



# Using decision theory to understand preservice teachers' implementations of mathematical modelling

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## Abstract

In this study, we aim to explain choices that groups of preservice teachers made to implement modelling tasks in primary school. We found that the preservice teachers decided to facilitate everyday-inspired contexts for the tasks and that this could be explained by their reasoning that modelling is supposed to be an activity close to reality. Essentially, the groups preferred two different approaches to introducing students to the tasks: (1) by inviting students to physically participate in situations related to their local communities that were familiar to them (e.g. trash picking), and (2) through pre-structured problems that were recognisable to students (e.g. estimation of the number of beads in an irregular box). We found evidence that the preservice teachers tended towards avoiding focusing on possible mathematical answers to the tasks. This could be explained by lacking experience with how to find approximate solutions, or that the preservice teachers were engaged in observing students' solution strategies and mathematical behaviour.

**Keywords** Mathematical modelling tasks · Preservice teachers · Primary grade · Decision theories · Orientations · Beliefs

## Introduction

Mathematical modelling is the process of transforming a real-world situation into mathematics and interpreting the mathematical solution in terms of the original situation. Modelling draws the attention towards mathematics as a tool to solve practical problems, and insight into models that influence society is assumed to facilitate responsible citizenship (Kaiser & Brand, 2015). Currently, there is consensus that modelling should play an important role in mathematics education for both schools and universities (Leung et al., 2021). The PISA 2018 Assessment and Analytical

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Framework (OECD, 2019) defines modelling as a central aspect of mathematical literacy and emphasises the role of strategic decisions in the selection of algorithms, concepts, and procedures in search of mathematical solutions to contextual problems (p. 82). Studies show that the authentic, interdisciplinary, community building, and socially interactive nature of mathematical modelling is beneficial for primary grade teachers' and students' learning (English, 2003, 2009; Fulton et al., 2019; Mousolides et al., 2008). However, large parts of the educational system hold traditional beliefs that mathematics is a fragmented body of knowledge best to be learnt by imitation of rules and facts (e.g. Phelps-Gregory et al., 2020; Tatto et al., 2012; p. 153).

Successful educational modelling requires careful attention to task design. According to Galbraith and Stillman (2006), educational modelling tasks should be designed to facilitate learning opportunities throughout the whole modelling process. Blomhøj and Jensen (2003) claimed that to develop competencies in structuring a complex domain of inquiry, students should not always work with pre-structured modelling problems (p. 128) and Blomhøj and Kjeldsen (2011) emphasised that students should reflect upon models in a variety of authentic contexts to experience how they function in actual or potential applications. From these views, we conclude that to facilitate for practical applications and responsible citizenship through modelling, teachers need the competence to design appropriate tasks and support students at different stages in modelling processes.

Research has demonstrated that, offered a well-adapted context and adequate supervision, primary grade students can work effectively with modelling tasks (e.g. English & Watters, 2004; Mousoulides & English, 2008; Stohlmann & Albarracín, 2016). For teachers, however, the unpredictable course of students' modelling processes leads to difficult choices regarding when and how to intervene (Blum & Ferri, 2009; Stohlmann & Albarracín, 2016). Blum and Ferri (2009) emphasised that teachers can strike a balance between minimising their guidance and maximising students' independence by performing strategic interventions in the modelling process and providing meta-level hints to students (p. 52). One of the conclusions in the literature review by Stohlmann and Albarracín (2016) was that more research is needed to develop elementary teachers' pedagogical content knowledge on how to respond to students' diverse thinking in modelling processes (p. 7).

In the Norwegian context, modelling was incorporated into the mathematics curriculum for compulsory school in 2020 as one of the core elements. For "modelling and applications" it was stated that "A model in mathematics is a description of reality using mathematical language. The pupils shall gain insight into how mathematical models are used to describe everyday life, working life, and society in general." (Ministry of Education and Research, 2019). For our university college, the curriculum renewal made it necessary to prepare preservice students for the teaching of modelling. Especially, our primary school preservice teachers (students educating to teach at the Norwegian grades 1–7) were unfamiliar with the modelling concept and how to implement modelling tasks. The literature review by Jung and Brand (2021) points to studies showing that careful planning of multitiered teaching experiments involving, for example, students, teachers, preservice teachers, and researchers fosters professional development of all participants (e.g. English, 2003; Watson & Ohtani, 2015). Thus, by exploring

preservice teachers' implementations of practice-based modelling tasks in primary school classrooms, teacher educators can improve their learning about how to develop the preservice teachers' teaching proficiency in this respect. As stated by Niss and Blum (2020), "the fact that there are many choices and decisions involved in establishing the model makes it clear that establishing a model is indeed a process" (p. 7). Insight into what choices preservice teachers make to plan, intervene, and assess students' modelling processes can inform teacher educators and researchers on how to design prospective modelling assignments that support strategic choices, challenge misunderstandings, and strengthen relevant subject and pedagogical content knowledge. We therefore decided to explore the choices our primary grade preservice teachers tended to make when planning and supervising modelling tasks. The problem is stated as follows: "How can choices that preservice teachers make to implement modelling activities in primary school be explained?"

A mathematics didactics course for primary school preservice teachers performed at our college in the autumn semester 2020 was the starting point to investigate the problem. An assignment ("modelling assignment") asking to implement (plan and carry out) a modelling activity during a 3-week practice teaching period was part of the course. To assist with the implementation, the assignment suggested a three-act procedure (Meyer, 2011): present a picture/film/story (act 1), formulate a hypothesis on this event (act 2), test the hypothesis (act 3). Furthermore, it asked to (1) select and document examples where students were working with modelling and (2) analyse an individual problem formulation related to the examples. The answers to (1) and (2) included descriptions and reflections of choices and decisions the preservice teachers had made to implement the modelling activities. We analysed some of the answers to the modelling assignment to investigate the problem. To analyse the assignments, we relied on descriptive, psychologically oriented, theories on decision-making (Schoenfeld, 2011; Simon, 1993).

