Why use a leek in mathematics teaching?

Teachers' choice of using practical activities

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From a system theoretically grounded perspective, a hierarchy of primary and secondary impact factors influencing the mathematics teacher's choice to use practical activities in mathematics teaching is initially suggested. This study, based on qualitative responses from acknowledged teachers of mathematics, then gives grounds for suggesting that a hierarchy of impact factors regarding the choice to use a practical activity in mathematics teaching must include more than just a binary clustering of primary and secondary impact factors. The teacher's everyday life experience, knowledge of pupils' everyday life experience, and conscience are examples of impact factors that call for the introduction of another level, as they are primary impact factors that depend on adaptation to responses at the secondary impact factor level.

Introduction

Practical activities[i], for visualisation, investigation and application of mathematics, are considered integral to school mathematics in Norwegian school policy documents (e.g., KD, 2006). The literature illuminates the influence of several impact factors on teaching (e.g., Shulman, 1987; Handal & Lauvås, 1987), and the teaching of mathematics in particular (Koehler & Grouws, 1992; Mosvold, 2005; Franke, Kazemi & Battey, 2007). However, the influence from elements, circumstances or qualities (here defined as *impact factors*) which may make the mathematics teacher choose to use a practical activity, like for instance to use a leek to concretise geometrically the algebraic deduction of the area of a circle or the essence of integration[ii], are less known.

Without discussing practical applications or the learning effects of practical activities, the aim of this article is to suggest a theoretically grounded hierarchy of impact factors that influence the mathematics teacher's priorities when choosing to use a practical activity. Next, it is an aim to identify discrepancies, in relation to the impact factors, between the theoretically grounded hierarchy and the experiences of mathematics teachers. Moreover, it is intended to discuss the ways in which any discrepancies call for an expansion of the suggested theoretically grounded hierarchy. In order to suggest a hierarchical approach to the issue of influence from impact factors that influence the teacher's choice of using a practical activity, a system theoretical point of view is used as a theoretical grounded approach. The possibility of a rather simplistic hierarchical categorising of clusters of impact factors seen through a system theoretical framework opens up for a necessary discussion of greater emphasis on some impact factors than others, based on qualitative, empirical data which consists of teacher experiences. Such an aspect regarding the issue of influence on teachers' choice of using of practical activities in mathematics teaching is not explicitly available in the current research. The research question is: *In what way do teachers' experiences call for an expansion of a system theoretically grounded hierarchy of impact factors regarding the choice to use practical activities in mathematics teaching?*

The use of practical activities in school, especially the uncritical and unquestioned use, has been criticised in the literature (Klette, 2003; Kjærnsli et al., 2004). The results of a comparison of experience-based and theoretically grounded hierarchies of impact factors will, in our opinion, have implications at two levels. Firstly, we believe that such a comparison will contribute to the understanding of just what levels of attention different kinds of impact factors ought to be given in the pre- and in-service education of mathematics teachers. This is likely to increase the quality of teaching with practical activities in school mathematics. Secondly, such a comparison will contribute to the discussion of what levels of reflection ought to be expected of teachers choosing to use a practical activity.

Theoretical background

From a *system theoretical perspective*, the totality of the teaching situation and the teacher's interaction with the surrounding conditions might provide understanding regarding the teacher's choices for his/her teaching. Within such a perspective, it is emphasised that the teacher interacts with a number of social systems. Thus, the common factor within social system theory is that the teacher is part of a system where he/she influences on the totality, and is influenced by this totality (Eide & Eide, 1996).

Relational communication theory, developed by Gregory Bateson (Eide & Eide, 1996), is based on the assumption that communication between participants establish and develop relations, and that the relations decide how the communication takes place. The communication consists of interaction and the interactional patterns make up the structure of the system (Littlejohn, 1992: in Eide & Eide, 1996). This interaction influences the system and the system influences the interaction. In the essay "The Logical Categories of Learning and Communication" Bateson (1972) links *learning* to the element of *change*. Through a logical division in levels of learning and communication, with the levels labelled 0, 1 and 2[iii], he suggests that the influence of personal features on learning processes should be organised hierarchically. With regard to learning, Bateson characterises learning at Level 0 as first-order learning, learning at Level 1 as second-order learning, and so forth. In the perspective of a practising teacher, Level 0 is about receiving, understanding and responding to signals and responses stemming from the experienced teaching, and the teacher's learning at this level will be about developing (more or less) automatic actions (or reactions) based on received signals and responses (Glosvik, 2000). Level 1 relates to the way the teacher acts, and is about changing actions to adapt to responses to the performed actions from other groups of people in the system constituted around the teacher. Second-order learning is thereby a revision of actions based on experiences provided by actions at Level 0, which again generate changes at Level 1, and consequently at Level 0. Level 2 will be influenced by the teacher's internal responses to experiences at Level 1 and consists of factors that control second-order learning. Hence, third-order learning is about the teacher's perceiving and interpreting of new experiences stemming from responses and learning at Level 1, with a subsequent development of alternatives that control changes in learning processes at this level. It can, for instance, be a subconscious change of the teacher's beliefs about how mathematics ought to be taught and learned, and about the teaching conditions necessary for such a change.

