Multilingual preservice teachers evaluating mathematical argumentation: Realised and potential learning opportunities

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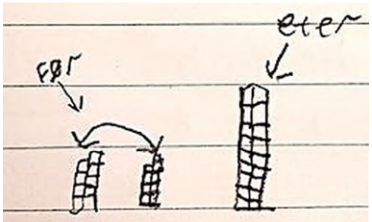
Preservice teachers' evaluations on school students' mathematical argumentation are rarely the focus of mathematics education research. Yet, understanding how they evaluate the quality of their future students' mathematical argumentation is important. In this paper, a discussion by a group of multilingual preservice teachers is analysed to determine the properties they considered to be connected to high-quality mathematical argumentation. The preservice teachers, who had two different dominant languages, discussed whether examples of Grade Four students' work, written in those two languages, displayed the qualities of being clearly written, mathematically correct and complete. From an analysis of this discussion, we identify how the preservice teachers' multilingual backgrounds provided potential and realised learning opportunities about students' mathematical argumentation.

Keywords: Multilingual preservice teachers, mathematical argumentation, multimodal representations

Introduction

Multilingual mathematics classrooms have been a research area for decades (see for example, Austin & Howson, 1979). However, it is only recently that mathematics teacher education for multilingual classrooms has been investigated (see Trinick, Meaney, & Fairhall, 2014). This lack of focus is also apparent in Norwegian teacher education, where preservice teachers [PTs] stated that they did not gain adequate knowledge on how to teach subjects, such as mathematics, in multilingual classrooms (Thomassen & Munthe, 2020). Nevertheless, the guidelines for the new teacher education programme clearly state, "teaching must be prepared from students' different needs, where different cultural, linguistic and social background are both taken into consideration, but also seen as a resource in the classroom" (UHR, 2016, p. 23, own translation). Viewing language as a resource has been promoted as a way of identifying a range of learning opportunities when different language backgrounds are included in mathematics lessons (Planas, 2014). In our project, *Learning About Teaching Argumentation for Critical Mathematics Education in multilingual classrooms* [LATACME], we aim to overcome both the lack of research and teacher knowledge by using PTs' discussions of school students' argumentation to gain understandings about using different language backgrounds as a resource.

In this paper, we explore how multilingual mathematics PTs, in a fifth-year Master level course, evaluated school students' argumentation about odd and even numbers. Our intention was to identify where their understandings, from different educational cultures, about mathematical argumentation provided both potential and realised learning opportunities about their future teaching. The course was part of an international programme and taught in English, with the PTs having either Norwegian or another European language (AEL) as their dominant languages. The examples, written either in Norwegian or AEL, were about adding two odd numbers together and came from Grade 4. We chose, these examples because we had incorporated them into a survey for our first-year PTs who we will survey again at the end of their second year. The survey responses suggested that we had a group of PTs who did not seem to make distinctions about the quality of children's argumentation. In the survey, the PTs were asked the level of agreement (or disagreement) with statements about whether they could follow an explanation, whether the explanation was incomplete and whether the explanation was mathematically correct. Figure 1 shows one of the four examples in the survey.



("før" means before and "etter" means after in Norwegian)

From Ure (2018)

Figure 1: A Year 4 student's argumentation for why adding two odd numbers makes an even number

In relationship to the explanation in Figure 1, 63% of the first-year PTs completely agreed that they could follow and understand it. 13% completely disagreed and 29% somewhat disagreed that the explanation was incomplete and 32% completely agreed that it was mathematically correct. Although A. J. Stylianides and Stylianides (2009) might suggest that the reason for such responses was because PTs lacked mathematical knowledge about proofs and mathematical argumentation, it was important for LATACME to investigate PTs' thinking about the students' argumentation. As the original PTs are to be surveyed at the end of their compulsory courses, we had to explore this with another group. This Master course was chosen as it included a group of international PTs who were native speakers of AEL, which was used in one of the argumentation examples. Although the first-year PTs were unlikely to know this language, similar percentages considered the example to be understandable, not incomplete and mathematically correct. It was, therefore, necessary to investigate how PTs evaluated the argumentations and whether being in multilingual groups supported the development of evaluation criteria and if this was the case, in what ways. The multilingual nature of mathematics teacher education courses is often ignored, with the focus being on the language of instruction of the course (Chitera, 2011). It was, therefore, important to see how multiple languages could support the PTs' possibilities for learning about school students' argumentation.