We present our theoretical framework in the "[Theoretical framework](#)" section, following the literature review in the "[Literature review](#)" section. After presentation and reflection on our results, we discuss implications for how to facilitate prospective modelling assignments at the teacher educator level. In the "[Discussion](#)" section, we also perform a discussion of the suitability of our theoretical framework.

## Literature review

Niss and Blum (2020) described a mathematical model as a representation of some aspects of an area outside mathematics (an extra-mathematical domain) by the means of mathematics (p. 6). Simple models build on direct and invertible transformations between the extra- and intra-mathematical domains (e.g. a street numbering), while more complicated models involve specifications and attention towards relations between elements in both domains (p. 6–17). Doerr and English (2003) defined models as "systems of elements, operations, relationships, and rules that can be used to explain, describe, or predict the behaviour of some other familiar system" (p. 112). Modelling processes can be expressed by flexible flow-charts describing essential subprocesses and possible workflows (e.g. Blomhøj & Jensen, 2003; Blum

& Ferri, 2009; Niss & Blum, 2020). Blum and Leiß (2006) suggested modelling processes to consist of the subprocesses (1) constructing, (2) simplifying, (3) mathematising, (4) working mathematically, (5) interpreting, and (6) validating. Referring to grades 8–10, Blum and Ferri (2009) pointed out that students had difficulties with how to relate to the stages 1, 2, and 6 (p. 48). Lesh and Fennewald (2013) stated that especially, young students' interpretation of what is being modelled often remains situated, piecemeal, and non-analytic (p. 7). Thus, the modelling subprocesses 1–6 may be incompatible with young students' modelling work. As an example of successful work with modelling at primary grade, Mousoulides and English (2008) referred to a group of 10-year-old students who worked on a lawn mower problem. On basis of different data tables, the students investigated the relative effectiveness of five different workers to mow lawns. They found that the students started with unsystematic approaches, for then to apply mathematical operations, like adding and averaging the data to compare different aspects describing the workers' effectiveness (e.g. kilometres driven, number of lawns mowed). The problem was introduced in three steps: presenting a newspaper article to familiarise the students with the problem, asking readiness questions about the article, and presenting the problem and corresponding data tables.

**Modelling with mathematics** Bleiler-Baxter et al. (2017) stressed the importance that teachers distinguish between “modelling mathematics” and “modelling with mathematics” to meet expectations in the 2010 curriculum (the Common Core State Standards Initiative). They claimed that using concrete materials or visual representations to illustrate mathematics (“modelling mathematics”) is not comparable to the activity of making assumptions and approximations to analyse and conclude about a real-world situation, the practice of authentic modelling. Observing how elementary grade students worked through the processes of simplification, making figures/tables, analysing, and concluding, to complete an unfinished table referring to waiting times for amusement park rides, they found it was critical that they valued student autonomy and had selected a task that represented a complicated real-world situation. Similarly, it has been shown that teachers and pre-service teachers have tendency to add non-relevant mathematics to real-world situations or adapt real-world situations to predetermined mathematics. Ng (2018) found that when designing modelling problems for their students, secondary mathematics teachers often started with adapting real-world contexts to pre-selected mathematical learning outcomes from the curriculum. The contexts, however, were often meaningful and relevant to the students. A similar result with pre-service teachers was found by Paolucci and Wessels (2017) for 1–3 grades. As a contrast, Villarreal et al. (2015) found that a group of secondary grade preservice teachers were able to design mathematical modelling projects that were useful in a socio-critical perspective, even though the mathematisation process was not building on advanced mathematics. The project relied on different representations of the amount of recyclable trash from an Argentinian city.

**Benefits of modelling experiments** Participating in well-planned multitiered experiments can empower teachers' learning of modelling and change their views on

student learning (English, 2003; Jung & Brady, 2016; Schorr & Koellner-Clark, 2003). English (2003) referred to two multitiered experiments that included students from fifth to eighth grade, preservice teachers, and teachers. The modelling problems were designed so that they provided meaningful and “experientially real” situations for the students (p. 232). In both experiments, the teachers introduced the modelling problems, while the other participants observed the student–teacher interactions. The results showed that the teachers welcomed the possibility to engage students in authentic problems that opened to various solutions. The preservice teachers were concerned with how the teachers responded to students. English (2003) concluded that the experiments had provided opportunities for teachers and researchers to interpret, describe, explain, and document students’ mathematical development (p. 240). A finding by Jung and Brady (2016) was that, from participating in a researcher-teacher multitiered teaching experiment involving model eliciting activities, a teacher realised that her perception of students’ work had changed. For example, the teacher started to perceive problem solving as work with problems that could have more than one answer. The teacher also appreciated the iterative approaches to approximate solutions that characterise modelling environments. Schorr and Koellner-Clark (2003) performed a large-scale study that was rooted in the perspective that when adopting new teaching practices, teachers often do this within their already existing teaching frameworks. To document how teachers develop new teaching models, they performed an in-depth analysis of the change that took place when twelve middle-school teachers were familiarised with a models and modelling perspective on teaching. The researchers found that from this experience, the teachers started to reflect more deeply on students’ thinking, asking more questions, and listening more closely to students. As an example, they referred to a teacher who started to “listen to students’ thinking” instead of asking the students to “explain their thinking”. Didis et al. (2016) investigated how 25 undergraduate prospective mathematics teachers responded to how secondary grade students had worked with six different modelling tasks. They suggested that insufficient subject matter knowledge could have hindered the teachers to recognise and interpret students’ mathematical understanding (p. 371). When engaging over time in students’ work, some student teachers improved their interpretations of students’ mathematical understanding.