The teacher's understanding of possible choices and the realisation of a choice depends on in what way the teacher observes his/her surroundings (Nordahl, 2007). The teacher has to make choices, related to his/her surroundings, which are recognised with a high level of complexity. When the surroundings are experienced as complex, the experience and reflection based choices made by the teacher to a large extent will be about reducing this complexity (ibid.). If the teacher becomes more complex through more knowledge, experience, expectations and so fort, the surroundings will be experienced as less complex (Rasmussen, 2004). According to a system theoretical perspective the complexity of the situation may be reduced through an increase in the teacher's complexity. Bateson's (1972) hierarchical categorisation of learning and communication offers such a perspective to the teacher's aim of reducing the surrounding complexity, through the introduction of levels with increasing complexity on the personal venue. Change on a level will generate change on the lower levels in the hierarchy, and due to the nature of system theory at least some impact higher up in the hierarchy.

A theoretically grounded hierarchy of impact factors

Teaching is influenced by several impact factors, and attempts to identify and systematise such factors have been made on several occasions (e.g., Shulman, 1987; Koehler & Grouws, 1992; Veal & MaKinster, 1999). Some impact factors are considered to be more influential than others, but since the experienced relevance of an impact factor is quite subjective, varying from teacher to teacher, the impact factors that appear to prevail in influencing teaching vary. Nevertheless, it is possible to suggest impact at different levels. A three-level hierarchy of factors with impact on the teacher's use of practical activities in mathematics can be seen as parallel to Bateson's hierarchy. In order to accomplish this, one need to divide possible impact factors into two categories: *primary* and *secondary* impact factors. These two categories represent, respectively, those impact factors that are teacher related and those stemming from sources external to the teacher. The impact factors are both discrete and intersecting, and hence it may be somewhat indiscriminate to cluster them in only two categories. But to suggest a Bateson inspired, theoretically grounded hierarchy of impact factors related to the teacher or sources external to the teacher as the origin of each impact factor, specific identification of a possible impact factor is not called for. The hierarchy is merely a categorisation of impact factors designed to show how Bateson's 1972 logical hierarchy of learning and communication can be interpreted as applicable for describing the influence and the possible change of influence of primary and secondary impact factors on teachers' choices to use a practical activity in mathematics teaching.

Secondary impact factors (Level 0) - impact factors from sources external to the teacher

Several impact factors stemming from sources external to the teacher influence both the teacher (Ernest, 1989; Koehler & Grouws, 1992; Frykholm, 1999; Mosvold, 2005) and the teacher's choice of using a practical activity in mathematics teaching. The teacher is, to a limited extent, in position to influence impact factors such as populist theories about how mathematics ought to be taught, the official curriculum and structure of schooling and evaluation, or the physical environment. The teacher is in a better position to respond to other factors stemming from external sources by making changes to the teaching. Examples of these are structuring factors, such as the number of pupils, access to equipment that makes it possible to use practical activities, work pressure, or available textbooks. There may also be impact factors of a more relational kind, such as recommendations of in-service education in mathematics, suggestions and inspirations from colleagues, comments on teaching by pupils and parents, or the teacher's experiences with time constraints. As all these impact factors described here stem from sources external to the teacher, and are directed towards the teacher either as guidelines or frameworks for the teaching, or responses to delivered instruction it is appropriate to cluster them in one category - *secondary impact factors*.

Primary impact factors (Levels 1 and 2) - impact factors related to the teacher

The teacher's disciplinary knowledge, didactical knowledge, and beliefs about mathematics and its teaching are put forward as crucial parts of the teacher's professional knowledge (Shulman, 1987; Ernest, 1989; Koehler & Grouws, 1992; Frykholm, 1999). Critical influences on the teacher's planning and teaching stem from these *primary impact factors*. Such influence is based on responses given to a teacher's teaching and his/her reflections about these responses (Level 1). Change at Level 1 leads to change at Level 0, according to Bateson. Hence, change in the teacher's disciplinary knowledge, didactical knowledge, or beliefs about mathematics and its teaching may lead to changes in practice related to responses stemming from secondary impact factors (Level 0). The literature reminds us that any change of practice will be temporary if it is not anchored in change on a higher hierarchical level of influence (Lloyd, 1999). Hence, teachers' beliefs need to be influenced in order for teaching to change permanently (Wilson & Cooney, 2003). Furthermore, attempts to influence teaching practice in a direction that requires a more developed disciplinary and/or didactical knowledge than that already possessed exposes teaching practice to responses from external sources (Haara & Smith, 2009). This requires the teacher to have developed a profound understanding of the beliefs that have been given the prevailing influence, for instance through an intervention process (Haara & Smith, in press). This accords with Bateson's logical necessity of making impact on a higher level in the hierarchy in order for changes to emerge.

