Kønig, Børsen, & Emmeche 2017, p. 3).

Kønig et al. (2017) identified 33 prototypical norms and values in documents that strived to practice PNS, such as: sustainability; quality; awareness; honesty; responsibility; and, equity. They argue that some of the underlying norms of an extended peer community are robustness, inclusiveness, and democratization of scientific expertise, and that these norms are interrelated and important for how trust can be established by dialogue through an extended peer community. They suggest TRUST as an acronym (Transparency, Robustness, Uncertainty management, Sustainability, and Tolerance), where all the identified norms and values are included and interrelated. Although this in meant as a reflective tool in science advice, I also see potential for this as a tool for educational purposes, with the same reflective goals, both for research purposes and in an educational setting.

Hulme (2010, p. 305) addresses three important questions for an extended peer community: who participates and how do they gain entry?; how is trust established within the community?; and, how does the extended peer community contribute to a democratisation (of science)? He reflects on these questions using Al Gore's environmental focus as an example, and looks at the different actors involved (e.g., Gore, IPCC, film-team, judges, experts, parents and school leaders). They can contribute with extended facts, such as local knowledge, critical and ethical considerations, investigative journalism or leaked documents (Funtowicz & Ravetz 2003). Hauge and Barwell (2017) argue that mathematics education can prepare students to participate in extended peer communities, and they raise some important questions as to what kind of competences one needs in order to participate. Participants need to understand and interpret complex scientific information, and they need mathematical literacy to understand and interpret data, graphs, statistics, probability, but also in order to be aware of the uncertainty and value-perspective of the problem.

Though critical citizenship and extended peer community perhaps reflect two different fields, they are interrelated. One might question whether one depends on the other: does an extended peer community rely on critical citizens, and does the extended peer community gives opportunities for critical citizenship?

Competences recognized as important (3)

Many people today celebrate and trust the rationality of mathematics. It is, however, important to question mathematical rationality. Skovsmose (2006, p. 267) asks: “Could mathematics education bring about competences which can be described as empowering, and as supporting the development of mathematical literary or a mathemacy, important for the development of critical citizenship?”. This is an important question, and leads to another, namely *what kind of competences do pupils need when reflecting on complex real-world problems in the mathematics classroom?* In the literature, there are many attempts to answers this. Here I will emphasize five concepts identified as important, namely: **mathematical literacy; reflective knowing; critical agency; critique;** and, **dialogue**.

Mathematical literacy: the definition from PISA states: “The capacity to identify and understand the role that mathematics plays in the world, make well-founded judgments, and use and engage with mathematics in ways that meet the needs of one’s life as a constructive, concerned and reflective citizen” (OECD 2003, p. 24). This emphasizes both identifying and understanding, and connects to a reflective citizenship. The term also involves social aspects, such as environmental and cultural awareness, competences to read socio-political settings as being open to change, and an evaluation of mathematics itself (Jablonka 2003 2015). The definition from PISA is not strongly connected to the notion of critique, therefore, I draw attention to mathemacy, which takes the element of critique into consideration to a greater extent. Skovsmose (1994, p. 27) states that mathemacy “…has to be rooted in the spirit of critique and the project of possibility that enables people to participate in the understanding and transformation of their society and, therefore, mathemacy becomes a preconception for social and cultural emancipation”. This dimension to competences required in or out of the classroom, is rooted in Freire’s term literacy as something more than just being able to read or write. Freire (1970) suggested that students needed to be thought of as conscious, thinking beings. He urged teachers to consider how students can change their social situations, and to reflect on social inequalities. Skovsmose (1994, p. 94) has embraced this, and defines mathemacy as “a capacity of making responses and as reading the world as being open to change”.