Also, the act of observing model eliciting activities is found to support teacher learning, for example, Doerr and English (2006) found that since the modelling environment offers students’ the opportunity to evaluate their own task responses, the teachers could analyse students’ understandings rather than their performance of the tasks. The teachers developed mathematical content knowledge when they observed students’ presentations of solutions to modelling problems and new pedagogical content knowledge by attending to the diverse solution strategies.

**Mathematical behaviour and decision-making** It is possible to understand mathematical behaviour from the perspective of psychologically oriented decision-making. In this respect, Simon (1993) defined decision-making as choosing which problems to attend to, and how to reach, evaluate, and eventually implement, solutions to the problems. He discussed how behaviours following a psychological

decision-making process could be either rational, non-rational, or irrational. A specific behaviour and the decisions underlying it were all to be considered rational if they were well adapted to goals that were necessary to solve the problem. Oppositely, a behaviour was considered irrational if it was poorly adapted. Non-rational behaviours were forced, because one had been presented with irrelevant goals by someone else, or because the behaviour was instrumental to reaching the goal (p. 393–394).

Schoenfeld (2011) assumed that if a goal-directed activity is familiar to a person, then the choices he makes to solve problems during the activity can be explained by three personal characteristics: goals, knowledge, and orientations. By interpreting teaching as a goal-directed activity, he used this idea to explain typical teaching behaviours. Applying a connotational top-down parsing method to transcribed classroom dialogues between students and teachers, the dialogues were decomposed in terms of the characteristics. The assemblage of *goals* set for the teaching activities were assumed tacit, explicit, conscious, or unconscious (p. 15) and the teachers' *knowledge* base could consist of facts, isolated pieces of knowledge, algorithms for how to do things, conceptual knowledge, and problem-solving strategies. *Orientation* was defined to encompass beliefs, values, tastes, and preferences (p. 29). The results of the analyses were used to discuss and model (predict) typical teaching behaviours. Findings were elaborated through subsequent interviews. From experimentation with the method to dialogues outside teaching (e.g. doctor vs. patient), it was argued that the theory could model (predict) behaviour of all sorts of experts in situations where they performed familiar practices.

In general terms, a belief can be understood as an acceptance that something exists or is true, particularly without formal proofs (Pajares, 1992; Ponte, 1994). Schoenfeld (1992) characterised beliefs towards mathematics as understandings and feelings that shape the ways one conceptualises and engages in mathematical behaviour. Goldin and Törner (2009) referred to several theoretical struggles around the issue of how to define beliefs in mathematics teacher education. By elaborating on research that had evolved around the questions that beliefs (1) are mentally structured, (2) are attached to observable or non-observable objects, and (3) possess normative aspects, like emotions, attitudes, and values, they pointed out these three aspects to frame a plausible interpretation of the concept of belief. For qualitative studies, evidence of beliefs can be sought in statements and mathematical behaviour of students and teachers (Goldin & Törner, 2009, p. 13).

**Positive influences of beliefs** To challenge a belief system, one should experience the usefulness of alternative perspectives (Schoenfeld, 1992). Goldin and Törner (2009) emphasised that as a contrast to the negative influences of beliefs found in many research papers, beliefs can also be guiding and inspiring, which can serve to improve mathematics teaching and learning (p. 6). Studies show that pre- and in-service teachers can develop positive values and change their view on mathematics learning by participating in learning situations involving modelling (English, 2003; Jung & Brady, 2016; Schorr & Koellner-Clark, 2003).

Stillman and Brown (2011) investigated whether insight into modelling could change teachers' beliefs about mathematics and modelling. As part of an international study evaluating the professionalism resulting from teacher-preparation programs at different universities, they asked 73 Australian participants to explain their position towards mathematics. They found that 68% of the teachers believed modelling tasks are part of mathematics, because mathematics is experimental and applied, 29% commented that mathematics also has a deductive nature (it's "dual"), and 3% viewed mathematics as only a deductive abstract science (p. 296). Preservice teachers having experienced practice teaching were not found as acknowledging the duality more than the ones having lesser professional experience.

## Theoretical framework

We decided to answer our research question by analysing some of the texts answering the modelling assignment. Preservice teachers from two (out of six) college classes were asked if their assignments could be part of a research study. After this, we selected five texts by 14 preservice teachers distributed on groups consisting of 3–5 individuals. To select a text, the content had to be informative to answer the research question. Thus, we ended up with three texts referring to grades 1–4 and two referring to the 6–7 grades. By overviewing a few texts, we found that three main themes appeared: descriptions of practicum episodes, reflections on the descriptions, and more general reflections. These contained examples of many different choices and decisions that were made to implement the various activities. We defined (cf. Table 1, headings) the descriptions of the practicum episodes as "micro-level descriptions" and statements that reflected on these as "reflections on the

**Table 1** Thematic categorisation of text content referring to a learning activity implemented in practice teaching. Headings: themes that were found in the content of the texts. Rows 2–5, refinement of the themes

Microlevel descriptions	Reflections on the microlevel	General reflections
Statements referring to choices concerning the contextualisation and initiation of the learning activity <sup>a</sup>	Reflections on the contextualisation and initiation of the learning activity	General reflections on how to contextualise and initiate mathematical learning activities
Statements referring to choices concerning the mathematical work with the learning activity	Reflections on the mathematical work with the learning activity	General reflections on how to work with mathematical learning activities
Statements referring to choices concerning the pedagogical guiding of the learning activity	Reflections on the pedagogical guiding of the learning activity	General reflections on how to guide mathematical learning activities
Retellings of practicum episodes	Reflections on practicum episodes	

<sup>a</sup>Because not all the activities in our data material could be understood as modelling activities, we use "learning activity" (instead of "modelling activity")



microlevel". Reflective statements with no relation to the specific learning activities were defined as "general reflections". These dealt with aspects like how to understand modelling or mathematics as sciences, and perspectives on the teaching and learning of mathematics and mathematical modelling.