A higher level of *primary impact factors* can also be identified (Level 2). The influence of the teacher's own responses to change depends on the teacher's reflection and self-confidence regarding his/her disciplinary and didactically knowledge and beliefs. Changes at this level may be identified through changes in interpretations and acknowledgement of experiences that the current state of the primary impact factors on Level 1 generates in practice. Such a change could, for example, be a change in the teacher's opinions about how mathematics is to be taught. Consequently, this may initiate efforts to make change at Levels 1 and 0. To Bateson (1972), learning at this level can have an impact on one's character. Hence, teacher learning at Level 2 changes the teacher identity (Korthagen, 2004).

Visualisation of a hierarchical structure

Based on the preceding clustering of impact factor a system theoretically grounded hierarchy of impact factors (Figure 1) that influence a teacher's priorities when choosing to use a practical activity in mathematics teaching can be suggested:

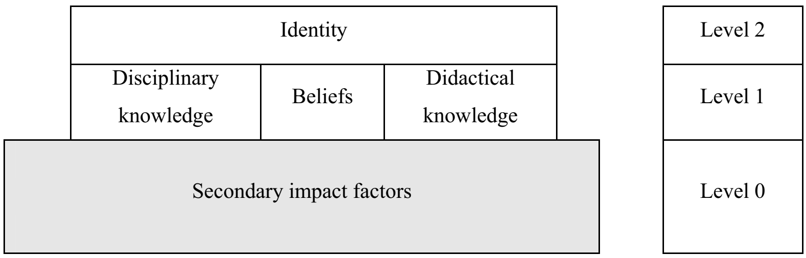

Figure 1: Suggestion of a theoretically grounded hierarchy of impact factors

Regardless of where an impact factor is recognised in this hierarchical structure, the derived impact on teaching practice varies. Hence, studies of the complexity of a specific impact factor and its implications for teachers' choices (for example, Sztajn's 2003 study of teachers' interpretations of the mathematical curriculum) may lead to various impressions of teachers' experience of the influence of the impact factor in question on teaching practice. However, by keeping the number of levels as low as possible, it is less likely to categorise any impact factor wrongly and one is able to pinpoint the qualitative difference between the origins of primary and secondary impact factors. The level of influence from secondary impact factors may be high, and this will probably vary for the individual teacher and from teacher to teacher. Hence, Figure 1 does not necessarily illustrate a hierarchy of significant qualitative relevance for teaching as planned and delivered. It is, for instance, always possible to ignore feedback and the context of teaching. Such attitudes to teaching, however, do not necessarily lead to development or enrichment of the teacher's professional knowledge, in the way Shulman (1987), for instance, suggests. Furthermore, they would hardly be conducive to changes through learning processes at Levels 1 and 2 in the Batesonian hierarchy, since the teacher in such instances does not allow change to occur.

The Study

The current study examines ways in which teachers' experiences with impact factors call for a hierarchy expansion, comparable to the Batesonian hierarchy visualised above. A qualitative methodology was chosen to allow mathematics teachers to present their experiences of how impact factors influence their choices to use practical activities in mathematics teaching. This provided an opportunity for an analysis that included the teachers' explanations and reasons. Data was produced through the responses of acknowledged teachers to an open questionnaire and structured interviews with two acknowledged teachers of mathematics[iv]. In Haara & Smith (2009), acknowledged teachers of mathematics are defined as *teachers who are viewed as competent mathematics teachers by the principal and earn respect from colleagues, pupils and other groups of relevance within the working environment*. Acknowledged teachers are often expert and/or experienced in the way these terms are defined (Berliner, 1986; Shulman, 1987), but they also add qualities to their positions through the positive impact they are recognised to have on their surroundings. They may, regardless of levels of formal disciplinary or didactical competence, or years of experience, have quite some authority with respect to eventual changes of direction or preservation of current directions in the content, methodology, and profession of mathematics teaching.