Several other authors have used related concepts, some of them described in the following. Jablonka (2015) elaborates on the evolvement of numeracy and mathematical literacy as models for curricula; numeracy can range from a narrow definition (basic calculation and arithmetic skills), to a wider definition like mathemacy. One of the aspects highlighted concerns critique and how this can be used as a tool to “…deconstruct, subvert and ultimately replace mainstream curricula” (p. 602). D’Ambrosio (2010 2015) uses the concept matheracy, and connects this competence to a deeper reflection about man and society. He suggests a trivium of literacy, matheracy and technoracy as providers of communicative, analytical and technological instruments for life in the twenty-first century. Gutstein (2012, p. 5) uses the concept critical literacy in mathematics, "to approach knowledge critically and skeptically". He furthermore elaborates on the relationships between ideas, underlying explanation, and questioning interests in play. Frankenstein (2010) refers to Freire when using the term 'criticalmathematical' literacy, and connects this to the desire "to read and re-write the world". She further develops this to criticalmathematical numeracy and relates this to a need for "*real* real-life word problems", and suggests several examples of how to use numbers, calculations and word-problems. She emphasizes that the main goal is not to understand mathematical concepts better, but rather how to use mathematical ideas to improve our society (pp. 1-2). Goos et al. (2014) argue that although rich definitions of notions such as mathematical literacy are useful, more direct guidance for teachers is necessary when planning for instruction that aims for such competences. They suggest a model for planning and reflection which emphasizes real-life contexts, mathematical knowledge, tools, dispositions, and a critical orientation. In a collaborative partnership with teachers, one of their findings was that development of critical orientation occurred to a lesser extent in the classroom and that teachers "…express low confidence in this aspect of their practice" (2014, pp. 100-101).

Reflective knowing: Skovsmose (1994) uses the notion 'knowing' to point toward a process of changes, avoiding using knowledge which is more associated with concepts such as truth, absolutism and authority. He describes three types of knowing: mathematical; technological; and, reflective knowing. Mathematical knowing is the mastering of algorithm and theorems as perhaps seen in "the traditional" mathematical classroom, and technological knowing is "the ability to apply mathematics and formal methods in pursuing technological aims" (pp. 100-101). Reflective knowing is "the competence needed to be able to take a justified stand in a discussion of technological question" (pp. 100-101), and are considered as important when reflecting on personal and sociopolitical matters. Hauge and Barwell (2017) highlight reflective knowing to be the most crucial competence (of the three knowings) when citizens participate in extended peer communities, "…in order to be able to critically engage with the information they receive or generate for themselves" (p. 7). They suggest that students learn about how uncertainty is interwoven in wicked problems, and of societal effects of different ways of handling uncertainty, and suggest several examples of how climate change can provide opportunities for mathematics education to address uncertainty through reflective knowing.

Barwell (2013) draws some parallels between Skovsmose's three kinds of knowing, and PNS. Mathematical and technological knowing can be a small-scale analogue to applied science and professional consultancy, and reflective knowing is the "small-scale analogy of post-normal science" (Barwell 2013, p. 12). In Figure 4 (see below), I have visualized this, and combined: the PNS-diagram from Funtowicz and Ravetz (2003) (Figure 3); Skovsmose's three knowing's; and, the three types of uncertainty highlighted by Hauge and Barwell (2017). Although Figure 4 illustrates some analogy between different concepts and perspectives exposed by Barwell (2013) and Hauge and Barwell (2017), it is important to emphasize the overlap between some of these, such as the mathematical and technological knowing.