Mathematical proof and argumentation

As Stylianides, Stylianides, and Weber (2017) noted, "proof is a mathematical argument" (p. 238), and in this paper, we use the terms interchangeably. G. J. Stylianides and Stylianides (2009) indicated that although proof in mathematics is a core activity, many students, including at university level, accepted empirical arguments as proof, suggesting this needs more focus in teacher education.

Balacheff (1988) distinguished between four types of proofs, based on school students' reasoning. These were 1) naïve empiricism where the student asserts a result and verifies this with some examples. 2) The crucial experiment, where the student deals explicitly with the problem of generality and resolves it by using the outcome of a particular case. Stylianides (2009) described the crucial experiment as a strategic search for an example that corroborates the claim. 3) The generic example is when the student explicitly identifies some common characteristics, as representative of a class, and uses an example to demonstrate the properties of the class. However, unless the interpreters of the proof share an understanding of the importance of the characteristics, then the example can be interpreted as simply illustrating a particular case, that is naïve empiricism. 4) The thought experiment is when the student's proof is detached from a particular example of a class, such as when a general algebraic solution is provided. Any of these four types can be considered as proofs, when "they are recognised as such by their producers" (Balacheff, 1988, p. 218). However, there is also a need for them to be accepted by the community, not just the producer. Van Bendegem (1993) stated "*the proof* as a mathematically accepted proof, *exists only on a social level. Hence the basic unit to consider is not the individual mathematician but the mathematical community*" (p. 32, emphasis in the original).

The social acceptance is also related to expectations about appropriate representations. Stylianides (2009) illustrated how a proof of the sum of two odd numbers always equals an even number could be achieved through three different representations: by everyday language; by algebra; and by pictures. In the picture solution, there are drawings of pairs of squares, making a rectangle, representing even numbers or pairs of squares with one extra square that represented odd numbers. These kinds of representations are common when teaching odd and even numbers in Norway and can be considered cultural artefacts, providing meaning to those familiar with them.

A. J. Stylianides and Stylianides (2009) explored elementary PTs' constructions and evaluation of proofs. Some proofs included empirical evidence, in the style of Balacheff's (1988) naïve empiricism. However, when their PTs evaluated them, they considered them invalid (A. J. Stylianides & Stylianides, 2009). Over the semester, the PTs collectively identified criteria of a "good" proof. The criteria included that the proof had to: be correct; address the question or the posed problem; be focused, detailed and precise; and be clear, convincing and logical. The PTs elaborated on these criteria by stating that the language, representations, definitions had to be understood by the people to whom the proof was addressed, that the proof could be used to convince a sceptic and did not require the reader to make a leap of faith, key points had to be emphasized, if applicable supporting pictures or other representations had to be used appropriately, coherent, clear with complete sentences, and that the proof could be used by someone to solve similar problems.

In our investigation, we wanted to see if the fifth-year PTs identified the same or different criteria in relationship to the Grade 4 students' written argumentation and how they valued the different representations. We also wanted to see if differences in expectations about acceptable representations, from different cultural/language backgrounds, could provoke discussions about evaluation criteria.

Data collection and analysis

In this paper, we analyse a transcript of an audio-recording of a group of three PTs discussing the four student examples, in regard to which ones could be considered clear/understandable, complete and mathematically correct. The PTs were also asked to describe the future teaching, the students would need, based on their analysis. The whole class discussions, which followed both episodes of small group work, are also analysed. The PTs used English to communicate together but none of them were fluent users of English for discussing mathematics education. At the beginning of the workshop, the PTs were asked to construct a proof for why two odd numbers added together produced an even number and then produce a proof which could be used to convince Grade 4 students. Of the collected examples, all except one provided correct algebraic proofs, with $2n-1$ or $2n+1$ being used to define odd numbers. In the examples for Grade 4 students, all included pictures with no words, usually of groups of pairs of squares. In this paper, the Norwegian preservice teachers are PTN 1 and PTN 2, etc and the international student with AEL, PTI 1. Tamsin was the teacher educator, in charge of this class, and Toril attended as an observer who was known to the PTs from previous interactions.

We began our analysis by identifying discussion points about mathematical argumentation being clear, complete and mathematically correct. We then looked for what had prompted the clarification or elaboration of these points, to see if cultural differences, such as whether the appropriateness of the representations contributed to these discussions points being made.

