Effective Fuzzy System for Qualifying the Characteristics of Stocks by Random Trading

Mu-En Wu, Jia-Hao Syu, Jerry Chun-Wei Lin*, and Jan-Ming Ho

***Abstract*—Trading strategies can be divided into two categories; i.e., those with momentum characteristic and those that appear contrarian. The characteristics of trading strategies have been widely studied; however, there has been relatively little work on the characteristics of stocks. Furthermore, there is no standard approach to the classification of stocks in terms of momentum and contrarian. This paper presents a fuzzy momentun contrarian uncertain characteristic system for the classification and quantification of stock characteristics. Random trading, stop-loss, and take-profit mechanisms are first used to identify characteristics, and then a novel profitability index with type-2 fuzzy-set module is used to quantify them. In the experiments, 41 stocks on the Taiwan 50 index were deemed suitable for momentum strategies, whereas 9 stocks were deemed suitable for contrarian strategies. An uphill relationship between profitability index and trading performance is observed, which produced correlation coefficients of 0.148–0.539, and classification accuracy of 52.0%–60.0%. However, the proposed system greatly improved classification performance, resulting in correlation coefficients of 0.572–0.722 with accuracy of 63.6%–84.5%. In the real-world application, the proposed system outperforms the benchmark among all datasets, and increases the profitability by 1.5 times on Taiwan 50 dataset. These results clearly demonstrate the efficiency of the proposed system in the quantification and classification of stocks suited to momentum- and contrarian-type trading strategies and also in the real-world applications.**

***Index Terms*—Profitability index, random trading, momentum, contrarian**

I. Introduction

The allocation of financial assets to companies or commodities in expectation of gaining a profit (i.e., investment) is crucial to economic growth [1], and trading strategies are crucial to investment performance. Overall, trading strategies can be divided into two categories; i.e., those with momentum characteristic and those that appear contrarian [2]. Momentum-type strategies are based on the belief that the price will follow recent trends [3]. Contrarian-type strategies are based on the belief that prices will move against recent trends [4]. These two types of strategy also tend to generate opposing trading signals.

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The characteristics of trading strategies have been widely studied [5]; however, there has been relatively little work on the characteristics of stocks. Furthermore, there is no standard approach to the classification of stocks as momentum-type or contrarian-type. Searching among the thousands of existing trading strategies is time-consuming and largely ineffectual. The adoption of an erroneous trading strategy or misidentifying the characteristics of the target stock can result in enormous losses. Investors need a system to facilitate the classification and quantification of stocks to inform their decisions with regard to trade strategies. In this paper, a system based on fuzzy analysis methods is presented, referred to as the Fuzzy mOmentun Contrarian Uncertain characteristic System (FOCUS). This paper makes the following contributions:

1) Random trading algorithms are designed to analyze the characteristics of stocks.
2) A profitability index, which uses a type-2 fuzzy-set is developed to quantify those characteristics.
3) An uncertainty factor in the system is devised to filter out stocks that resist classification.
4) The proposed system helps to elucidate the characteristics of stocks and thereby eliminates the time wasted assessing unsuitable trading strategies.

Douglas [6] defined random trading as the poorly-planned process of making trades without the guidance of a plan based on informative data (i.e., prices or market information). Nonetheless, a random trading strategy can be used to reveal investment behaviors and the characteristics of stocks and trading strategies [7]. Among the thousands of trading strategies that have been developed in the field of finance, stop-loss [8] and take-profit [9] are two common momentum-type and contrarian-type strategies. Several studies investigated for the momentum and contrarian effect [10] through the stop-loss and take-profit mechanisms [11]. In this paper, a random trading based on stop-loss and take-profit strategies is employed to investigate the characteristics of stocks. A Profitability Index (PI) is then created to indicate the trading performance of a given stock under momentum- and contrarian-type strategies. The proposed PI aims to quantify the degree of the suitability of target stock to momentum- and contrarian-type trading.

Unfortunately, the intangibility of momentum and contrarian concepts hinders the task of quantify the degree of these characteristics. In addition, the degree of characteristics is not absolute and well-defined, but relative and uncertain. Fuzzy-set theory [12] is used to model situations in a manner that makes it easier for humans to make rational decisions in uncertainty and imprecision environments. Type-2 fuzzy-set [13] can handle the rule uncertainties and capture more information for further process rather than the traditional type-1 fuzzy-set by the crisp value. They can be used to classify results into binary situations and transform values into linguistic terms based on their membership degree. In this study, a type-2 fuzzy-set [13] is designed to interpret the PI and characteristics of stocks. A third characteristic, uncertainty, is also defined to filter out stocks that resist classification. Essentially, the proposed FOCUS characterizes stocks in terms of momentum and contrarian using a random trading algorithm in conjunction with the profitability index (PI) and a type-2 fuzzy-set module.

FOCUS was applied to stocks on the Taiwan 50 index (TW50). Experimental results revealed that 41 of the stocks presented a positive momentum PI and negative contrarian PI, indicating that they are relatively suitable and profitable under momentum trading strategies, and vice versa. The weak and moderate uphill relationships between momentum (contrarian) PIs and the trading performance of momentum (contrarian) strategies with correlation coefficients of 0.148 to 0.539 are also observed, and classification accuracy is from 52.0% to 60.0%. The designed type-2 fuzzy-set module with uncertainty factor greatly improved classification performance, resulting in correlation coefficients of 0.572 to 0.722, and classification accuracy of 63.6% to 84.5%. In addition, the proposed FOCUS is applied to the real-world applications, that is using the FOCUS classification for stock selection (trade the momentum/contrarian stocks with momentum/contrarian type strategies). The results show that FOCUS-selected outperforms the benchmark among all datasets, and increased the profitability by 1.5 times in the opening gap strategy [14] on TW50 dataset. These results demonstrate the effectiveness of the proposed FOCUS in the quantifying and classifying stocks as contrarian- or momentum-type and the real-world applications.

II. Literature Review

In this section, background of momentum and contrarian characteristics are introduced in Section II-A. The commonly-used indicators for financial trading are studied in Section II-B. Furthermore, the fuzzy-set theory is stated and discussed in Section II-C.

A. Momentum and Contrarian Characteristics

Momentum and contrarian are usually used to describe the investment behavior of the trading strategies [2], and up to now there is no standard strategy to determine those characteristics for stocks. Momentum strategy refers to a situation in which price movements are driven by momentum. Chan et al. [15] believed that past returns and earnings are efficient to help for predicting large drifts in future returns, and it indicates that the future price trend of the stock is similar as its past trend. Opening range breakout is a common momentum strategy that believes when the price exceeds a given threshold, the price continuously moves toward to the same trend in the near past. Holmberg et al. [16] discovered that the normally-distributed returns can thus be obtained by the opening range breakout strategy. Tsai et al. [17] and Syu et al. [18] showed that the OBR strategy obtains better profitability when it is utilized in Taiwan financial market, especially when used in conjunction with the evolutionary algorithm [19] and multi-objective optimization [20].