From the microlevel descriptions and reflections on the microlevel, we found that essential stages for decision-making had been when deciding how to contextualise the activity, and during initiation, mathematical, and pedagogical work with it. To keep track of descriptions and reflections referring to these stages, we classified Table 1 accordingly.

## Method

Our analysis was based on first reading each text carefully, creating a narrative retelling of its content. We paid particular attention to how the groups had proceeded to plan and carry out the activities. This information was found in the microlevel reflections and reflections on the microlevel. We started with identifying moments in the process of implementing the activity, where it was evident that it had been necessary for the group to make a choice. Having identified the choice made, we organised it in accordance with the categorisation in the first column in Table 1 (that is, whether the choice was made to initiate, contextualise, and work mathematically, or progressing pedagogically, the activity). After this, we investigated whether some statement in the text directly explained it. An example of a directly explainable statement was "We started the lesson by turning on the music video about the bus drive. This was because the practicum teacher had the habit of doing this with the students to draw their attention". This reflection on the microlevel explained the choice to initiate the learning activity by showing a video.

To be able to explain the decisions made, we assumed that they were rational (Simon, 1993), thus expressing the preservice teacher groups' deliberately chosen approaches for the various problems arising when implementing the modelling activities. The written texts were not always exhaustive, and we found it important also to consider how choices that were not directly explained could be interpreted, thus enabling us to decode a broader spectrum of behaviours than collecting information from the explicit statements only. One example of a vague reflection was the following: "As mentioned, modelling in the subject of mathematics is about making teaching close to reality". To interpret and analyse such vague statements in the reflections, we relied on the basic idea in Schoenfeld (2011) that choices can be explained from personal characteristics. Different from Schoenfeld (2011), we did not have direct access to classroom dialogues that could serve to connect personal characteristics to instantaneous choices. Thus, we could not use the connotational top-down parsing method. Instead, we applied a critical content analysis of statements identified (confer Table 1) as "reflections on the microlevel" or "general reflections". Here, we assumed that each group text reflected a single collective voice. To apply the idea in Schoenfeld (2011), we decided to identify goals (G),



knowledges (K), beliefs (B), values (V), and preferences (P). Regarding beliefs, we relied on the understandings in Goldin and Törner (2009), Pajares (1992), and Ponte (1994) stated earlier. We related the findings of [G, K, B, V, and P]'s to the choices by organising them as in the second and third column of Table 1.

To elaborate on how we classified a statement according to [G, K, B, V, or P] and used the result to explain decisions, we present an excerpt from one of the texts. Regarding the pedagogical work with the activities, one of the groups described that most of the decisions concerning which calculations to be made were left to the students. Recounting some of the calculations, the group reflected as follows: That is, they [the students] have understood that mathematics is not just some assignment in a workbook but is actually used for everyday and outside the math class". The formulations that mathematics "is not just some assignment" but "is actually used for everyday" can be interpreted as that the preservice teachers assumed that the students would value (V) the everyday use of mathematics more positively than textbook practices of mathematics. We interpreted this as reflecting the preservice teachers' view on mathematics, as well. Thus, we explained the decision of not interfering with students' calculations by the value (V) (Table 3, row 6, column 2).

## Results

The next section contains summaries of the narrative retellings we constructed from the texts.

### Narratives and results from analysis

**Group 1** Lesson started by the group presenting a music video about a bus drive. After having shown the film, it was decided, together with the class, to recreate the bus context by placing six pairs of classroom chairs in a row. One of the preservice teachers "drove" the imaginary bus, and students were invited to play out bus rides by occupying the chairs and suggesting calculations by observing different numbers of "passengers" entering and leaving the bus.

**Group 2** The preservice teachers introduced a context for the modelling activity by creating a story about three jugs of beads found in an attic. They then presented three jugs of different sizes and with non-uniform cross-sections. Each jug contained many beads. Students were divided into groups and asked what they would like to find out about the beads. In cooperation with the preservice teachers, this resulted in the general problem formulation to find out how many beads altogether.

**Group 3** The project involved station work. Informal and formal representations of bar graphs were the focus of the teaching. The modelling activity was introduced by counting the number of girls, boys, and adults in the classroom, where the preservice teachers took the responsibility for creating a bar graph that represented the

resulting numbers. After having discussed this diagram with the class, the station work started. At one station, students had been asked to sort different classroom materials (toy blocks, stones, pencils, and bits of fabric) and represent them in informal bar graphs.

**Group 4** The context was initiated by showing a video about waste generation. This had inspired students to ask how much plastic waste one could find at a public square. It was reported that this became the overall modelling problem. The problem was investigated by letting student groups pick garbage at an area close to the school.

**Group 5** The activity was introduced by showing a self-made video of two persons who measured a distance by using their arm spans. Then, the preservice teachers and students agreed to measure the circumference of a nearby football field. First, the preservice teachers invited the students to suggest a too large, and a too small, circumference. Then, the class worked with measurements and measurement units.

The learning activities are summarised in Table 2. In the fourth column, we have marked whether we considered the activities to have potential for working with modelling, and in the fifth whether they were carried out as modelling or applications of mathematics/modelling mathematics. How we came to these conclusions is elaborated in the “[Discussion](#)” section.

The findings coming from the analysis are given in Table 3.