The PTs were provided with mathematical argumentations from four Grade 4 students: Example 1 (E1) in Figure 1; Example 2 (E2) had a claim and $3 + 3 = 6$; with the two other ones having more elaborated arguments, one in Norwegian (E3) and the other in AEL (E4). Unlike Figure 1, E3 and E4 used both diagrams and written words to show that an odd number added to an odd number always resulted in an even number. There were also differences between E3 and E4. In the Norwegian student's argumentation (E3), an odd number was described as a number with a hole and there were pictures with three examples of odd numbers with arrows pointing at the hole and even numbers with arrows highlighting "no" holes. The drawing and explanation correspond to an algebraic definition of odd numbers as 2n-1. The child also wrote, "So if we have three and nine, we then can do like this so you fill the hole and then it will be an even number." E4 also included drawings of three different diagrams to illustrate even numbers, similar to those in E3. E4 stated that an odd number had an extra block, with two rows of squares forming a rectangle with one square on the top. This explanation corresponds to the algebraic definition of odd numbers as $2n + 1$. As well as a drawing, similar to Figure 1 (E1), this solution included the symbols $+$ and $=$ between the diagrams as a support to the argumentation that two odd numbers added together produced an even number.

Preservice teachers' evaluation criteria for mathematical argumentations

In the following sections, we describe the PTs' criteria for what made mathematical argumentations clear, mathematically correct and complete. Where possible, we then identify what seemed to prompt them to discuss these criteria. In the final section of the paper, we discuss how the different language backgrounds of the PTs provided both potential and realised mathematical learning opportunities and the implications of these opportunities for teacher educators.

Clear

The PTs in the small group questioned what it meant for a mathematical argumentation to be clear. One of the Norwegian preservice teacher, PTN 1, described E2 as, "Just giving an example, instead of doing a mathematically correct proof. So, the category would be clearly written or mathematically? It's mathematically correct but it's not clearly written." E2 could be classified as naïve empiricism (Balacheff, 1988), and although Balacheff (1988) did suggest that some producers would consider it a proof, these PTs did not see it as being sufficient. PTN 2 queried if the student actually understood what they had written, "It looks like they just read this somewhere and just wrote it on their own, but it doesn't necessarily mean they understood." So, it seemed that for the PTs clarity in the mathematical argumentation did not always indicate that a student had a deep understanding of a topic. In the whole class discussion, there was a similar response to a direct question from Tamsin about what clarity was, PTN 7 stated, “If they had examples and had clear reasoning for their claim, we said it was clear.” Clarity was, thus, connected to reasoning which had to be understandable by others, including teachers, with group members suggesting that there should not be a gap in the reasoning, which could be the case if a student just repeated someone else’s words. This description seems similar to those in A. J. Stylianides and Stylianides (2009) where the PTs considered that a proof had to be clear, convincing and logical, without requiring interpreters to make jumps of faith to understand what was happening.

Mathematically Correct

In the small group work, the PTs suggested that mathematically correct was the opposite of mathematically incorrect. Consequently, they did not discuss what would make a mathematically correct proof, but only about whether the symbolic, written or diagrammatic mathematics was correct. From this perspective, the PTs considered that all the examples were correct as included “incorrect” mathematics. This was exemplified in the plenary discussion when PTN 4 stated that to be mathematically correct, the example could not be wrong. Thus, there seemed to be a consensus at the social level of the PTs about the criteria to do with mathematical correctness.

However, when queried by Tamsin in the plenary session, PTN 5 stated, “that the equation has to be right at least or correct if there is an equation in the example and also the argumentation should be good.” This, then, switched the focus from the mathematics in the argumentation to what constituted a correct mathematical argumentation. However, for the PTs this seemed to be connected more to the criteria about completeness and so is discussed in the next section.

Complete

During the discussion, it became clear that for the argumentation to be considered complete, it had to be mathematically correct and clear. In discussing the elaborated Norwegian example (E3), the PTs distinguished between what other fourth grade students would understand and what was required of PTs providing a mathematical proof. This suggests they understood that the social group for whom the mathematical argumentation was produced would affect whether it was considered sufficient.

In discussing E2, PTI 1 stated, “It’s just an example, so is it mathematically correct but not complete.” This reinforces the earlier point from the whole class discussion about the difference between between being mathematically correct and being a mathematically-correct argumentation. PTN 2, in the small group, considered that complete argumentations often (if not always) included visualisations. The proofs with visualisation were acknowledged as “coming a bit further”, and PTI 1 reinforced this by stating that “they show the concept of even and odd number.” However, in discussing the argumentation in Figure 1 (E1), PTN 2 acknowledged that just having a visualisation was not sufficient, “It just shows that these two if you add them, they become an even number but it doesn’t say anything … It’s just two bits put together, it’s nothing.”