Participants

The recruitment of acknowledged teachers started with a randomised selection of three Norwegian counties. Next, a random selection of schools was carried out within the three counties. The selection process was monitored by an independent observer. For each county, a letter in the alphabet was randomly selected, and the principals of the ten first elementary schools starting with that letter, sorted alphabetically, were contacted by the researcher. Thirty principals were contacted. Each principal was informed about the study and asked to participate by recruiting from the school one teacher recognised by the principal as an acknowledged teacher of mathematics, and who would agree to respond anonymously to an open questionnaire. I wanted to recruit teachers from a group of potential informants whom I supposed could offer data relevant to the question of examining ways in which teachers' experiences call for an expansion of a theoretically grounded hierarchy of impact factors regarding the choice to use practical activities in mathematics. All the principals agreed to participate in the study, and acknowledged teachers of mathematics were recruited.

In addition, six (6) teachers initially recruited as acknowledged teachers of mathematics to another study on teachers' choice of practical activities (Haara & Smith, 2009)[v] were asked to respond to the questionnaire. The intention was to interview two acknowledged teachers of mathematics about hypotheses formulated on the basis of the analysis of data from the open questionnaire, and it was reckoned as important that these two interviewees were familiar with the questionnaire when being interviewed. To maintain the anonymity of the respondents' recruited by principals, the two teachers to be interviewed were selected from the group of six additional acknowledged teachers.

Each of the thirty recruited principals received an envelope containing the questionnaire, a letter of information to the principal, a blinded letter of information about the questionnaire, and a stamped envelope. The principal was asked to give the questionnaire and the blinded letter of information to the teacher. The teacher returned the completed questionnaire to the principal who then mailed the questionnaire to me. All thirty principals assured that they would do their best to support the study, but eleven (11) schools did not participate, in the end. Hence, I received twenty-five (25) completed questionnaires (70% response rate) including the questionnaires from the additional six teachers, as they had confirmed in advance their agreement to participate in the study. Their anonymity remained the same as that for the other nineteen questionnaires; they were instructed to follow the same mailing procedure as the principals.

The two interviewed teachers (called here Xantippe and Zoltan) were randomly selected among the six acknowledged teachers, and the selection was approved by an independent observer. Both teachers have worked as mathematics teachers in Norwegian elementary schools for more than 25 years each, Zoltan mainly in lower secondary school and Xantippe at the primary level.

The open questionnaire

The initial questions of the questionnaire examined the responding teacher's teaching of mathematics and personal definition of what characterises a practical activity. This was followed by questions enquiring about the influence of primary and secondary impact factors on the teacher's planning and actual teaching, and questions regarding the influence of secondary impact factors on primary impact factors. The last part of the questionnaire contained four questions about demographic facts concerning the teacher's age, formal education, and teaching experience. The questionnaire was validated by three researchers and three experienced teachers of elementary school mathematics, who read through the questionnaire and commented on the clarity of each question.

Analysis

The questionnaire produced hand-written answers from the responding teachers. The answers were structured and compared through a matrix (Grønmo, 2004) based on a systematic extraction of meaning of the answers to the questionnaires. A phenomenological analysis was conducted, in the sense that the analysis was based on the participants' experiences with impact factors influencing the use of practical activities in mathematics teaching (Grønmo, 2004; Kvale, 2006). The analytic approach was chosen to focus on an illumination of the participants' own impressions and interpretations of their experiences. This analysis showed that in addition to impact factors already labelled as primary or secondary, the teachers put emphasis on some impact factors that can be identified as *primary impact factors depending on responses from secondary impact factors*. Three such impact factors were identified: the teacher's everyday life experiences, the teacher's knowledge about the everyday life of pupils, and the teacher's conscience. They are beliefs and value-related qualities, and to a certain extent they are dependent on sources external to the teacher.

Interviews

Each interview consisted of four clusters of questions. Initially there was a cluster of questions regarding the influence of impact factors related to the teacher's use of practical activities. The main part of the interviews consisted of three clusters of questions, each with regard to one of three hypotheses formulated on the basis of the analysis of the questionnaires:

- *The teacher's everyday life experiences influence the frequency of possibilities for using practical activities in mathematics teaching.*
- *The teacher's knowledge about pupils' everyday life experiences influences the frequency of possibilities for using practical activities in mathematics teaching.*
- *The teacher's use of practical activities is related to the teacher's conscience.*

The structured interview was piloted with a teacher from the remaining group of four teachers not selected for interview.

Analysis

The interviews were fully transcribed. Extracts of meaning (Kvale, 2006) were identified from the transcriptions and interpreted to find points of support, or otherwise, of the proposed hypotheses. A hermeneutical approach (Grønmo, 2004; Flick, 2006) was then applied in the analysis of the data from the questionnaires and interviews, through a more holistic interpretation of the influence of impact factors. This increased the viability of the first interpretations made of data from the questionnaires. Hence, the second round of analysis increased the viability of the hypotheses formulated from the analysis of the questionnaires.