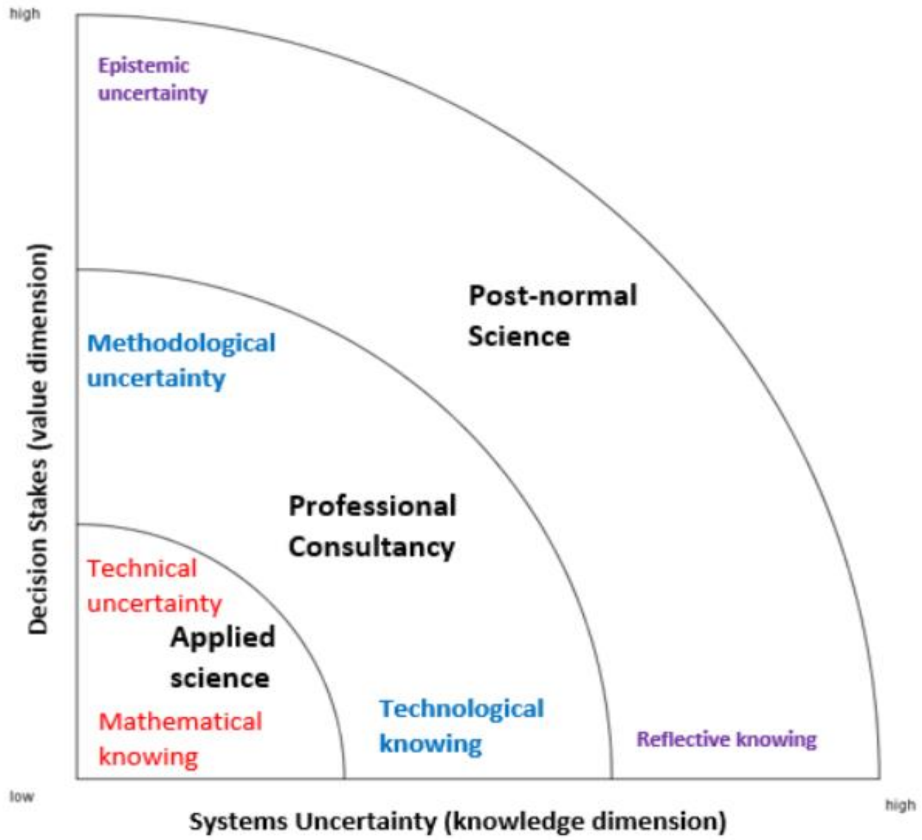

Figure 4: PNS model – Modified with Skovsmose's three knowings (c.f. Barwell 2013), and three types of uncertainty (Hauge & Barwell 2017).

Critique: Skovsmose (2011, p. 19) summarises a modernist conception: "a critique should prepare the ground for obtaining *certainty* and *truth*; it should be an expression of *rationality*; and it should include a search for the proper *foundation* of knowledge". This definition of critique as the pursuit of truth, certainty and foundations has been challenged by several, and Skovsmose and Greer (2012b, p. 369) question whether it is time to move away from providing critique with certainty, and forward to more post-modern conditions. They elaborate the notion of critique as a logical, epistemological, political and certain endeavour, and interpret the notion with respect to theories of Descartes, Kant, Marx, and Modernity respectively, and argue that they all unite in attempts to form an alliance with certainty. Skovsmose (2011) sees this as a possibility for opening to different approaches in CME.

He highlights critiquing in terms of imaginations as important to reveal that situations can be different from the present (Skovsmose 2016). Similarly, Pais, Fernandes, Matos, and Alves (2012) highlight that critique originates from the Frankfurt School and Freirean Pedagogy and had strong social and political aspects; it has lost its most radical meaning in the sense that critiquing is now done within the frame of the system. They exemplify this by stating that CME is only possible within the confines of the capitalistic school's credit system, as long as it is not undermining the foundations of the system (p. 32). In the field of PNS, Funtowicz and Ravetz (2003, p. 2) suggest that the search for the "truth as the goal of science is a distraction, or even a diversion from real tasks". Instead they propose the search for quality, as a guiding principle, and "as a contextual property of scientific information".

PNS has itself been criticised for embracing a non-scientific perspective (e.g., by stressing the uncertainty in climate change), perhaps partly due to the critiquing element of PNS (Turnpenny 2012). Another critique of PNS highlighted by Turnpenny (2012), concerns how anecdotal evidence can give value to the problems. Goeminne (2011, p. 629) also critiques PNS by questioning if science ever has been normal, and by arguing that normal science is a constructivist account of science, but Goeminne agrees there is a change from the "…dispute of truth of scientific answers to a discussion about the way science frames its issue". Hauge and Barwell (2017, p. 3) emphasize the role of critique in CME in how students can engage in social, environmental or economic problems, and argue for both learning mathematical methods and about which roles mathematics plays in these situations (both in creating, identifying and how to reflect and discuss the problems).

Critical agency: Brantlinger (2014, p. 201) understands critical agency as "…the sense that students can fight for justice and make a difference in the world". Skovsmose and Greer (2012b) use the term critical agency, as a generalization of mathematical agency. The latter can be seen as how students can improve their life situation (social empowerment) and how they consider their role in creating and evaluate knowledge (epistemological empowerment) (Stemhagen 2009). By emphasizing the critical part of it, Skovsmose and Greer (2012a 2012b) draw attention to the political aspect of mathematics education, and the importance of critiquing mathematics education. The dialectic relationship between reflection and action is emphasized as being a crucial part of this, with references to Freire "reflection – true reflection – leads to action" (p. 6 & 382). Relatedly, Gutstein (2006, p. 334) suggests developing social agency as a "beginning step in writing the world" and developing a "sociopolitical consciousness" as "reading the world (with mathematics)". This was a part of normalizing politically taboo topics in the classroom. This can mean being able to understand, examine critically, and to act where mathematics is interwoven (visible or not) with one's immediate life or society in general (Gutstein 2012). This understanding of the formatting power of mathematics (see main area 4), is crucial in critical agency.