The importance of understanding how context affected whether a mathematical argumentation was complete came up both in the small group session, around whether just a visualisation was sufficient, and also in the plenary session. As a result of a direct question from Tamsin about how they knew that E1 showed a proof of how adding two odd numbers always resulted in an even number, the Norwegian PTs stated that they could use their background knowledge about these representations from textbooks, “I would say our background knowledge because we have seen this picture of the blocks many times before, but we have no idea what the students are thinking about this, so there is no way of knowing.” They used their background knowledge to understand that E1 (Figure 1) could be a generic example (Balacheff, 1988), where they could interpret essential features of the sets of odd and even numbers from the representations. Still, they also acknowledged that they could not be sure that the student intended it to be viewed in this way. It was their background and knowledge of common cultural artefacts that allowed them to interpret whether mathematical argumentations were complete or not.

For PTI 1, the example provided in their language was the clearest. Nonetheless, they insisted that it needed a definition of an odd number for it to be complete. In the example, even numbers were defined with a series of diagrams where boxes were paired to show 2, 4, and 6. PTI 1 seemed to value the use of mathematical symbols, $+$ and $=$, which provided more information about how to interpret the arrows in the diagrams. Neither of the other two PTs in this small group reacted to this point, by showing either agreement or disagreement. In contrast to the Norwegian PTs, PTI 1 struggled to consider E1 as being mathematically correct, let alone a complete mathematical argumentation. In the task about how to work with the students to improve their mathematical understanding, PTI 1 stated that as a teacher, she would have the student talk about what odd and even numbers were and what their diagram meant. She seemed to be indicating that she saw it only as a single case, naïve empiricism (Balacheff, 1988), rather than a generic example (Balacheff, 1988) which represented a whole group of numbers.

In the plenary, there was agreement that for a mathematical argumentation to be complete, it had to be both clear and mathematically correct. However, what this meant still seemed to be interpreted differently amongst the PTs, especially when they had different language backgrounds. In the task, where the PTs had to decide what they would do as teachers with a class that included the four students whose examples they had analysed, the Norwegian PTs described E3 written in Norwegian as being exemplary. By mentioning a “hole”, E3’s diagrams suggested a 2n-1 understanding of odd numbers. In contrast, PTI 1 suggested that the student, who had produced E2, could be shown E4, which included a diagram with one extra to represent odd numbers, to illustrate how to elaborate their mathematical argumentation of a statement and a symbolic example ($3 + 3 = 6$). PTN 2 agreed with this point because they stated that the student who had made E2 had a similar approach to the student who produced E4. This led to a change in thinking for PTN 1 and PTN 2 who had originally placed E4 with E1 and E2 as needing improvement. However, it was not until just before the final plenary, that the PTs realised that they had not discussed how to teach all students so that their contributions could be valued, including E4 written in AEL.

Potential and realised mathematical learning opportunities.

In the data, it is possible to see that the PTs were able to identify a range of criteria for evaluating mathematical argumentations, many of which had similarities with those identified by A. G. Stylianides and Stylianides’ (2009) PTs. This was perhaps not surprising considering that these PTs were in their fifth year of their teacher education. The different language backgrounds of the PTs also provided some opportunities for enriching these discussions. For example, in the discussion in the small group and the plenary, different interpretations of E1 (Figure 1) provided an opportunity to raise issues about how cultural artefacts provide meaning only to those who are from the same education systems. In the small group work, it was the PTI 1, who spoke AEL, who initially queried whether E1 was mathematically correct. Although the Norwegian PTs did not state that they disagreed with PTI 1, they seemed to be challenged by this suggestion and this led to a discussion about the importance of context. In the plenary, it was Norwegian PTs from another group, who had work in a group with another PT who spoke AEL, who voiced similar issues to do with the need for context to interpret the mathematical argumentation. Tamsin, who also did not have previous experiences of using drawings of blocks as representations of odd and even number, queried in the plenary whether E1 could be considered mathematically correct. When teacher educators and PTs share the same backgrounds, then expectations about mathematical ideas, including mathematical argumentation can go undiscussed. It was when expectations. connected to the different language backgrounds, were raised that there were increased possibilities for richer discussions and the development of broader understandings. Unlike most of the previous research which has shown that the multilingualism of PTs in mathematics teachers education is often ignored (see Chitera, 2011), multilingual PTs' knowledge about cultural artefacts for teaching mathematics made possibilities available to query assumptions connected to just one language/culture about what were acceptable mathematical argumentations.