Contrarian strategy is based on the concept of mean reversion [21], which supports to the market over-reaction and delayed-reaction [22] hypotheses. Under this scenario, the pricing trends tend to be remained within a certain range. Thus, when a price drifts from the mean (i.e., beyond a certain range), it is assumed that the price will return to the mean. In other words, the concept of contrarian strategy is contrast to the concept of momentum strategy. Bollerslev et al. [23] proposed the mean reversal strategy to measure the volatility and price trends of stocks over time. After that, Bollinger bands became a well-known contrarian technical index in the field of empirical trading and research [24].

Stop-loss [8] and take-profit [9] are two commonly-used mechanisms in the financial trading. Stop-loss mechanism shows when you invest in a market and once your unrealized losses are greater than a given threshold, you should clear all of your position (the stocks you hold). It believes that if your losses are larger than a threshold, your losses will become larger and larger, therefore, the stop-loss mechanism is considered as momentum strategy. Take-profit mechanism states that once your unrealized gains are greater than a given threshold, you should clear all of your position. It believes that if your gains are larger than the threshold, your gains will start to fall down; therefore, the take-profit mechanism is considered as contrarian strategy. Several studies are investigated for the momentum and contrarian effect through the stop-loss and take-profit mechanisms [10]. Wu and Chung [11] used the stop-loss and take-profit mechanisms to evaluate the effects of momentum in the empirical studies.

B. Financial Trading Indicators

In the financial field, there are some commonly-used indicators to evaluate the trading performance. All of the indicators focus on measuring profitability and risk [25]. To measure the profitability, **total profit** and **annual return** are utilized in financial trading. Total profit is the sum of profit during the entire trading period, which is affected by the length of the trading period. Annual return refers to the annualized rate of return [26], which is the total profit divided by the average costs during the trading period, and then divided by the number of the years.

To measure the trade-off between profitability and risk, **profit factor** [27] and **Sharpe ratio** [28] are used in financial trading. Profit factor is the net profits divided by the absolute value of the net losses [29], which indicates how much profit can be earned in the face of a dollar loss. Sharpe ratio is the total revenue divided by the standard deviation of the daily profits, which is used to indicate the trade-off between profitability and risk; i.e., how much profit can be earned under a unit of risk (volatility) [30]. Generally, a strategy with higher Sharpe ratio is more attractive to investors. The **annual return** is utilized to evaluate the profitability since it standardizes the trading period and make it be a generic performance indicator.

Also, the **Sharpe ratio** is also used to evaluate the trade-off between profitability and risk in the following experiments, which is one of the most commonly statistical indicator in financial field [30].

C. *Fuzzy-set Theory*

Fuzzy-set theory [12] aims at modeling the imprecise situations for reasoning that helps human make rational decisions in uncertainty and imprecision environments. It does not only classify the results into two situations (yes or no) but transform the values into the linguistic terms with their membership degrees. Fuzzy-set theory has been widely used in engineering [12], finance [31], and even expert systems [32], and helps us efficiently solve the limitation of crisp-set. Buckley [31] employed the fuzzy present value, fuzzy future value, and fuzzy interest rates in the mathematical finance. Yu et. al [33] proposed a fuzzy-neuro system, which expresses the probabilities and system parameters through fuzzy sets, and inherits the advantages of both fuzzy-set theory and neural networks.

Most conventional fuzzy-set theory belongs, however, to type-1 fuzzy-set, which indicates that the uncertainty does not really take into account in conventional and classic type-1 fuzzy-set theory. Karnik et al. [13] introduced a type-2 fuzzy-set system, which can handle rule uncertainties and capture more information than defuzzified value (a crisp fuzzy number). According to the definition in [34], a type-1 fuzzy-set A can be expressed as:

$$A: \ X \rightarrow I,$$

where X is the universe of discourse (independent variable), and I is the universe of fuzzy degree, $[0, 1]$.

Let μ_A be the membership function of type-1 fuzzy-set A, which can be expressed as:

$$\mu_A(x) = u,$$

where $x \in X$ and $u \in I$.

For the type-2 fuzzy-set, a type-2 fuzzy-set $\tilde{A}$ can be expressed as:

$$\tilde{A}: \ X \rightarrow I^I,$$

where I is the universe of the range of fuzzy degrees, $I \rightarrow I$,$[0, 1] \rightarrow$ [0,1].

Let $\mu_{\tilde{A}}$ be the membership function of type-2 fuzzy-set $\tilde{A}$, which can be expressed as:

$$\mu_{\tilde{A}}(x) = v,$$

where $x \in X$ and $v \in I^I$.

The bounded region between upper and lower membership functions can be presented by the footprint of uncertainty (FOU) [34], which is a measurement of uncertainty for type-2 fuzzy-set [35]. The type-2 fuzzy-set theory is able to cope with the uncertainties which is suitable to characterize the stocks. Thus, it is suitable to determine an interval of fuzzy degree for a stock.

In fuzzy-set theory, membership functions are used to transform the inputs to the linguistic terms with the corresponding degrees [36]. Commonly-used membership functions includes triangular, trapezoidal, Gaussian, R- and L-functions [37]. R-function contains two thresholds for independent variables. L-function is almost symmetrical to the R-function in the horizontal direction. R- and L-functions can describe degrees from small to large (large to small) as independent variable increases, and they are approximately linear transformations that convert a independent variable to a fuzzy degree between 0 and 1. Trigonometric function [38] contains a vertex shape and two thresholds for the independent variable. If the independent variable is not between the thresholds, it will be set to 0. Otherwise, if the independent variable is between the thresholds and is closer (farer) to the vertex, the dependent variable is considered as a large number.

III. Proposed Fuzzy Momentun Contrarian Uncertain Characteristic System

In this section, the proposed FOCUS is outlined, and its flowchart is presented in Fig. 1. FOCUS comprises three main modules as: random trading algorithm (**RTA**), profitability index (**PI**), and fuzzy-sets quantification module (**FQ**), which will be respectively discussed in the Sections III-A, III-B, and III-C. In addition, there is an illustrative example of the designed FOCUS presented in Section III-D.

FOCUS first executes **RTA** by historical price data, which includes the momentum RTA (MomRTA) and contrarian RTA (ConRTA). The **RTA** generates two distributions of annual returns, which are then used to calculate the **PI** for use in assessing overall trading performance and quantifying various stock characteristics. Then, the **FQ** module takes the **PI** as input, and transforms it into the fuzzy degrees of each characteristic. The outputs of FOCUS are the degrees of momentum, contrarian, and uncertain characteristics, which are subsequently used for stock classification and selection.