Dialogue: dialogue is considered as important in all of the above competences, but it can also be considered as a competence of its own. It is frequently mentioned in the literature, both of PNS and of CME. Skovsmose (2016, p. 11) emphasizes the important role dialogue has in CME, with inspirations from Freire, and it is considered as "…one way of developing broader critical competences related to mathematics". In PNS, dialogue can be seen as an attempt to integrate and reconcile different sorts of knowledge from different fields, and can have a variety of forms. Some of these include methodologies for structuring dialogue with an extended peer community, such as community research and democratising technology (Funtowicz & Ravetz 2008). A characteristic of a dialogue in a PNS perspective is "negation in good faith" (Ravetz 2007, p. 278). In addition, Kacem and Simonneaux (2009) highlight argued debates as a positive contribution towards a scientific citizenship, and suggest controversial issues to motivate pupils to engage in socio-scientific issues. Colucci-Gray et al. (2013) claims by offering students opportunities for dialogue, they can express and evolve personal opinions and through this also fulfil their role in society. An inclusive dialogue, instead of being two fronts of rights and wrongs facts, can be a tool for different legitimate perspectives to be elaborated. Colucci-Gray (2009) mention two different perspectives of communication in educational context: discussion and dialogue. The first can be seen as "to shake and break something apart", while the latter is "a verbal exchange between two or more people involved in conversation" (pp. 194-195). Colucci-Gray et al. (2013, p. 129) emphasize "the linguistic reflections that can enable epistemological awareness", by e.g., seeing metaphors (such as ecological footprint, mother Earth, the Earth as a goddess versus as a machine) as important in the process of knowledge production in science.

Alrø and Johnsen-Høines (2010) reflect on a critical dialogue when developing critical democratic competences in student teachers. They highlight several aspects of dialogue, e.g. the evaluating approach of the traditional mathematics classroom, dialogue with elements of inquiring and cooperation, and the notion on subject-based reflective dialogue. The latter is an "educative approach that aims to explore how the situation might generate discussion for further development; hence, it has a future-oriented perspective" (pp. 13-14). Omuvwie (2015, p. 59) suggest the aim of students and teacher-dialogue guided by "…a combination of *context, meaning, the mathematics in action and critique*". He uses Bakthin's (1981) dialogic philosophy, and sees dialogue as an on-going and social process of opposing concepts and meanings in order to develop students' critical mathematical thinking.

Wals and Schwarzin (2012) has researched how dialogic interactions can be a key mechanism for supporting group learning when wicked problem of sustainability is dealt with in society. Several important aspect of dialogue was highlighted, such as diversity of perspectives, mutual respect and trust, emphatic listening, assertive but not aggressive, anticipatory thinking, but also phases of conflict and instability was seen as important (pp. 17-20). Furthermore, Frame and Brown (2008, p. 226) emphasize that dialogue is an "irreducible plurality of perspectives and modes of understanding", and not reducible from a combination of many voices into a single consensual view. This agrees with perspectives of CME, and Hauge and Barwell (2017, p. 8) argue that if the dialogue in mathematics education is about teaching facts and procedures, this is insufficient. Instead, they highlight the notion of dialogue as what Skovsmose refers to as knowledge conflict and negotiation, and leave behind any conception of the homogeneity of knowledge. Hauge and Barwell (2017, p. 8) furthermore link dialogue in CME with "issues of uncertainty and the need for negotiation through extended peer communities", and suggest this type of dialogue as a classroom activity.

The competences recognized as important in this main area are by no means exhaustive. They do have some common features and some differences, but are perhaps not the competences traditionally associated with mathematics. Then again, neither are wicked problems.

Aspects of democracy (4)

From the included items, some concepts and perspectives stood out as significant from the perspective of democracy. These concepts can to a certain extent be an answer to the question: *Why is it important to bring real-world problems into mathematics education from a democracy perspective?* Some of the concepts that were frequently mentioned, were: **power; formatting power of mathematics; responsibility; ethics; values and morals; and, democratization**.