Findings

The attempt to distinguish between primary and secondary impact factors on a theoretically grounded basis implies a rigid bilateral classification of identified impact factors. The collected data confirm that mathematics teachers associate several distinct impact factors with influence on use of practical activities in mathematics teaching, and that the experienced influence from an impact factor is individually weighted by each teacher, regardless of the categorisation of that impact factor as primary or secondary. The following two sequences illustrate this interpretation:

> T1: Resources, materials, time + the number of teachers present. This might on occasion limit my use of practical activities.
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> T2: As I have a broad disciplinary knowledge in mathematics, it is always easy to use practical activities in teaching. You have to be discipline confident to rely on practical activities.

It is therefore not possible to compare an isolated experience of the influence of an impact factor to a hierarchical clustering of impact factors. This impression was confirmed by Zoltan and Xantippe through the interviews in, for example, the following:

> X: As a teacher I am facing great expectations. I am responsible to my superiors … I am supposed to work towards measurable aims [given through the national curriculum]. Hence, I cannot leave out anything, because then I would not be doing my job. But then it is up to me to find ways of teaching … if one wants to do practical activities, I do not see any problems in doing so, but I do have to make a plan [for the teaching] …
>
> Z: A factor which I have to consider when it comes to practical applications … is time. There are activities, I have to admit, that … I choose not to include in the teaching …. My experience says that the effect of some activities in proportion to the required time makes it difficult for me to defend any use of them.

The collected data did not challenge Bateson's idea of the necessity of influencing primary impact factors (Level 1) for changes to occur regarding the influence of secondary impact factors (Level 0) as a hierarchical consequence. The two interviewed teachers confirmed rather than discounted this through a specific emphasis on the importance of primary impact factors when deciding to use or not to use a practical activity:

> X: … [related to an example from a lesson outdoor on a windy day in which paper planes were made from folded A4-sheets] … it is about seizing the opportunity, right? One should not have everything planned before the lesson … but have the opportunity to see that … Wow! They [the planes] went

> straight up, and then backwards … may we then introduce negative numbers? You cannot be too caught up in the plans you have made…
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>
> Z: My experiences from years of teaching … dictate my decision of using a practical activity or not … in situations in which I see possibilities to do so and so…

Once again the relation to Bateson's interpretation of second-order learning is clear. Primary impact factor revisions are made based on experiences "with variations in the terrain".

The attention can now be turned to the three hypotheses proposed on the basis of the analysis of the data from the questionnaires, and their position and influence in the Bateson-inspired hierarchical visualisation.

Influence from the teacher's everyday life experiences

Data from the questionnaires indicates that teachers look, among available sources, to their personal experiences and interests for examples or activities by which to introduce mathematical content. One teacher (T3) saw numerous possibilities to combine his experiences from farming with mathematical topics, while another teacher (T4) stated that it is easy to use personal everyday life experiences:

> T3: I am also a farmer, and as a farmer, I apply practical mathematics fully. Today, many children do not participate in many such practical activities that are related to mathematics (digging ditches, carpentry, butchering, carrying water and firewood, and so forth).
>
> -----
>
> T4: It is easy to use personal experiences as examples when relating theory to everyday life, and thereby create a greater understanding for "why we need math".

In the interview, Zoltan focused on his interest in carpentry as a highly valued area for making relevant connections to the organisation of practical activities, to integrate mathematical theory and application. He was clear about the possibilities that were revealed in prioritising his everyday experiences from house building and arts and crafts:

> Z: … I actually use woodwork and carpentry on occasions. That is, I introduce old-fashioned carpentry to apply working drawings, and integrate mathematics in the work we do. And that is something that the pupils appreciate. We work with scales and other geometrical topics through an approach that is completely different from the theoretical approach we apply in the mathematics classroom.

Xantippe, on the other hand, was slightly more restrained regarding the possible influence of the teacher's everyday life experiences. She pointed out that it should not be required to have a specialised or rich level of experience from outside school to be a qualified mathematics teacher:

> X: I can see the point in having a hobby or a profession outside school, something that can be appropriate to bring into teaching, but it is not crucial. I do not see that teaching requires having … to do something special in your spare time to bring it into teaching. You can find topics and other stuff as well!