The performance of FOCUS was examined by conducting the collected TW50 dataset from Sep. 2015 to Dec. 2019, which was provided by Taiwan Stock Exchange (TWSE). The data period from Sep. 1, 2015 to Sep. 30, 2019 (1,000 trading days) is used as the training data by performing the **RTA**, **PI**, **FQ**, and parameter fitting. The data period from Oct. 1, 2019 to Dec. 20, 2020 (300 trading days) is used as

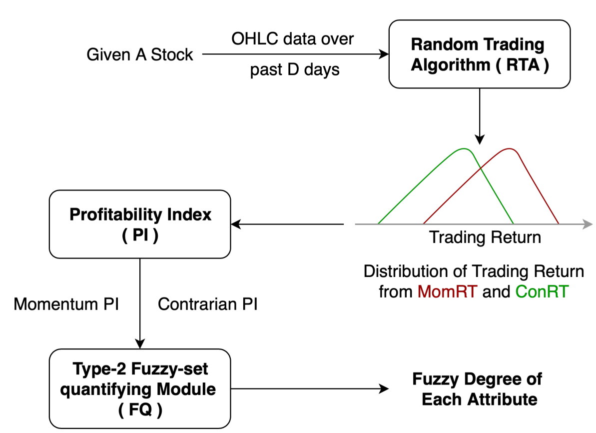


Fig. 1: The flowchart of the proposed FOCUS.

the testing data for effectiveness evaluation of the proposed FOCUS. TW50 data includes the Opening, Highest, Lowest, and Closing price on each trading day (OHLC) [39], which are four most informative prices in a trading day and roughly describe the volatility and trend of a day. Each module in the FOCUS is described below.

A. Random Trading Algorithm

Random trading generates trading signals without referring to any other information [7], which makes it ideal to investigate pure momentum and pure contrarian characteristics of a stock. In this section, two random trading algorithms (**RTA**) are outlined in Algorithm 1. The first momentum algorithm (MomRTA) uses random trading with a stop-loss mechanism to evaluate momentum characteristics. The second contrarian algorithm (ConRTA) uses random trading with a take-profit mechanism to evaluate contrarian characteristics. $OHLC_D$ refers to OHLC data of D days, and is used as the input of RTA.

Algorithm 1 RTA for evaluating momentum (contrarian) characteristic, called MomRTA (ConRTA)

Input: N, δ_{SL} (δ_{TP}), $OHLC_D$
Output: A distribution of N annual returns

```
1:  while Repeat N times do                              ▷ N times sampling
2:      for d from 1^st to D^th day do                   ▷ each trading day
3:          Randomize a 0 or 1;                          ▷ random Trade
4:          if 1 then
5:              d^th position → long at opening price;
6:          else
7:              d^th position → short at opening price;
8:
9:          During the day,
10:         if unrealized loss >δ_SL then
11:             Clear d^th position immediately;
12:                                          ▷ stop-loss, only in MomRTA
13:         if unrealized gain >δ_TP then
14:             Clear d^th position immediately;
15:                                        ▷ take-profit, only in ConRTA
16:         else
17:             Clear d^th position at the market closes;
18:         Accumulate the return of d^th day;
19:     Record a overall annual return of the sampling.
```

In Algorithm 1, N is the number of sampling times, and $OHLC_D$ is the OHLC data of a D-day period, and δ_{SL} and δ_{TP} are threshold of stop-loss and take-profit. All trading associated with MomRTA and ConRTA involves intra-day strategies, in which the algorithms take (perform) and then clear a position on a given day, and then repeat this strategy on every trading day. **RTA** first randomly determines whether to take a long (buy) or short (sell) position when the market opens (at opening price) on each trading day (Algorithm MomRTA and ConRTA, Lines 3 to 7). MomRTA employs a stop-loss mechanism in which an unrealized loss exceeding δ_{SL} (threshold of stop-loss) at any time triggers the immediate clearing of the position (Algorithm MomRTA, Lines 10 to 12). If the stop-loss mechanism is not triggered, then the position is cleared when the market closes (Algorithm MomRTA, Lines 16 to 17). ConRTA employs a take-profit mechanism in which an unrealized gain exceeding δ_{TP} (threshold of take-profit) at any time triggers the immediate clearing of the position (Algorithm ConRTA, Lines 13 to 15). If the take-profit mechanism is not triggered, then the position is cleared when the market closes (Algorithm ConRTA, Lines 16 to 17). Two thresholds (δ_{SL} and δ_{TP}) are set to 1% (without a loss of generality), and set the D to 1,000 days (roughly four years), which should be long enough to observe the characteristics of a stock.

The random trading mechanism was implemented for the 1,000 trading days to calculate the a **annual return** (Algorithm MomRTA and ConRTA, Line 19). The 1,000-day random trading process is referred to as a sampling (i.e., one sample). Due to the randomness in the developed algorithm, the annual return can be attributed solely to a distribution. After implementing sampling several (N) times, it is possible to obtain the realistic annual return distribution (Algorithm MomRTA and ConRTA, Output). The experiments used to determine an appropriate N value are outlined in Section IV-A.

B. Profitability Index

A profitability index (**PI**) is first designed here to evaluate the overall trading performance of a return distribution. The PI value should be proportional to the expected return (profitability) for a given distribution, and inversely proportional to the standard deviation (risk) of the distribution. This is essentially the same idea as the Sharpe ratio [28] (expected profitability divided by risk). Thus, PI is defined as the mean of the distribution from RTA divided by the standard deviation (SD) of the distribution, as shown in Equation (1).

$$PI = \frac{\frac{1}{N}\sum_{i=1}^{N} Ret_i}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(Ret_i - \overline{Ret})^2}} \tag{1}$$

$$Ret_i = \sum_{j=1}^{D} DR_{i,j}$$
$$DR_{i,j} = \begin{cases} \delta_{SL} & \text{stop-loss first} \\ \delta_{TP} & \text{take-profit first} \\ \frac{Close-Open}{Open} \text{ or } \frac{Open-Close}{Close} & \text{neither occurred} \end{cases} \tag{2}$$

Suppose that the **RTA** generates a set of cumulated returns $[Ret_1, Ret_2, \ldots, Ret_N]$ from N times sampling. Ret_i is the i-th cumulated return of D-day random trading, $DR_{i,1}, DR_{i,2}, \ldots, DR_{i,D}$, as shown in Equation (2). The rules of $DR_{i,j}$ are also shown in Equation (2). It shows that if the stop-loss (take-profit) comes first before the market is closed, the return of i^{th} day for $DJ_{i,j}$ is δ_{SL} (δ_{TP}), since the loss (gain) is realized immediately. If neither the stop-loss nor take-profit has occurred before the market is closed, the return of i^{th} day is $\frac{Close-Open}{Open}$ for long position or $\frac{Open-Close}{Close}$ for short position. Since the trading price and rules are given, the $DR_{i,D}$ is determined for both the long and short position of each day. The only randomness is to take a long or short position on a day, which is a fixed and independent 50-50 chance in **RTA**. Therefore, the $DR_{i,D}$ is designed as an independent and identically distributed ($i.i.d$) and equiprobable random variable.