Power: the connection between mathematics and power has been emphasized by researchers, e.g., Gutiérrez (2013); Skovsmose (1994); Valero (2009). Valero (2009) asks (rhetorically) what power has to do with mathematics education, and how it relates to democracy and political affairs. She reflects on three different notions of power: the intrinsic power of mathematics; power as structural imbalance of knowledge control; and, power as distributed positioning. The first notion deals with how society considers mathematics as empowering for both individuals and society, and therefore crucial to master. Valero problematizes this, and argues that this conceptualization does not bring us further in a modern understanding of society. The second notion concerns who controls the knowledge, how society is class-divided and unequal, and the cultural aspect of mathematics and society. Also this notion has challenges, one is to give importance to the dissonance between mathematics education, power and democracy (Valero 2009). The last notion sees power as a relational capacity, where actors can actively choose different resources, and by this, move between different positions. This view on power is more dynamic, not as a permanent characteristic, and can also be considered as both a constructive and destructive force (Valero 2009).

Gutiérrez (2013, p. 40) uses the term sociopolitical turn to refer to "… a growing body of researcher and practitioners who seek to foreground the political…", which signals a shift in how one sees "…knowledge, power, and identity as interwoven and arising from (and constituted within) social discourses". She highlights two major constructs of power in relation to mathematics education: the power of mathematics; and, the power associated with being successful in mathematics (p. 46). The former concerns a conception of mathematics as having a privileged position of truth, while the latter can function as a gatekeeper for those who master mathematics well enough. Gutiérrez (2013, pp. 48-49) emphasizes that the sociopolitical turn, in mathematics education, has challenged these (and other related) concepts of power, moving away from a rational and universal logic, and toward something "being created discursively through practices".

Kollosche (2016, pp. 75-76) provides an overview of how Foucault's ideas have inspired mathematics education, and highlights how knowledge, subjectivity and power are interrelated. Furthermore, he suggests a framework for research on socio-political studies in mathematics education where this interrelation can be explored, and how thoughts from critical theory can be broadened with ideas from Foucault. Mathematics and power can also be seen from another perspective, and this leads me to the notion of the formatting power of mathematics.

Formatting power of mathematics: Yasukawa, Skovsmose, and Ravn (2012, p. 268) highlight that the idea of the "formatting power of mathematics" is that mathematics itself is neither value-free nor neutral; mathematical thinking affects how we view, interpret and negotiate our surroundings, and in many ways (sometimes invisible) shapes our society. Furthermore, they connect the formatting power of mathematics to the notion of mathematics in action, which they understand as how mathematics influences, is influenced by, and interacts with, society. Skovsmose (1994, p. 207) states that "…mathematics has an important social influence; it follows that to understand this formatting power becomes an essential aspect of critical mathematics education". It is a challenge for mathematics education to enable students (as citizens), to de-mask this sometimes invisible "pervasive social influence" (p. 82). Ernest (2010, p. 23) suggests that being a critical citizen involves "critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems", and that "Every citizen needs to understand the limits of validity of such uses of mathematics, what decisions it may conceal, and where necessary reject spurious or misleading claims". He argues that this capability "is a vital bulwark in protecting democracy and the values of a humanistic and civilized society" (p. 23).

As our society has been more and more mathematized, the amount of implicit mathematics embodied into different objects (realized abstraction) has increased accordingly, and this has led to a demathematization of the objects (Gellert & Jablonka 2009). Gellert and Jablonka (2009, p. 23) emphasise that demathematization has impeded citizenship, in that it is has become difficult to unmask the "frozen" mathematics behind objects in society. They refer to notions from Skovsmose of the constructors, the operators and the consumers, with the first being the ones exercising power over the other two, since they are the ones in control (2009, p. 23). Similarly, Straehler-Pohl (2017, p. 38) highlights the phenomenon or notion of de|mathematization, to show the dialectical relationship of mathematization and demathematization, and suggest that the entanglement of this phenomena with capitalism should be on the agenda in a mathematics classroom. Hauge and Barwell (2017, p. 6) connect the idea that mathematics formats our society with the ideas of PNS and the "uncertainty in the problem framing". They exemplify this by saying that our understanding of climate change is mainly built on mathematical analysis and models (associated with uncertainty), and question how we transform these descriptive analyses to prescriptive ones.