Influence from the teacher's knowledge about the pupils' everyday life experiences

Responses to the questionnaires show that the teacher's knowledge about pupils' everyday experience is recognised as an influential resource for the teacher. This counts in a teacher's efforts to get to know each pupil, and in this particular case, when relating mathematical content to practical situations and activities relevant to the pupils' experience outside school:

> T5: [In answer to a question about how the teacher experiences his/her didactical knowledge as influencing the choice to use a practical activity in the teaching] … I encourage pupils to solve the problems and to choose the approach to each problem themselves. They get to use their everyday experiences.

Both Xantippe and Zoltan confirm the importance of the teacher's knowledge about pupils' everyday life experiences:

> R: How important do you find pupils' everyday life experiences in the planning of practical activities?
>
> X: To me this is almost more important than anything, really. That we feel that they develop an ownership of what they do, because it is familiar to them.
>
> Z: … perhaps the … most important thing you can achieve in mathematics teaching, I would say, would be if the pupils could bring their everyday lives, their practical experience, into the mathematics class.

Influence from the teacher's conscience

Data from the questionnaires indicates that a teacher's bad conscience is a recurrent theme regarding the use of practical activities in mathematics teaching, especially in terms of the influence of the effects that ever-present dilemmas related to time constraints have on the conscience:

> T6: I experience several factors that prevent the use of practical activities; for instance, allocation of substitute teacher lessons, a lot of papers to mark, and little time for preparation.
>
> T7: A lot of factors have to be taken into consideration regarding the choice to use a practical activity. But the most important is, by far, time. Practical activities take time. The pupils are supposed to go through a huge quantity of mathematical content …. You do the math.

Zoltan and Xantippe also recognised conscience as an influencing impact factor that is almost always present when setting their mathematics teaching priorities:

> X: (… it was not so easy to do really, teaching a mathematics lesson outside. Maybe we as teachers had too little experience, so that … you had maybe just two pupils with you and the rest of the class was pretending to be Indians in the woods ….
>
> Z: I never have a good conscience, because it is always something that you should have done, but which you cannot do. If you use a lot of practical activities … I have done that sometimes, and I have thought that now I am really going to … and then I have arranged for a project, and spent much time on it, and then I have often felt that, Oh, my goodness! Did this produce any results at all? ... conscience rebounds that way! So you constantly experience dilemmas, it is part of the job.

Following his indication of a dilemma in relation to the impact of bad conscience, Zoltan continued by pointing to primary impact factors as an important counter weight against influence stemming from secondary impact factors such as time constraints. When time constraints prevent a practical activity, a secondary impact factor opposes the primary value-based impact factor that the teacher sees in the practical activity. The tension arising from the contradiction between the experienced time constraints at Level 0 in the hierarchy of impact factors and the Level 1 based wish to use a practical activity emerges as bad conscience. Hence, the influence of primary impact factors is necessary for dealing with both dilemmas and the considerable conscience challenges that may arise from choices made in the face of dilemmas about whether to use a practical activity or not:

> Z: …in a way I feel that there are nuances related to being a very professionally confident mathematics teacher … that is, I think that conscience, it can become intolerable, almost detrimental, if you are a mathematics teacher who is not disciplinary or didactically confident, because … I have experienced these dilemmas myself, that you can have a bad conscience no matter what you do … but this only gets worse if you are discipline or didactically unconfident. Then you will almost develop a bad conscience no matter what. You will, probably, if you use a lot of practical activities … you have been to courses and you have heard that it is the right thing to do … and then you do it, and realise that you are losing time, and feel badly about that … because you believe that you have done much wrong …. Or you can choose to keep up a steady pace getting through the whole textbook, and you cannot do [practical activities] … and then you get a bad conscience because of that. So I feel that disciplinary and didactical overview is extremely important in mathematics. In a way it is not enough to master what is presented in the textbook ….

Zoltan describes a situation that suggests that the teacher’s conscience is influenced by both primary and secondary impact factors, and that conscience as an impact factor leads to responses that influence both primary and secondary impact factors.

Discussion

The findings confirm that the teachers' everyday life experiences, their knowledge about the pupils' everyday life experiences, and their conscience are impact factors with a potentially high level of influence when it comes to using practical activities in mathematics teaching. Furthermore, according to the definitions of primary and secondary impact factors these three impact factors cannot be treated solely as neither of them. They are based on knowledge and beliefs, but are at the same time dependent on secondary impact factors, such as, for instance, pupils' qualifications and interests, time constraints, and curriculum demands. Hence, it would be necessary to make an expansion of the Batesonian inspired hierarchy based on teacher experience (Figure 2):