TABLE I: The PI_Mom and PI_Con of TW.2330

	Mean of Return	SD of Return	PI_Mom & PI_Con
MomRTA	0.01252	0.07430	0.16850
ConRTA	-0.01145	0.06618	-0.17301

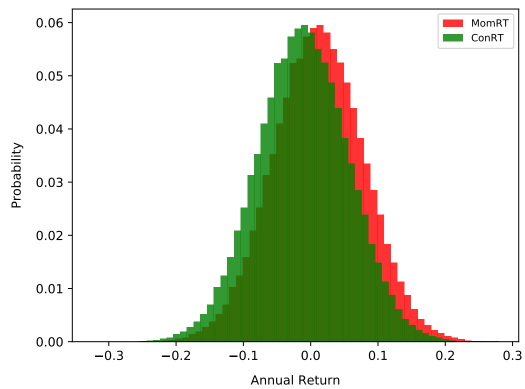


Fig. 2: The histograms of the return distributions generated by MomRTA (red) and ConRTA (green) on TW.2330.

In Equation (2), the molecular of PI is the mean of the return distribution, which indicates the profitability of the **RTA**. A higher mean indicates higher profitability and produces a higher PI value, and vice versa. The denominator of PI is the SD of the distribution, where $\overline{Ret}$ is the average value of all Ret_i. The SD indicates the concentration and volatility of the **RTA**, and a higher denominator states higher risk and lower PI. An ideal strategy produces a higher mean and lower SD; i.e., trading efficiency is proportional to the value of PI. A PI based on MomRTA distribution is referred to as PI_Mom. A PI based on ConRTA distribution is referred to as PI_Con. The PI_Mom and PI_Con obtained for each stock can be used to represent the momentum and contrarian characteristics of that stock.

Take TW.2330[1] as an example to illustrate the steps involved in implementing the proposed FOCUS. Fig. 2 shows the histograms of annual returns (Ret) sampled by MomRTA distribution (red) and ConRTA distribution (green) under 10,000 (N=10,000) sampling runs. Note that Ret is a continuous random variable sampled from Equation (2). However, to illustrate the Gaussian-like distribution, the histograms in discrete buckets (bucket size of 1%) are presented. As shown in Table I, the mean and SD of the MomRTA (red) distribution are 0.01252 and 0.07430, respectively. Thus, PI_Mom is 0.16850 (i.e., 0.01252/0.07430 = 0.16850). The mean and SD of the ConRTA (green) distribution are -0.01145 and 0.06618, respectively. Thus, PI_Con is -0.17302 (i.e., -0.01145/0.06618 = -0.17301). The positive PI_Mom of TW.2330 is also larger than PI_Con, which indicates that TW.2330 would be profitable under MomRTA and is therefore more likely to be a momentum-type stock.

[1]TW.2330 is the company with the largest capital value on the Taiwan stock market

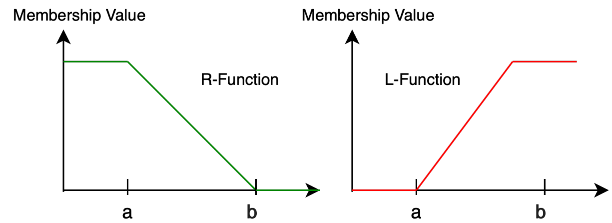


Fig. 3: Membership function of R- and L-functions.

C. Developed Fuzzy-set Quantifying Module

In the fuzzy-set quantifying module (FQ), a type-1 and a type-2 **FQ** are developed, respectively presented in Sections III-C1 and III-C2. Furthermore, in order to well handle the uncertainty in the designed system, a third characteristic is proposed in the type-2 **FQ**, namely uncertainty, presented in Section III-C3.

1) Type-1 fuzzy-set quantifying module for momentum and contrarian: PI_Mom and PI_Con are used to evaluate momentum and contrarian characteristics; however, setting an appropriate threshold by which to classify the characteristics of a stock is not a trivial matter. Thus, a fuzzy quantification model (**FQ**) based on type-1 fuzzy-set theory to facilitate the interpretation of stock characteristics is firstly designed. **FQ** takes PI_Mom and PI_Con of a stock as inputs, from which it respectively generates fuzzy degrees for momentum and contrarian characteristics.

Two membership functions are used in type-1 **FQ** such as R- and L-functions to obtain two change points, which controls the slope of the membership functions, as shown in Fig. 3. Both of them are the two special cases of trapezoidal membership functions that are commonly used membership functions in the fuzzy-set theory [37]. When the input of the R-function (L-) is less than the lower change point a, the membership value of the R-function (L-) will be 1 (0); otherwise, when the input of the R-function (L-) is greater than the higher change point b, the membership value of the R-function (L-) will be 0 (1). If the input is between two charging points, the slope of the R-function (L-) will be $\frac{-1}{b-a}$ ($\frac{1}{b-a}$). The R- and L-functions represent the linear relationship between the input and the degree of membership within two change points, and simplify the strong and weak input signals to 0 or 1.

Since the FOCUS produces two PI values, two type-1 membership functions respectively for PI_Mom and PI_Con, are defined which are momentum membership function (**MMF**) and contrarian membership function (**CMF**). Both of them are L-functions. The type-1 membership functions map PIs to type-1 fuzzy degrees, which is denoted as:

$$\textbf{MMF}: PI \to I, \qquad PI = \mathcal{R},$$
$$\textbf{CMF}: PI \to I, \qquad I = [0, 1],$$

where PI is the universe of the PIs (the real numbers) and I is the universe of fuzzy degree. The **MMF** and **CMF** respectively generate the type-1 momentum and contrarian fuzzy degrees

(*Mom* and *Con*) under the given PI_Mom and PI_Con, which are respectively denoted as:

$$\begin{aligned} \mathbf{MMF}(\text{PI\_Mom}) &= \mathit{Mom}, \\ \mathbf{CMF}(\text{PI\_Con}) &= \mathit{Con}, \end{aligned}$$

where PI_Mom, PI_Con $\in PI$ and *Mom*, *Con* $\in I$.

The two membership functions used in the type-1 **FQ** are shown in Equations (3) and (4). Since momentum and contrarian characteristics are symmetric, therefore, a regulation of $\alpha = \beta$ is set to ensure that the membership functions are also symmetric (only one parameter is to be optimized). The **MMF** (**CMF**) states that if the input PI_Mom (PI_Con) is larger than a given threshold α (β), the ouput type-1 momentum (contrarian) degree is 1. If the input PI_Mom (PI_Con) is smaller than a given threshold $-\alpha$ ($-\beta$), the output type-1 momentum (contrarian) degree is 0. Otherwise, the output type-1 degree is $(\text{PI\_Mom} + \alpha)/2\alpha$ ($(\text{PI\_Con} + \beta)/2\beta$).