**Table 2** Learning activities implemented by different preservice teacher groups in five primary school classes

Group	Grade	Activity	Modelling potential (Y: yes, N: no)	Modelling (M) versus applications of mathematics (A)
1	2	Making calculations by observing the number of passengers entering and leaving an imaginary bus (made of classroom chairs)	Y	A
2	3	Determining the number of beads in three jugs with non-uniform cross-sections (the number of beads was large)	Y	A
3	3	Creating formal and informal bar graphs based on different classroom materials or leisure activities	Y	A
4	6	Determining how much plastic waste one could collect at a familiar, public square	Y	M
5	7	Measuring the circumference of a nearby football field by informal methods (arm spans)/formal measurement scales (e.g. metres)	Y	A

The five activities are listed in the second row of Table 3 and the main categories for stages of decision-making are given in the first column (they are identical to the categories in our theoretical framework (Table 1)). In rows 4–6, choices that the groups made within each category of decision-making are specified. Bold face text indicates that the content is to be understood as a choice. The text beneath the bold face text is our explanation of the choice. Explanations are given in terms of goals (G), knowledge (K), beliefs (B), values (V), and/or preferences (P). We defined something as lack of knowledge (K-) if we considered that it would have been natural to demonstrate this knowledge in the context referred in the text.

## Elaboration of results

In this section, we elaborate on the results in Table 3 by showing examples of various reflections and how we interpreted them to explain decisions. We have applied the structure of the three main categories for decision-making to organise the content.

**Explanations of choices connected to contextualisation** Group 1 decided to use an imaginary bus ride as context for the activity (Table 3, fourth row). A general reflection that could explain this was the following: “When we create a mathematical model, we try to describe reality using mathematics. Modelling is therefore close to reality and can be linked to topics that interest students”. The first sentence showed that the preservice teachers possessed the knowledge (K) that a purpose of a mathematical model is to use mathematics to describe some reality. Interpreting “modelling” as something one does, the first part of the second sentence indicated that from this knowledge, the group made the conclusion that modelling, as an activity, was “close to reality”. Continuing by expressing the possibility that modelling “can be linked to topics that interest students” can be interpreted as that the preservice teachers valued (V) this quality of modelling. This shows that from perceiving modelling as an activity that is close to reality, the group derived that the pedagogical teaching of modelling could be realised by facilitating for classroom situations that resemble everyday situations. This interpretation was supported by their concluding remark that “... one has, as a student teacher, achieved insight into how to integrate real and everyday situations, that students easily can relate to, into the mathematics teaching”. But there can also be additional explanations, and the group wrote a reflection that showed this: “At the start of the work [i.e., the assignment] we were not entirely sure how to integrate modelling in mathematics teaching in a 2nd grade, as the degree of difficulty and relevance of a possible model/ practical task to students in a 2nd grade was unclear to us. How to formulate a possible task was also a challenge, as we had no relationship with the students, thus we did not know what level of knowledge they were at”. The text further informed that the class teacher had explained that the students were familiar to the music video about the bus ride. Thus, an additional explanation of why the preservice teachers chose the bus context can be that they lacked pedagogical content knowledge (K-) of students’ interests and academic level and therefore asked the practice teacher about this. We concluded

**Table 3** Possible explanations for choices primary grade preservice teacher groups made when implementing learning activities in practice teaching. Explanations are denoted as G, goals; K, knowledge; K-, lack of knowledge; B, beliefs; V, values; P, preferences. (-), insufficient information to draw conclusions

Main category of choice ↓	Activity (group)				
	<i>Bus passenger calculations</i> (1)	<i>Bead jug problems</i> (2)	<i>Bar chart problems</i> (3)	<i>Garbage picking</i> (4)	<i>Informal and formal circumference measurements</i> (5)
<i>How to contextualise and initiate the activity</i>	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices
	<b>Bus role play</b> Three-act modelling (K). Insufficient knowledge of student's interests and academic level (K-). Modelling tasks can be everyday-inspired, and problem contexts integrated into the teaching (P, V)	<b>Exiting story</b> Three-act modelling (K). Modelling tasks can be everyday-inspired (P, V)	<b>Use objects from classroom</b> Modelling tasks can be everyday-inspired (P, V)	<b>Film about waste generation</b> Three-act modelling (K). Modelling tasks can be everyday-inspired (P, V)	<b>Self-made video about informal measurement methods</b> Three-act modelling (K). Modelling tasks can be everyday-inspired (P, V)
<i>How to work mathematically with the activity</i>	(-)	<b>No focus on possible answers</b> Fixed answers to modelling tasks (B). Approximation methods not referred (K-)	(-)	<b>Minimised their involvement in pupil's work processes</b> Modelling facilitates relational understanding (B, V) and comparison with real-world objects promotes it (B)	<b>Not solve the modelling problem</b> Focus on distributive law, and exact measurements (G). Other mathematical goal preferences than to solve the modelling task. Valued (V) relational understanding. Believed in experiences of distance (B)

Table 3 (continued)

Main category of choice ↓	Activity (group)				
	<i>Bus passenger calculations (1)</i>	<i>Bead jug problems (2)</i>	<i>Bar chart problems (3)</i>	<i>Garbage picking (4)</i>	<i>Informal and formal circumference measurements (5)</i>
<i>How to pedagogically guide the activity</i>	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices	<b>Choices made</b> Explanation of choices
	<b>Left to students what calculations to perform</b> Modelling not a textbook activity (V). One does not solve math tasks in everyday contexts (B)	<b>Ignored students' informal approx. methods</b> Fixed answers (B). Approximation methods (K-). Lack of pedagogical content knowledge to interpret students' work (K-). Other goal preferences (G)	<b>Jumped back in the modelling cycle when teaching</b> Valued (V) this knowledge of modelling cycle (K)	<b>Minimised their involvement in student's work processes</b> Modelling facilitates relational understanding (B, V). Comparison with real-world objects promotes it (B). Observation was a goal (G)	<b>Changed their teaching approach</b> Valued (V) and knew about (K) the flexibility of the modelling cycle. Opportunity to observe students (G)

that the decision to contextualise the activity within a bus ride could be explained by the earlier discussed preference (P), the preservice teachers' knowledges (K, K-), and values (V).