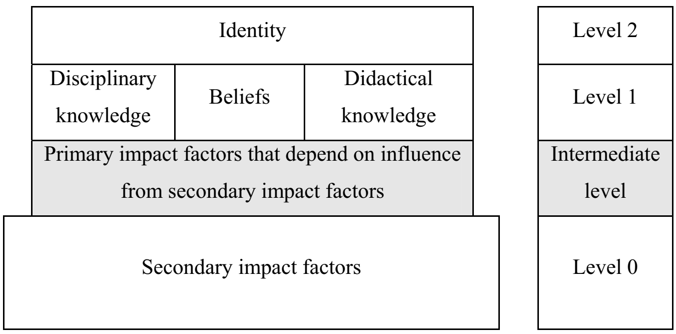

Figure 2: An influence-based expansion of the suggested hierarchy of impact factors

The intermediate level introduced in Figure 2 represents impact factors that in some cases can be identified as primary and in other cases as secondary. The inclusion of such a level requires the same width as Levels 1 and 2 in the hierarchy, since secondary impact factors are not affected by teacher-related impact factors. On the other hand the introduction of an influence based intermediate level in the hierarchy would abort the thought of aligning a Bateson inspired hierarchy of clusters of impact factors and teacher learning, if the level requires a separate order of learning. But since they are defined as primary impact factors that depend on influence from secondary impact factors both first- and second-order learning would in that case be recognisable.

The importance of the teacher's everyday life experience is a variety of tacit teacher knowledge, which to some extent have been overlooked when researching, reviewing, and suggesting models for teachers' knowledge (e.g., Fennema & Franke, 1992; Koehler & Grouws, 1992). The focus has been mainly on disciplinary and didactical knowledge components, in addition to teacher beliefs (e.g. Philipp, 2007). Gudmundsdottir and Shulman

(1987), on the other hand, call attention to the impact of the teacher's knowledge of topics that pupils find interesting as an example of pedagogical content knowledge influenced by knowledge about the students. The topic is also emphasised by Sowder (2007). For teachers with a developed professional knowledge in mathematics, everyday life experiences may represent a rich source of relevant situations and examples for teaching mathematics through practical activities. This might be the case for both the teacher's everyday life experiences and the teacher's knowledge about pupils' everyday life experiences, although these impact factors might influence instructional decisions in different ways. When it comes to what influences the choice to use a practical activity, the teacher's main attention must be at the pupils' perceptions of what is being taught. If the teacher refers to something pupils do not recognise or find interesting, the practical activity, for motivational reasons, will probably not generate the teacher's intended impression and effect. According to Xantippe, the teacher's attention to pupils' everyday experiences is a more important resource than the teacher's personal everyday experience. Pupils' experiences ought to be prevailing when it comes to what issues to emphasise when choosing to use a practical activity in mathematics teaching. At the same time this should not prevent the teacher from introducing mathematics through practical activities based on experiences with which the pupils are unfamiliar. After all, it is the school's responsibility to provide the pupils with knowledge and skills relevant to life. This calls for introduction of themes still unfamiliar to the pupils.

The teacher's conscience is an impact factor that is found to be influential in the teacher's choice to use a practical activity, and it is an impact factor that is almost solely referred to as "having a bad conscience" (Mellin-Olsen, 1996; Frykholm, 1999). In this study, answers to the questionnaire, and Zoltan when asked about the influence of his conscience, attach conscience considerations to the tensions that teachers experience when they are confronted with decision dilemmas (Lave, 1988; Mellin-Olsen, 1996). Zoltan suggests that a possible way of trying to handle the influence of conscience is for the teacher to change his/her primary impact factors, for instance, by further development of disciplinary and/or didactical knowledge. Seen in relation to the Batesonian inspired hierarchal structure of impact factors, this is followed by changed influence from secondary impact factors. Such a process means that the teacher's experiences with practical activities in mathematics teaching are systematically used to adjust the foundation for teaching by changes on a higher level in the hierarchy. Changes made by the teacher on the basis of his/her bad conscience, however, are made without making an impact on Level 1, and are merely adjustment activities[vi] within unchanged conditions.

Conclusions

The aim of this article has been three-sided. First, I wanted to suggest a theoretically grounded hierarchy of impact factors that influence a mathematics teacher's priorities when choosing to use a practical activity. Second, I wanted to identify discrepancies between the suggested theoretically grounded hierarchy and experiences of mathematics teachers, through the voices of acknowledged teachers of mathematics. And last, but not least, I wanted to examine ways in which teachers' experiences call for an expansion of the suggested theoretically grounded hierarchy. Responses from acknowledged teachers were given through open questionnaires and structured interviews. The teachers' responses to the questionnaire did not invalidate the initially suggested Batesonian hierarchical clustering of impact factors, but led to the formulation of three hypotheses regarding impact factors that did not fit the suggested system theoretically grounded hierarchy. The hypotheses were confirmed by two acknowledged teachers of mathematics through structured interviews.