For example, consider a stock with PI_Mom of 2.5 and PI_Con of -2.7, under the assumption that $\alpha = \beta = 5$. The PI_Mom of 2.5 is converted to *Mom*: 0.75 by Equation (3), and the PI_Con of -2.7 is converted to *Con*: 0.23 by Equation (4). In summary, the stock is with the type-1 momentum degree of 0.75 and with the type-1 contrarian degree of 0.23. Note that the parameter α (β) should be selected through the training data in the following experiments.

$$\mathbf{MMF}(\text{PI\_Mom}) = \begin{cases} 0 & \text{PI\_Mom} < -\alpha \\ \frac{\text{PI\_Mom}+\alpha}{2\alpha} & -\alpha < \text{PI\_Mom} < \alpha \\ 1 & \text{PI\_Mom} > \alpha \end{cases} \tag{3}$$

$$\mathbf{CMF}(\text{PI\_Con}) = \begin{cases} 0 & \text{PI\_Con} < -\beta \\ \frac{\text{PI\_Con}+\beta}{2\beta} & -\beta < \text{PI\_Con} < \beta \\ 1 & \text{PI\_Con} > \beta \end{cases} \tag{4}$$

2) Type-2 fuzzy-set quantifying module for momentum and contrarian: To better handle uncertainty and make full use of the two PIs, a type-2 **FQ** is then designed to facilitate the interpretation of stock characteristics. The type-2 momentum and contrarian membership functions ($\widetilde{\mathbf{MMF}}$ and $\widetilde{\mathbf{CMF}}$) are also developed containing R- and L-functions. The type-2 membership functions map PI pairs to type-2 fuzzy degrees, which are respectively denoted as:

$$\begin{aligned} \widetilde{\mathbf{MMF}} &: (PI, PI) \rightarrow I^I, \qquad PI = \mathfrak{R}, \\ \widetilde{\mathbf{CMF}} &: (PI, PI) \rightarrow I^I, \qquad I^I = I \rightarrow I \;=\; [0,1] \rightarrow [0,1], \end{aligned}$$

where PI is the universe of the PIs, and (PI, PI) is the universe of the PI pairs. In addition, I^I is the universe of type-2 fuzzy degree, and is the range $I \rightarrow I$, $[0,1] \rightarrow [0,1]$. The $\widetilde{\mathbf{MMF}}$ ($\widetilde{\mathbf{CMF}}$) takes both PI_Mom and PI_Con as inputs, and generates the type-2 momentum (contrarian) fuzzy degrees $\widetilde{Mom}$ ($\widetilde{Con}$), denoted as:

$$\begin{aligned} \widetilde{\mathbf{MMF}}(\text{PI\_Mom}, \text{PI\_Con}) &= \widetilde{Mom}, \\ \widetilde{\mathbf{CMF}}(\text{PI\_Mom}, \text{PI\_Con}) &= \widetilde{Con}, \end{aligned}$$

where PI_Mom, PI_Con $\in PI$ and $\widetilde{Mom}, \widetilde{Con} \in I^I$.

Since the concept of momentum and contrarian is completely contrary to each other, the momentum with low linguistic term is considered as the contrarian, and vice versa (the contrarian with low linguistic term is considered as momentum). Thus, the type-2 $\widetilde{\mathbf{MMF}}$ converts a PI_Mom by **MMF** and a PI_Con by **CMF**, and the type-2 momentum fuzzy degree is defined as $\mathbf{MMF}(\text{PI\_Mom}) \rightarrow 1 - \mathbf{CMF}(\text{PI\_Con})$, as shown in Equation (5). Similarly, the type-2 $\widetilde{\mathbf{CMF}}$ converts a PI_Con by **CMF** and a PI_Mom by **MMF**, and the type-2 contrarian fuzzy degree is defined as $\mathbf{CMF}(\text{PI\_Con}) \rightarrow 1 - \mathbf{MMF}(\text{PI\_Mom})$, as shown in Equation (6).

$$\begin{aligned} &\widetilde{\mathbf{MMF}}(\text{PI\_Mom}, \text{PI\_Con}) = \\ &\qquad \mathbf{MMF}(\text{PI\_Mom}) \;\rightarrow\; 1 - \mathbf{CMF}(\text{PI\_Con}) \end{aligned} \tag{5}$$

$$\begin{aligned} &\widetilde{\mathbf{CMF}}(\text{PI\_Con}, \text{PI\_Mom}) = \\ &\qquad \mathbf{CMF}(\text{PI\_Con}) \;\rightarrow\; 1 - \mathbf{MMF}(\text{PI\_Mom}) \end{aligned} \tag{6}$$

A simple example of the designed type-2 **FQ** is shown in Fig. 4. For example, consider a stock with PI_Mom of 2.5 and PI_Con of -2.7, under the assumption that $\alpha = \beta = 5$. The $\widetilde{\mathbf{MMF}}$ maps the PI_Mom of 2.5 to 0.75 (by **MMF**) and the PI_Con of -2.7 to 0.77 (by 1 - **CMF**). The type-2 momentum fuzzy degree is thus $0.75 \rightarrow 0.77$. Similarly, the $\widetilde{\mathbf{CMF}}$ maps the PI_Con of -2.7 to 0.23 (by **CMF**) and the PI_Mom of 2.5 to 0.25 (by 1 - **MMF**). The type-2 contrarian fuzzy degree is thus $0.23 \rightarrow 0.25$.

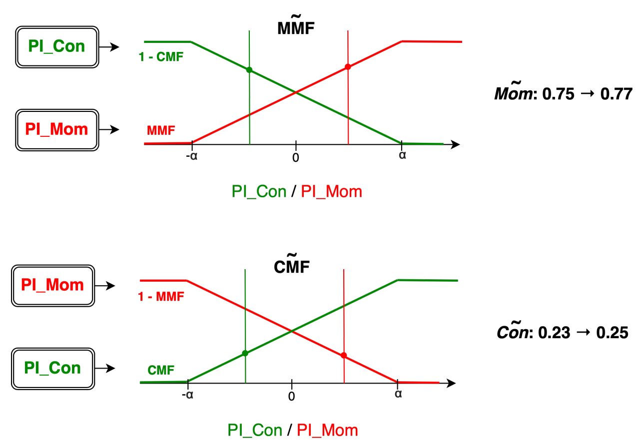


Fig. 4: Two membership functions of **MMF** and **CMF**.

Fig. 5 shows the footprint of uncertainty (FOU) of the designed type-2 **FQ** with $\alpha = \beta = 5$. The *y*-axis represents the fuzzy degrees, and the *x*-axis represents the PI_Mom, and the gray area is the FOU (distance of fuzzy degrees, uncertainty). Since the proposed type-2 **FQ** has two input values, the PI_Con to 2.5 is fixed set in Fig. 5 to present the FOU under single variable (PI_Mom). It can be found that there is the lowest and zero uncertainty when the PI_Mom is -2.5. Moreover, the FOU is composed of a L-function and a horizontal line, which come from the **MMF** and the constant PI_Con of 2.5 (mapped by 1-**CMF**).