Responsibility: Skovsmose and Greer (2012a, p. 4) argue "Of profound political importance is the challenge to mathematicians and mathematics educators to accept ethical responsibilities". What might such an ethical responsibility look like in mathematics education? In matters of climate change, there is a duality with respect to responsibility, where some consider it to be an individual responsibility and others consider government or other countries as the ones with the collective responsibility to act. In the latter way of thinking, responsibility then can get detached and depersonalized. Atweh (2012) highlights the complex relation between democratic participation and mathematics education, and argues for a socially response-able approach. He suggests an ethical perspective that complements the existing perspectives, and address three complexities in the relationship between mathematics education and democratic participation: "the uncertainty in the relationship, the question of power, and the elusive nature of democratic participation in globalized pluralistic times" (p. 325). Some of these complexities are: intrinsic resonance/dissonance between mathematics and democratic participation; the politicization of mathematics education; and, our understanding of modern democratic participation in a globalized and pluralistic society. Furthermore, Atweh (2012, p. 331) emphasises an understanding of an ethical/responsibility perspective not as a set of rules of behavior, but as an inescapable responsibility towards others. This also needs a shift in how mathematics is being perceived in education (e.g., curriculum changes, content-task versus problem solving, modelling and real-world activities, etc.). Atweh and Brady (2009) argue that ethical responsibility can provide a common ground for students' and teachers' engagement in socio-political matters. Ernest (2009, p. 211) highlights that "social constructivism regards mathematics as value-laden, and sees mathematics as embedded in society with social responsibilities".

Kønig et al. (2017) claim that responsibility and safety can no longer be conceptualized in terms of control and predictability, due to the extent of uncertainty that permeates our lives, and they argue for a redefined responsibility and safety to cohere with the norms of adaptability and honesty. The notion of responsibility can then be understood as “a commitment to do good and preserve and extend humanity” (Kønig et al. 2017, p. 10). An example of this is when scientists give policy advice that will preserve humanity. Renert (2011) argues for recognizing the responsibility mathematics educators have with respect to preparing students for future challenges such as environmental problems. He proposes a stage model of approaches to teaching about sustainability in mathematics education, which consists of: accommodation (education *about* sustainability); reformation (education *for* sustainability); and, transformation (education *as* sustainability). McGregor (2013, pp. 3563-3564) raises a critique of education *for* sustainability as stated by the United Nations (UNESCO 2006); the critique involves concerns such as indoctrination of certain values and ideas, an anthropogenic orientation, and the role of education as the saviour of sustainability. McGregor (2013) identifies seven approaches to global problems of unsustainability that challenge prevailing thoughts about normative concepts of sustainability, and seven overarching alternative messages for sustainable education.

Values, ethics and morals: dilemmas of value, ethics and morals are often considered as separated from mathematics education, that is, if one considers mathematics education to be value-free and neutral. Gray and Bryce (2006, p. 179) refer to several findings showing that teachers avoid confronting political interest and values, and feel unable to discuss ethical issues in the classroom. Real-world problems, unlike “traditional” mathematics tasks, can surface actuality, to consider ethics and values as important parts of mathematics education. Several have argued for this, e.g., Atweh and Brady (2009); D’Ambrosio (2010); Ernest (2012). Ernest (2009, p. 215) calls for more ethics in mathematics education, to acknowledge the social responsibility in mathematics. Boylan (2016, p. 401) suggests reflecting on ethical issues in mathematics education through four dimensions: “relationships with others, the societal and cultural, the ecological and the self”. Due to the complexity of real-world problems such as climate change, they can bring up ethical reflections in several dimensions. The concept of values gets a special emphasis in the field of PNS: values and facts are brought together into a unified conception. By doing so, problems that are value-loaded and involving uncertainties, can be managed through a plurality of legitimate perspectives which are mutually respected (Funtowicz & Ravetz 2003). Van der Sluijs et al. (2010, p. 413) highlight that “climate policies can be justified in moral terms without any need for recourse to abstract climate or economic models”. Similarly Rommetveit, Funtowicz, and Strand (2010) suggest an ethics of knowledge and action to meet the challenges appearing in problems such as the climate change, e.g., the lack of effective action and agency because of alienation. They argue that the issue is presented in a scientific context, detached from people in communities, and that due to lack of technological fixes, citizens need to get involved in a civilisation change. Furthermore, this should focus on what is happening in the present, rather than the future, with, e.g., such as excessive use of limited and non-renewable resources, extinction of living species, and pollution of air, soil and water.