In alignment with this group, we found that three other groups utilised their knowledge (K) of the three-act method to decide how to initiate the activity. They also valued (V) that educational modelling tasks can be everyday-inspired. Group 3 relied on this fact, only (cf. Table 3).

**Explanations of choices connected to mathematical work** The text describing the bead jug problem did not focus on possible answers to the task. To explain this, we analysed the groups' reflections on a microlevel description selected to answer the second question in the modelling assignment:

1. Student 1: We can count how many beads that cover the top, to find out how many beads there are in one layer.
2. Student 2: Hmm... yes, we can.
3. Student 1: And then we can count how many layers there are downwards in the jug. If we multiply the numbers, then we can find out how many beads altogether in the whole jug [points to beads downwards along the side of the jug].
4. Student 2: Yes [looks at the jug]. But it is a little strange shape on it. Will it not become a little wrong...? When it [the jug] is not everywhere of the same size?

In a reflection on this episode, the group wrote "The fact that [student 1] suggests counting all the beads in one "layer" for then to multiply it all downwards, is a good idea. This is a method one could use to find an approximate number of beads in the vase". This shows that the preservice teachers interpreted student 1's suggestion as relevant if one wished to find an approximate number of beads. The excerpt continued as "But when the vase is not a straight cylinder, the situation becomes a little different. [Student 2] understands that there are more beads in the layer in the area where the vase is wider, than where it is narrower. Therefore, the answer will not be correct". Here, the group ignored student 1; instead, they commented that student 2 had understood the difficulty arising from the irregularity of the cylinder. The group finished its analysis by expressing that using student 1's method would have led to an incorrect answer. From the reference to counting and the non-uniform distribution of beads, we first interpreted the group as thinking that the students had to count the beads in each layer to have the answer as a fixed number. If so, this could explain why they ignored student 1's suggestion. At the other hand, the group was critiquing the suggestion through the phrase "But when the vase is not a straight cylinder..." They also used the expression "the layer [...] where the vase is wider". This indicated that the group was concerned with the non-uniform cross section of the cylinder and the possibility to separate the beads into layers. The choice to not involve in further mathematical considerations related to this understanding could be explained by lacking knowledge (K-) of how to perform geometrical calculations,

mathematical approximations, or both. We registered the explanations as either beliefs (B) about fixed answers or lack of knowledge (K-) (Table 3).

For the class that had worked with circumference measurements, the text showed that the students decided not to focus uniquely on solving the original modelling problem. To explain this choice, we interpreted the reflections on a situation where the preservice teachers explained that the students had started by measuring the lengths of the sides using arm spans. Then, a discussion of whether to first multiply or first add together the lengths of the parallel sides, for subsequent addition of the resulting numbers, had taken place. In their reflections, the preservice teachers interpreted the proposal to multiply the sides as if the students were able to “use both mathematics and reality” but that they failed to “link them to each other”. They reported that the students finally realised that the sides had to be added together and were able to discuss that  $76 + 76 + 33 + 33$  was identical to  $2(76 + 33)$  where 76 and 33 were the number of arm spans contained in the respective side lengths. Our interpretation of these reflections was that the preservice teachers had additional goals (G) with the activity than just finding the circumference. Another interpretation was that they valued (V) that through this collective experience the students had achieved relational understanding towards mathematics.

The next microlevel description showed that the group tended to overlook students' informal methods. To explain why, we studied their reflections on an episode starting with this dialogue:

1. Student: I want to measure the court by metres.
2. Preservice teacher: Yes, but do we know how long a meter is?
3. Student: Yes. Because I know approximately how long a meter is. And then I simply can take a step of one meter and count.
4. Preservice teacher: [Turns to the other students.] Do we all agree that we can measure by using steps? [Here some students agree, others do not.]

The group indicated that the episode continued with the preservice teacher inviting all students to walk one step from the wall, which made the students observe that the positioning from the wall now differed between participants. Then, the students who had suggested the approximate method in line three had said that “No, it doesn't work. I must use something that I know is exactly one metre each time to get the correct answer”. The preservice teachers analysed the episode as a physical experience that had assisted students with how to deal with the task. Because the perception that authentic experience of distance would make students aware of the importance to be precise when taking measurements was accepted without formal proof, we characterised it as a belief (B) (Pajares, 1992; Ponte, 1994). To focus on precise measurements and measurement units had been the mathematical goal (G) with the activity. A combination of the belief and this goal could explain why the group overlooked the informal method. The decision to turn to the preciseness of mathematics in the above examples can also be explained by the impression that



teachers often change their practices slowly and within their established teaching frameworks (Schorr & Koellner-Clark, 2003).

**Explanations of choices connected to pedagogical guidance** When analysing the bead jug problem, we found that the preservice teachers did not reflect on how student 1's informal method could have been supported (see the first transcript in the previous section). To explain this, we studied the following reflection where the group stated that the two students had "...worked towards gaining experience with multi-digit numbers and the positioning system". From this, one may conclude that the preservice teachers had perceived the students' goal as purely mathematical. Another interpretation was that they preferred this goal (G) for the modelling activity themselves (Schorr & Koellner-Clark, 2003). For the first interpretation, the preservice teachers' pedagogical content knowledge to interpret the students' goal was insufficient (K-). For the second, we can also explain this as the preservice teachers were busy with observing students' solution strategies and therefore did not reflect on the pedagogy.