In order to investigate the influence of two variables, a two-dimensional heatmap whose color indicates the distance of fuzzy degrees (uncertainty) is presented, as shown in Fig. 6.

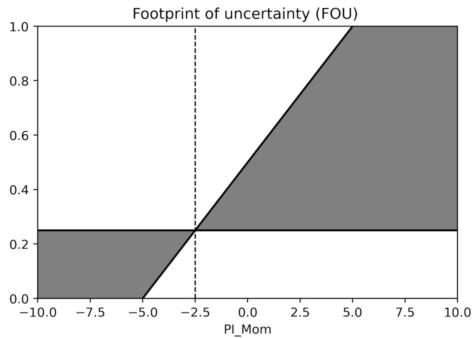


Fig. 5: Footprint of uncertainty with $\alpha = \beta = 5$ and PI_Con of 2.5.

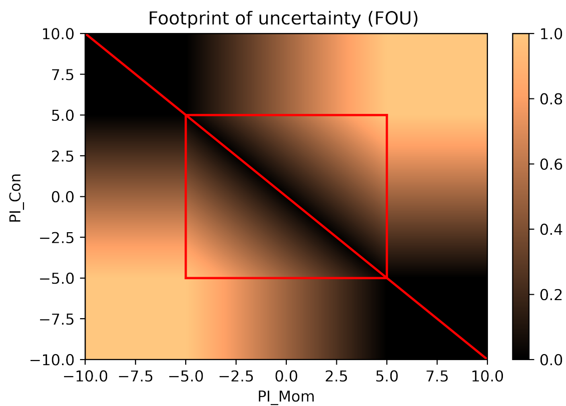


Fig. 6: Distance of fuzzy degrees (uncertainty) with $\alpha = \beta = 5$.

The two axes are the input variables (PI_Mom and PI_Con). The heatmap is symmetrical to the 45-degree line, therefore, PI_Mom and PI_Con can be placed on either axis. In addition, the red rectangle shows the boundary of β (within the inclined region of the R and L-functions), and the 225-degree red line represents the PI_Mom = - PI_Con. It can also be observed that the FOU for the red line with 255-degree is 0, and the higher FOU is obtained if the values of two axes are close to the boundary. Although FOU is usually used to express the uncertainty of type-2 **FQ**, it cannot, however, perfectly state the uncertainty in the proposed system. For example, if a stock with PI_Mom = PI_Con = 0, its FOU is zero since the PI_Mom = - PI_Con. However, the stock is neither profitable in momentum nor in contrarian trading strategies, and it is highly uncertain in our purposed model which cannot be quantified by the generic FOU. Thus, an effective uncertain characteristic is defined as follows.

3) Type-2 fuzzy-set quantifying module with uncertain characteristic: Due to the limitations of R and L-functions (only provide linear transformation) used in the fuzzy-set theory, unfortunately, the used membership functions are unable to deal with stocks that present only a slight difference between momentum and contrarian fuzzy degrees. Note that even a slight difference can have a profound impact on the final classification results. For example, a stock with a momentum degree of $0.53 \rightarrow 0.55$ and a contrarian degree of $0.45 \rightarrow 0.47$ from type-2 **FQ**, where the difference between momentum and contrarian is small. The results show much uncertainty, and there is less confidence in classifying this stock as a momentum and contrarian stock.

On the other hand, the designed PIs are generated from the random trading algorithms including several randomness. Although the following experiments show that enough simulation times can reduce the change of PI, the randomness and uncertainty in the proposed system can still not be ignored. Thus, it is possible to improve performance by incorporating an uncertainty factor within the developed type-2 **FQ**. Essentially, this term, uncertainty, is used to identify and filter out stocks that resist binary classification.

Triangular functions [38] were included in the uncertainty membership function, namely $\widetilde{\textbf{UMF}}$. When the independent variable is closer to the center, the dependent variable will be larger, and vice versa. Thus, when the input PIs (PI_Mom and PI_Con) are closer to 0, the dependent variable (fuzzy degree of uncertainty) becomes larger. A triangular function includes two parameters. The γ determines the position of the top vertex, whereas δ presents the base of the triangular function on the x-axis, which is within $\gamma \pm \delta$. Note that the γ and δ are subsequently optimized by using a grid search in training data.

Since the FOCUS produces two PI values, two type-1 uncertainty membership functions are respectively defined using PI_Mom and PI_Con, which are **UMF_Mom** and **UMF_Con**. The type-1 membership functions map PIs to type-1 fuzzy degrees, which are respectively denoted as:

$$\begin{aligned} \textbf{UMF\_Mom} : PI \rightarrow I, \qquad PI = \mathcal{R}, \\ \textbf{UMF\_Con} : PI \rightarrow I, \qquad I = [0, 1], \end{aligned}$$

where PI is the universe of the PIs (the real numbers) and I is the universe of fuzzy degree. The **UMF_Mon** and **UMF_Con** respectively generate a type-1 uncertainty fuzzy degree (*Uncertain*) under the given PI_Mom and PI_Con, which are respectively denoted as:

$$\begin{aligned} \textbf{UMF\_Mon}(\text{PI\_Mom}) &= \textit{Uncertain}, \\ \textbf{UMF\_Con}(\text{PI\_Con}) &= \textit{Uncertain}, \end{aligned}$$

where PI_Mom, PI_Con $\in PI$ and *Uncertain* $\in I$.

The proposed type-1 uncertainty membership functions (**UMF_Mom** and **UMF_Con**) based on triangular functions are shown in Equations (7) and (8). Due to the fact that the independent variables (PI_Mom and PI_Con) are approximately symmetric to zero, the **UMF_Mom** and **UMF_Con** are designed to be symmetrical to the y-axis Thus, the two functions are set to share the same δ as:

$$\textbf{UMF\_Mom}(\text{PI\_Mom}) = \begin{cases} 0 & \text{PI} < \gamma - \delta \\ \frac{\text{PI} - (\gamma - \delta)}{\delta} & \gamma - \delta < \text{PI} < \gamma \\ \frac{(\gamma + \delta) - \text{PI}}{\delta} & \gamma < \text{PI} < \gamma + \delta \\ 0 & \text{PI} > \gamma + \delta \end{cases} \quad (7)$$

$$\mathbf{UMF\_Con}(\text{PI\_Con}) = \begin{cases} 0 & \text{PI} < -\gamma-\delta \\ \frac{\text{PI}-(-\gamma-\delta)}{\delta} & -\gamma-\delta < \text{PI} < -\gamma \\ \frac{(-\gamma+\delta)-\text{PI}}{\delta} & -\gamma < \text{PI} < -\gamma+\delta \\ 0 & \text{PI} > -\gamma+\delta \end{cases} \tag{8}$$