De Freitas (2008) highlights the ethical dimension in problem solving in mathematics education, and suggests an emphasis on plurality of perspectives being explored by students, including ethical and environmental considerations. One of the aspects she highlights, is an “attention to the language of uncertainty…”, with key words such as probably, almost, speculation, and less emphasis on that certainty that normally is associated with mathematics. Furthermore, she suggests a method of “think aloud” so as to model humility and to “…help students embrace/celebrate a language of uncertainty…” (2008, pp. 89-90). Wicked problems can be polarized in the public debate. Ravetz (2012, p. 146) suggests approaches such as “reconciliation and non-violence” in such debates, in order to respect contested views in controversies, as a practical and effective way to strengthen the quality of problems.

Mathematics education, democratization and global society: the democratization of science that critical citizens in an extended peer community may contribute to, can destabilize knowledge as much as legitimize it (Hulme 2010). Yasukawa et al. (2012, p. 265) argue that although the relationship between mathematics and society has implications greatly beyond the classroom, there has been relatively little attention from sociology in order to understand this relationship compared to, e.g., science and technology. Many mathematics researchers argue that mathematics education is crucial for the socio-cultural, political and economic scenario of our global society (Sriraman, Roscoe and English 2010). Sriraman et al. (2010) question if politicizing mathematics education has gone too far – or not far enough – and how CME can improve the democratization of society. They highlight how mathematics is both a means for empowerment as well as a means to oppress, and how it can be both used as a democratizing force for improving, and as a vehicle for maintaining the current situation (p. 623). They further emphasize how “…mathematics education has much more to do with politics, in its broad sense, than with mathematics, in its inner sense” (pp. 627-628), and this would have important consequences for how educators perceive pedagogy in mathematics education, moving away from a more dominant “…’industrial consumer state’ model of education”.

The concepts identified in this main area, are again perhaps not the ones traditionally associated with mathematics classrooms, but they are recognized as important in the fields of CME and PNS, and I find them crucial for reflecting on wicked problem as a critical citizen in an extended peer community.

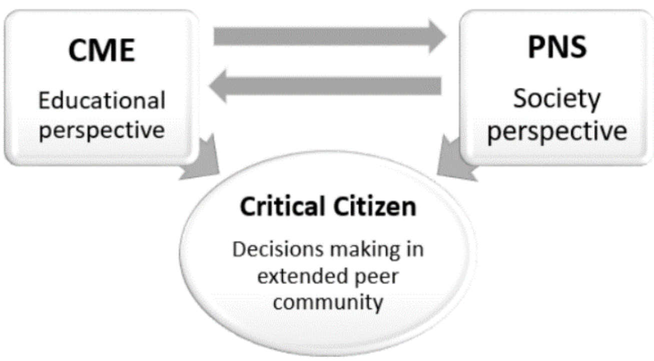

Figure 5: Concepts from CME and PNS contribute both to develop thinking about the concept of critical citizenship, but also develop each other’s conceptual and theoretical perspectives.

Concluding remarks

In this literature overview, concepts and perspectives from CME and PNS, important for a critical citizenship, have been identified and critically reflected upon, with both a philosophical and educational perspective. The concepts were structured in four main area with accompanying questions. The literature from PNS contributed to describing characteristics of real-life problems and pupils' role in the society, and through that, developed the philosophical perspective of CME; the literature from CME contributed to identifying competences recognized as important in a democracy, and through that, developed the philosophical perspective of PNS. The combined philosophical perspective contributes further to our understanding of how educational perspectives can influence mathematics classrooms towards a critical citizenship.

The overview presented in *Figure 2: Conceptual mapping of identified concepts and perspectives*, may work as a starting point when considering how mathematics education can continue to develop along with society. Although the figure portrays CME and PNS as two separated field on different sides, there are overlaps and connections between the concepts and perspectives. For instance, the concept of critical citizen from CME is strongly related to extended peer community from PNS. Furthermore, both fields emphasize embracing a pluralistic dialogue and critiquing with multiple perspectives. Both CME and PNS question how society deals with mathematical or scientific "facts": and while CME uses the notion of the formatting power mathematics, PNS argues for embracing uncertainty and not relying solely on "scientific fact", but including an extended peer community. The questions raised in *Table 4: Overview over four main areas*, may contribute to ideas on why, and how, mathematics education should/could deal with wicked problems. The perspectives and concepts identified in this literature overview (although not considered as a final and exhaustive list), can be useful for teachers, researchers, curriculum-designer and policy-makers by giving an overview of important questions and findings related to sociopolitical issues. The perspectives and concepts might help us consider what types of problems our students can learn to deal with, what role students and/or citizens could play in society, which competences are important when facing wicked problems, and why the perspectives of democracy can be important to include in mathematics education.

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