Regarding the work with creating bar charts, the group commented that the students were not able to draw mathematically correct diagrams: "The starting point and the thoughts behind the diagram were correct, but the height of the columns did not match the number it represented". In the following, they related to one of the modelling articles from the course literature. This article described that if one had difficulties with solving the mathematised problem, one should formulate and attack the problem in another way. Referring to this consideration, the group decided to change their teaching approach to statistics: "We observed that the students did not understand how to attack the problem, and we then had to jump back to step 2 [the formulation of the modelling problem] to find another way to give them the knowledge they needed to solve the problem. Here we also emphasised the importance that the height of the columns represented the correct number, so that it could all become correct". The group decided to work with creating a new bar graph, this time based on the distribution of leisure activities in the class. Afterwards, the students were asked to return to the original problem, concerning sorting of classroom materials. The preservice teachers valued (V) the possibility to jump between stages in modelling processes, and this knowledge (K) triggered them to change their teaching approach.

In the assignment referring to the garbage problem, it was described that in modelling, students "are not given leashes when deciding what they want to investigate, and as teachers we didn't interfere with how the pupils came up with an answer". This showed that this group also had decided to minimise involvement in students' work with solution methods. Again, the preservice teachers were busy with observing the students (G): "A little later, a student surprised us student teachers with his answer to an additional question we had for their problem".

Preservice teacher: 'How much garbage would we have found if we had picked for a time twice as long?'

Here, the preservice teacher had expected that the student would just multiply the amount of waste they had found by two, but this did not happen.

Student: 'Then we could find bottles, more boxes, cardboards, etc.'

Because of the word "surprised", we concluded that the preservice teachers believed (B) that students were accustomed to instrumental teaching, and that such teaching experience would characterise their relation towards mathematics. Somewhat later in the text, we found this macrolevel reflection: "Many teachers make teaching purely instrumental, where the focus is on practicing and mastering an algorithm. By working with modelling, a relational understanding of various mathematical problems can be promoted, since there is no focus on how students should do things". We interpret this statement as that the preservice teachers believed (B) that in work with modelling, there is no requirement for the teacher to focus on which method is used. But one could also think that the preservice teachers valued (V) relational understanding and believed (B) modelling to be a teaching method that could promote it. Some of the students had made a line out of garbage, and according to this, the preservice teachers wrote: "The line with garbage can make the students achieve relational understanding of the measurement concepts metres, centi-metres and millimetres because the line is a concrete to compare with". This excerpt shows that the preservice teachers valued (V) that the line was something concrete and visible that could be measured directly. Thus, they believed (B) that comparison with real-world objects was necessary for relational understanding.

## Discussion

### Discussion of results

In line with other studies, we found that the groups were proficient in facilitating for contexts that were familiar and recognisable to students (Ng, 2018; Paolucci & Wessels, 2017). Most of the contexts also had potentials for working with modelling (Table 2). The groups *preferred* to initiate the activities by either showing a film/video (cartoon, waste generation, arm spans) telling a story (findings in an attic) or present concrete objects. These choices were obviously rooted in the preservice teacher's *knowledge* of the three-act process. All texts showed evidence that the preservice teachers interpreted modelling as an activity that can be based on everyday experiences. Based on their reflections on this, we concluded that this made them make the choice to select modelling contexts that were experientially real to students and that they *valued* this opportunity. The first group concluded that modelling could be facilitated by acting out episodes from everyday life in the classroom, and the preservice teachers explored this in their teaching. Group six had implemented a modelling task in a similar socio-critical perspective as found in Villarreal et al. (2015).

We did not interpret the groups to make full use of the contexts. The last column in Table 2 shows that we interpreted the performance of most of the activities as applications of mathematics (or "modelling mathematics" (Bleiler-Baxter et al., 2017)). Many of the essential modelling subprocesses (Blum & Leiß, 2006) lacked in the descriptions of the implementations. The definition that modelling can be the

activity to use operations, relationships, and rules to describe, or explain, a familiar system (Mousoulides & English, 2008) was carried out in an early phase. Many of the texts referred to situated understandings and piecemeal modelling processes (Lesh & Fennewald, 2013). Several studies conclude that teachers need to engage over time and deeply in students' work to improve their understanding and evaluative skills towards modelling (Didis et al., 2016; Doerr & English, 2006; English, 2003; Jung & Brady, 2016; Schorr & Koellner-Clark, 2003).

The groups basically chose two different ways to involve students in the activities. The first approach was by inviting to actively participate in various local community contexts to collect data. This approach was chosen by three of the groups, who invited students to act out a bus scenario, pick garbage, and measure the circumference of a yard. The second approach was to let students work with pre-structured problems that they were supposed to understand. Here, the groups provided the students with data (jugs with beads, objects to systematise). Separately, these two approaches can be understood as representing different orientations towards educational modelling. The essence of the first approach is to perceive modelling as a holistic process (e.g. Blum & Leiß, 2006) in which students should participate from the start. The mathematisation is supposed to take place after the data collection (like in the garbage picking and circumference problems) or it can be integrated along the way (as for the bus context problem). Teacher educators who would like their students to experience how to implement fully integrated modelling processes can design assignments based on this approach (Blomhøj & Jensen, 2003; Blomhøj & Kjeldsen, 2011; Galbraith & Stillman, 2006). For the second approach, modelling is perceived as the application of mathematics to explore understandable pre-structured situations. To solve the problems does not require fully integrated modelling processes. The flexibility of this approach can be increased by inspiring preservice teachers to invite students to structure the problem situation differently. Here specific problems and modelling stages can be emphasised. The two approaches require completely different preparations for how to invite students into the modelling context.