The type-2 uncertainty membership function are also developed, namely $\widetilde{\mathbf{UMF}}$. The type-2 membership functions map PI pairs to type-2 fuzzy degrees, denoted as:

$$\widetilde{\mathbf{UMF}} : (PI, PI) \rightarrow I^I,$$
$$PI = \mathfrak{R}, \quad I^I = I \rightarrow I \;=\; [0,1] \rightarrow [0,1].$$

The $\widetilde{\mathbf{UMF}}$ takes both PI_Mom and PI_Con as inputs, and generates the type-2 uncertainty fuzzy degrees ($\widetilde{Uncertain}$), denoted as:

$$\widetilde{\mathbf{UMF}}(\text{PI\_Mom}, \text{PI\_Con}) = \widetilde{Uncertain},$$

where PI_Mom, PI_Con $\in PI$ and $\widetilde{Uncertain} \in I^I$.

Finally, the type-2 uncertainty membership function is defined as:

$$\widetilde{\mathbf{UMF}}(\text{PI\_Con}, \text{PI\_Mom}) = \mathbf{UMF\_Mom}(\text{PI\_Mom}) \;\rightarrow\; \mathbf{UMF\_Con}(\text{PI\_Con}). \tag{9}$$

A simple example of the designed type-2 uncertainty fuzzy degree is shown in Fig. 7. For example, consider a stock with PI_Mom of 0.3 and PI_Con of -0.35, under the assumptions that $\gamma = 0.5$ and $\delta = 1$, the $\widetilde{\mathbf{UMF}}$ maps the PI_Mom of 0.3 to 0.8 (by **UMF_Mom**) and the PI_Con of -0.35 to 0.85 (by **UMF_Con**). The type-2 uncertainty fuzzy degree is thus $0.8 \rightarrow 0.85$. If the fuzzy degree of uncertainty is larger than the fuzzy degrees of momentum or contrarian, such that the stock is identified as uncertain and subsequently filtered out. When dealing with stocks that are difficult to classify as momentum- or contrarian-type, investors should adopt a neutral strategy or ignore them.

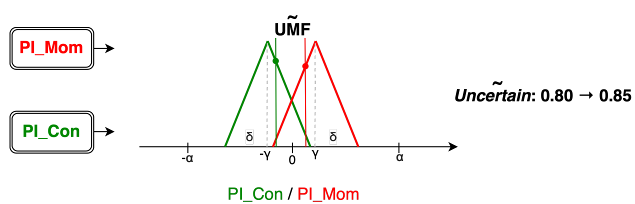


Fig. 7: Two membership functions that is incorporated with uncertain factor with **MMF** and **CMF**.

D. *Illustrative Example*

In the following, a brief example is presented to illustrate the implementation of the proposed scheme step-by-step. Take TW.2330 as an example for the following progresses. The historical OHLC data of TW.2330 for the period between Sep. 1, 2015 to Sep. 30, 2019 (1,000 days) for use as an input of the **RTA** module is collected. RTAs (MomRTA and ConRTA) repeatedly executes the random trading strategy to generate the momentum and contrarian distributions of annual returns for use as inputs of the **PI** module (see Fig. 2). The **PI** is used to evaluate the overall performance obtained using the two distributions and generate two PI values: PI_Mom (for MomRTA) and PI_Con (for ConRTA). As shown in Table I, PI_Mom (0.16850) and PI_Con (-0.17301) indicated that TW.2330 would be slightly profitable under momentum trading strategies. Using the **PI**s as inputs of the type-2 **FQ**, the **PI**s are converted into the fuzzy degrees of $\widetilde{Mom}$: $0.53 \rightarrow 0.55$, $\widetilde{Con}$: $0.45 \rightarrow 0.47$, and $\widetilde{Uncertain}$: $0.8 \rightarrow 0.85$ characteristics. Here, the fuzzy degree of uncertainty dominates other two characteristics, indicating that it cannot be sure which type of trading strategies would be better for TW.2330, and TW.2330 is identified as an uncertain stock.

IV. Experimental Results and Analysis

In Section IV-A, it determines the number of samples that should be obtained in **RTA** to achieve a stable **PI**. The **PI**s of stocks in the TW50 are then calculated to verify the effectiveness of the proposed **PI** in Section IV-B. In Section IV-C, the type-1 **FQ** and type-2 **FQ** with uncertain characteristic are employed to describe the characteristics of a stock as the final robust output of FOCUS. Finally, in Section IV-D, FOCUS is applied on trading strategies (for stock selection and classification) to show the usefulness and effectiveness in real-world applications.

A. *Samples Required for Random Trading Algorithm*

RTAs (MomRTA and ConRTA) was applied to data obtained over a period of 1,000 (D) trading days, in which long or short position (2 options, 50-50 chance) were taken at the daily opening of markets. This analysis faced the combinational explosion problem resulting from 2^{1000} trading possibilities, which cannot be resolved in a reasonable amount of time. Sampling is meant to approximate the results that would be obtained when sampling a large real-world database N times. Experiments are therefore performed N times sampling of the random trading and determined whether N = 100, 1,000, 10,000 or 100,000 would be sufficient to represent the actual distributions. This was achieved by observing changes in the 1st moment to the 4th moment of the distributions.

The TW.2330 dataset was adopted as the running dataset. For each N, sampling was repeated 100 times (i.e., $N \times 100$ simulations) to obtain 100 return distributions. The 1st to the 4th moments of each distribution are then plotted, as shown in Fig. 8. The distributions obtained using MomRTA were similar to those obtained using ConRTA; therefore, only the results of MomRTA is used, as shown in Fig. 8. Ideally, a line close to the horizontal would indicate that the number of samples was sufficient to represent the actual distribution.

Fig. 8 presents the four moments of the return distributions from MomRTA with the sampling times (N) of 100, 1,000, 10,000 and 100,000. The blue line indicates the mean (first moment), the orange line indicates the variance (second moment), the green line indicates the Skewness (third moment), and the red line indicates the Kurtosis of the distribution (fourth moment). Small changes in every moment were used to

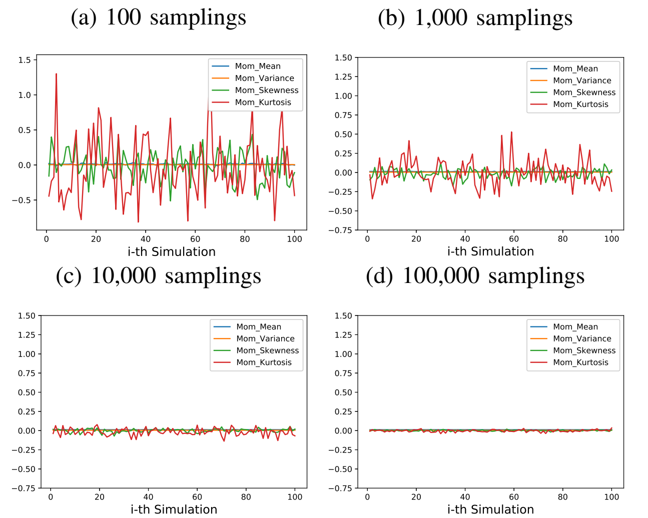


Fig. 8: Four moments of return distributions of TW.2330 between simulations from different sampling times (N).