Nevertheless, the analysis shows that there were many other possibilities where the PTs' backgrounds were likely to produce alternative interpretations that could have led to richer discussions about the evaluation criteria. This occurred, for instance, around the discussions of the two most elaborated examples where the differences between them, one connected to defining odd numbers as $2n - 1$ by highlighting the "hole" and the other defining odd numbers as $2n + 1$ by focusing on the extra box, were not described and discussed. The different definitions could have led to richer discussions about multiple representations–verbal, diagrammatic, symbolic– and thus may have contributed to clarifying what made mathematical argumentations clear, mathematically correct and complete.

There is likely to be several reasons why these potential mathematics education learning opportunities were never realised. One of them was that all the PTs were discussing these ideas, probably for the first time, in English, a language. This suggests that using an unfamiliar language can result in assumptions about mathematical argumentation not being raised. It also seemed that when the PTs did not know each other well, it was perhaps difficult for them to query each other's understanding. The teacher educator, therefore, has a role in moving the discussions beyond the hindrances caused by a lack of fluency and familiarity with each other. It is also important to explicitly bring the discussions with PTs back to what could be learnt about teaching mathematics in multilingual classrooms. As Thomassen and Munthe (2020) found, Norwegian PTs have noted a lack of input about how to teach subjects like mathematics in multilingual classrooms. It cannot be sure that just participating in this workshop would have made PTs aware of what they could have facilitated students to learn from seeing their language/cultural backgrounds as resources in mathematics lessons. As our project develops, we, as teacher educators, will explore how to support PTs to reflect on and query their own assumptions about mathematical argumentation, even when they speak the same language and have similar previous experiences and what this reflection could contribute to their teaching in multilingual mathematics classrooms.

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References

Austin, J.L., & Howson, A.G. (1979). Language and mathematical education. *Educational Studies in Mathematics*, *10*(2), 161–197.

Balacheff, N. (1988). Aspects of proof in pupils' practice of school mathematics. In D. Pimm (Ed.), *Mathematics, teachers and children* (pp. 216–235). London, UK: Hodder & Stoughton.

Chitera, N. (2011). Language of learning and teaching in schools: An issue for research in mathematics teacher education? *Journal of Mathematics Teacher Education*, *14*(3), 231–246.

Planas, N. (2014). One speaker, two languages: Learning opportunities in the mathematics classroom. *Educational Studies in Mathematics*, *87*(1), 51–66.

Stylianides, A.J. (2009). Breaking the equation "empirical argument = proof." *Mathematics Teaching*, *213*, 9–14.

Stylianides, A.J., & Stylianides, G.J. (2009). Proof construction and evaluations. *Educational Studies in Mathematics*, *72*(2), 237–253.

Stylianides, G.J., & Stylianides, A.J. (2009). Facilitating the transition from empirical arguments to proof. *Journal for Research in Mathematics Education*, *40*(3), 314–352.

Stylianides, G.J., Stylianides, A.J., & Weber, K. (2017). Research on the learning and teaching of proof: Taking stock and moving forward. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 237–266). Reston, VA, USA: National Council of Teachers of Mathematics.

Thomassen, W. E., & Munthe, E. (2020). A Norwegian perspective: Student teachers' orientations towards cultural and linguistic diversity in schools. *Teaching and Teacher Education*, *96*, 1-11.

Trinick, T., Meaney, T., & Fairhall, U. (2014). Teachers learning the registers of mathematics and mathematics education in another language: An exploratory study. *ZDM*, *46*(6), 953–965.

Ure, F.K. (2018). *Argumenterande skriving på barneskulen: Ein analyse av elevar sine argumenterande matematikktekstar på 4. og 7. trinn* (Argumentative writing in primary school: An analysis of students' own mathematical argumentation texts for Grades 4-7). (Master's thesis, Western Norway University of Applied Sciences, Norway).

Van Bendegem, J.P. (1993). Foundations of mathematics or mathematical practice: Is one forced to choose? In S. Restivo, J.P. van Bendegem, & R. Fischer (Eds.), *Math worlds: Philosophical and social studies of mathematics and mathematics education* (pp. 21–38). New York, USA: State University of New York Press.