Many of the texts both presented an acceptable modelling problem *and* pure mathematical questions related to the problem context. For example, for the waste generation problem, the preservice teachers, in addition to presenting the modelling problem as finding the amount of plastic at an open square, had asked students to answer simple math tasks which we did not consider as mathematical modelling. Firstly, these findings support the discussion that teachers need more experience with the constructing stage (Blum & Leiß, 2006) of the modelling process and that the finding in Blum and Ferri (2009) seems to apply to primary grade preservice teachers as well. Secondly, these findings are illustrative for how the preservice teachers interpreted the mathematising stage in the modelling process. Even for those problems that could be characterised as having potential to become "good" modelling problems, the preservice teachers seemed to have chosen simple mathematical tasks. For the waste generation problem, the preservice teachers could, for example, have integrated an approximation problem based on predictions from collecting the amount of plastic at a small part of the square. Instead, they presented mathematical tasks in their assignment. A similar approach was found for the circumference

problem (to find exact measures for the yard). For the bead context, the preservice teachers could have guided the students towards approximating the beads in the jug by measuring the diameter of different cross-sections or approximating the jug by relevant geometrical objects (e.g. cones). We explained these choices by the preservice teachers not expressing sufficient relevant mathematical content knowledge to make full use of the contexts to explore the problems. On the other hand, our research showed that the preservice teachers often had been busy with observing the students and exploring their mathematical strategies. This is shown by research to be valuable for teachers and preservice teachers' empowerment towards modelling (Doerr & English, 2006; English, 2003; Jung & Brady, 2016; Schorr & Koellner-Clark, 2003).

Regarding the pedagogical guidance of the activities, four of the groups reported that they handed over most of the responsibility for working with solution strategies to the students. The decision to transfer some control of the modelling process to students is in alignment with the impression that modelling supports a constructivist perspective on how to work with mathematics (Stillman & Brown, 2011). As discussed in the previous paragraph, the preservice teachers did not always notice students' informal reasonings and not demonstrated knowledge of how to find approximate solutions. This can also explain why they involved very little in the students' modelling processes. In addition, the results showed that many of the groups had the orientation that direct contact with tangible or directly measurable objects, like a line of garbage (group 4) or footsteps (group 5), would promote relational understanding of mathematics. From the overall analysis, we concluded that the belief that observations are important for relational understanding, the valuation of outside-classroom learning experiences, the tendency to strictly adhere to pure mathematical goals, and the business with observing students and their mathematical solution strategies made the preservice teachers leave many decisions to the students. The results regarding the benefits of observational experiences through the modelling environment are supported by other studies (e.g. English, 2003; Jung & Brady, 2016; Schorr & Koellner-Clark, 2003).

Our findings have several consequences for how to design prospective modelling assignments at the teacher education level. Firstly, assignments need to be constructed such that preservice teachers notice the difference between modelling and applications of mathematics, alternatively "modelling mathematics". Examples of how to introduce learning activities may contribute to illustrate these differences. Secondly, assignments should emphasise that approximation methods are common in modelling and for example demonstrate the use of such methods. Thirdly, the concept "relational understanding" and how to support this understanding by facilitating relevant modelling tasks and adequate guidance can be addressed. Fourthly, assignments could ask to facilitate specific mathematical knowledge for students and encourage preservice teachers to solve tasks before implementing them in practice teaching. This is for preparing to guide different mathematical approaches. Finally, we think it is important that college teachers are aware that preservice teachers may have additional goals implementing a modelling activity than just solving the modelling problem. Thus, assignments can ask to prepare for specific situations or phenomena to observe.

## Discussion of theoretical framework

Because the texts responded to an official assignment, one can discuss to what extent the preservice teacher's thoughts and reflections were authentic, or whether they were strongly adjusted to the requirements in the assignment. The strategy to use "reflections on the microlevel" and "general reflections" to explain what were described to have taken place at the microlevel must be understood considering the given conditions. To follow the flow of the modelling processes, the texts were less efficient as data sources, because they were focusing on single episodes and only through the eyes of the preservice teachers. An advantage of the framework was that it organised different types of explanations for each choice.

A criticism towards relying on the theory in Schoenfeld (2011) was that modelling was only to some extent familiar to the preservice teachers. Furthermore, we assumed that the decisions were rational (Simon, 1993). If we had included non-rationality, then we would have opened for the possibility that a choice was made to, for example, satisfy some requirement in the modelling assignment or a request from the practice teacher. Our assumption was that the vague statements were important to identify goals, knowledge, and orientations.

## Conclusion

Our study was based on a specific training program, which involved a particular modelling assignment. By categorising preservice teachers' reflections that answered the assignment into goals, knowledge, and orientations, we could explain various choices they had performed during the implementations of the modelling activities. The preservice teacher's orientation towards how to integrate modelling into teaching had a large impact on how the modelling problems were introduced.

It is interesting that the preservice teachers sometimes had mathematical goals with the activities which deviated from students' goals and hindered processes of solving the modelling problems. An explanation for this can be that they avoided approximation and estimation methods because of insufficient subject matter knowledge of this mathematical topic. Other explanations can be that they believed in firm and exact answers to modelling tasks (e.g. Jung & Brady, 2016; Ng, 2018). For prospective task designs, our research indicates that to exemplify how to use approximation methods could improve the completion of educational modelling processes, also for the primary grades. In addition, it is important to be aware of preservice teachers' needs to observe the many instructive situations that can arise in educational modelling.

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**Code availability** Not applicable.

## Declarations

**Ethics approval** Ethics approval for this study is given by the Norwegian Centre for Research Data (NSD).

**Consent to participate** Permission to use texts for research was given by the Norwegian Centre for Research Data (NSD) on condition of each student teacher's consent per email or SMS.

**Consent for publication** Yes.

**Conflict of interest** The author declares no competing interests.

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