In spite of the limitations of this study, it confirms that clusters of impact factors influencing teachers' choices to use practical activities can be organised in a hierarchical structure. Furthermore, I acknowledge that changes ought to occur on a higher level in a hierarchy of impact factors in order for the teacher to develop further the reasons for choosing to use practical activities in mathematics teaching. This is in accordance with the Batesonian hierarchical thinking about learning. But such a binary clustering of impact factors does not treat all kinds of impact factors properly. The teacher's everyday life experience, the teacher's knowledge of pupils' everyday experiences, and the teacher's conscience are impact factors related to the teacher's use of practical activities that are capable of having a considerable impact on whether the teacher decides to use a practical activity or not. In my opinion, these impact factors cannot be categorised solely as primary or secondary. The influence of everyday life experiences is, on the one hand related to beliefs and knowledge, and on the other hand dependent on relations between teacher and pupils. The influence of the teacher's conscience is related to beliefs and knowledge, but it is also dependent on how much the teacher lets secondary impact factors, such as time constraints or curriculum demands, influence the teaching. Hence, a model based on a Batesonian hierarchy of learning and communication does not completely satisfy the experience-based impressions of impact factors that influence the teacher's choice of practical activities. The incorporation of an intermediate level of impact factors in the hierarchy is required, and this might call for a more complex visualised structure of impact factors.

On the basis of this study, priority given to primary impact factors such as disciplinary and didactical knowledge in mathematics in teacher education and in service-education of teachers is likely to increase the quality of practical activity use in school mathematics. Such a development is likely to increase the frequency with which teachers might refer to everyday experience in deciding to use practical activities in mathematics teaching, and to result in better conscience among teachers regarding their use of practical activities. Moreover, in accordance with the suggested model expansion (Figure 2), a focus in teacher education on the resources hidden in teacher's everyday life experiences and knowledge about the pupils' everyday life experiences can influence teachers' beliefs about using practical activities. But such change will be a double-edged sword, since they call for a balanced application of practical activities in teaching to avoid any influence from bad conscience.

In this article we have drawn attention to three impact factors located in the intermediate level of the suggested hierarchy visualisation. The influence of each of these impact factors should be examined individually, and in larger and more longitudinal studies to establish more thoroughly their influence on teachers' use of practical activities in mathematics teaching and in order to suggest a more complex and proper model for influence from impact factors on teachers' choice of using practical activities.

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Sammendrag

Ut i fra et systemteoretisk fundamentert perspektiv blir det innledningsvis foreslått et hierarki av primære og sekundære påvirkningsfaktorer som påvirker matematikklærerens valg av å bruke praktiske aktiviteter i matematikkundervisning. Studiet som deretter presenteres, som er basert på kvalitative data fra matematikklærere som i sine arbeidsomgivelser regnes som anerkjente matematikklærere, gir argumentasjon for å hevde at et hierarki av påvirkningsfaktorer i forhold til valget av å bruke en praktisk aktivitet i matematikkundervisning må inkludere mer enn en binær samling av primære og sekundære påvirkningsfaktorer. Lærerens hverdagserfaringer, lærerens kunnskap om elevenes hverdagserfaringer og lærerens samvittighet er eksempler på påvirkningsfaktorer som krever at det blir introdusert et nytt nivå i et slikt hierarki, siden de er primære påvirkningsfaktorer som er avhengige av tilpassing til respons som blir gitt på nivået for sekundære påvirkningsfaktorer.

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[i] In Haara & Smith (2009), a practical activity is defined *to include all forms of engagement where the pupil uses physical concretes while carrying out the activity at hand.* This means that the opportunity for physical activity is included, and not just the use of artefacts or material which may be found in the nature.

[ii] The essence of integration can be concretised by showing how the almost rectangular areas of the rings that make up the cross section of the leek are the area between the curve of the circumference of the circle with a variable radius and the "radius-axis" (the first-axis). This represents the integration of the circumference of the circle in the interval [o, r] (e.g., Myklebust, 1994).

[iii] Bateson (1972) suggests that the number of levels might continue, but it becomes more and more difficult to see how a higher level will materialise.

[iv] All the teachers who participated in this study teach mathematics in Norwegian elementary schools (pupils aged from 6 till 16 years old).

[v] These teachers participated in a study of how acknowledged teachers of mathematics acknowledge practical activities in mathematics teaching, seen in relation to their level of disciplinary and didactical knowledge.
[vi] This refers to how Glosvik (2000, p. 56) describes adjustment activities in a Batesonian understanding of learning processes.