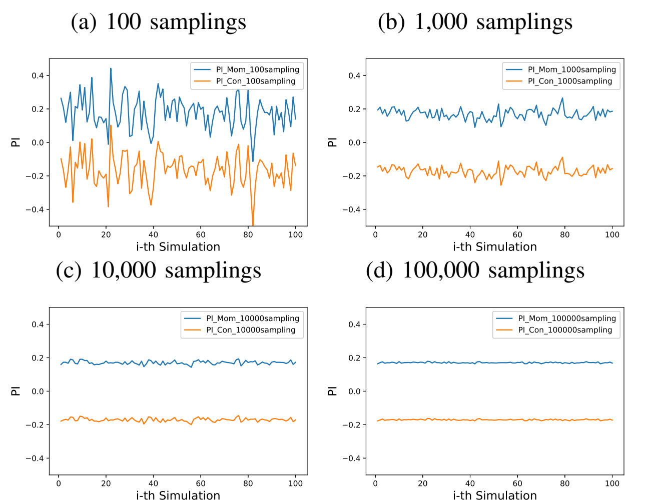


Fig. 9: Changes in TW2330's PI_Mom and PI_Con between simulations with different sampling times (N).

indicate the stability of each sample distribution with enough samples, where a stable distribution should have similar mean, spread (variance), symmetry (Skewness), and tailedness (Kurtosis) between samplings. As shown in Fig. 8, it can be found that the more samples are adapted, the smaller the changes and fluctuations in each moments, especially at lower moments. When the number of samples increased to 100,000, the changes of four moments (lines) are almost unobservable. It shows the high stability of the simulation, and indicates that 100,000 random samples should be sufficient to represent the original database.

The PIs (PI_Mom and PI_Con) in terms of stability using various numbers of samples are also evaluated, the results are shown in Fig. 9. It can be found that a large number of samples resulted in stable PI values, regardless of which from two algorithms was used. Note that 100,000-sample scheme ($N = 100,000$) provided the most stable results and was therefore adopted for all subsequent experiments. In summary, the designed **RTA**s reduce the computation from 2^{1000} possible paths to only 100,000 samples with suitable results and significant efficiency.

TABLE II: PI_Mom and PI_Con for constitutional stocks of TW50

Stock ID	PI_Mom	PI_Con	Stock ID	PI_Mom	PI_Con
TW.3008	2.834	-2.832	TW.2882	0.566	-0.574
TW.2408	2.651	-2.667	TW.2382	0.530	-0.530
TW.2454	2.625	-2.629	TW.2883	0.487	-0.480
TW.2327	2.301	-2.311	TW.1326	0.439	-0.444
TW.2888	1.680	-1.671	TW.2886	0.407	-0.396
TW.2633	1.616	-1.626	TW.2207	0.370	-0.367
TW.6505	1.574	-1.566	TW.3045	0.366	-0.362
TW.2474	1.458	-1.466	TW.2881	0.324	-0.319
TW.1102	1.245	-1.257	TW.2412	0.278	-0.283
TW.2317	1.237	-1.233	TW.2880	0.232	-0.240
TW.2912	1.226	-1.225	TW.2002	0.196	-0.172*
TW.5871	1.094	-1.094	TW.2330	0.168	-0.173*
TW.2823	1.071	-1.074	TW.2357	0.068	-0.059
TW.1101	0.985	-0.982	TW.1301	0.049	-0.053
TW.9904	0.879	-0.887	TW.1216	0.039	-0.031
TW.2308	0.872	-0.865	TW.2891	0.016	-0.014
TW.2890	0.844	-0.849	TW.2892	-0.141	0.147
TW.2303	0.836	-0.831*	TW.3711	-0.180	0.190
TW.4938	0.835	-0.841*	TW.9910	-0.192	0.197
TW.2301	0.809	-0.804	TW.2884	-0.216	0.214
TW.1402	0.802	-0.803	TW.1303	-0.298	0.295
TW.5876	0.787	-0.779	TW.2395	-0.397	0.395
TW.2885	0.661	-0.665	TW.2887	-0.439	0.438
TW.2105	0.612	-0.620	TW.4904	-0.656	0.659
TW.2801	0.586	-0.578	TW.5880	-0.689	0.688

B. Profitability Indexes and Trading Performance of Stocks

As shown in Table II, **RTA** was applied to the constitutional stocks of TW50 to obtain the PI_Mom and PI_Con values. Note that the table is sorted from large to small based on PI_Mom values. Note also that the order of PI_Con values would be the precisely the opposite with the exception of four stocks indicated by *.

It can be observed that the PI_Mom and PI_Con of each stock were roughly symmetric to zero. Among the 50 stocks examined here, 41 presented positive PI_Mom and negative PI_Con, which means that they generated positive average returns under MomRTA, and would therefore be suitable and profitable for momentum trading strategies. Among the 50 stocks examined here, only 9 stocks presented a positive PI_Con and negative PI_Mom, which means that they generated positive average returns under ConRTA, and would therefore be suitable and profitable for contrarian trading strategies. The sorted results in Table II could be used by investors to identify stocks that would benefit from a momentum or contrarian strategy.

To evaluate the effectiveness of the proposed **PI**, four common strategies are adopted to intraday trading from the perspectives of momentum trading as well as contrarian trading. The correlation between PI (simulated through training data) and the trading performance (in the testing data) of each stock are then observed to evaluate the effectiveness of the proposed **PI**. The first approach was the opening gap strategy (**GAP**) [14]. On any given trading day, if the opening price of a stock is higher than the closing price on the previous day, then adopting a momentum-type GAP (GAP_Mom) would take a

TABLE III: Correlation coefficient and accuracy between the PI and trading performance of strategies

Strategy	Mom_Ret		Mom_Sharpe		Con_Ret		Con_Sharpe		Average	
	CC	Accuracy	CC	Accuracy	CC	Accuracy	CC	Accuracy	CC	Accuracy
GAP	0.558	60%	0.519	60%	0.558	60%	0.520	60%	0.539	60%
1H1L	0.483	52%	0.454	52%	0.484	52%	0.456	52%	0.469	52%
3H3L	0.250	58%	0.222	58%	0.250	58%	0.223	58%	0.236	58%
5H5L	0.170	58%	0.126	58%	0.169	58%	0.126	58%	0.148	58%

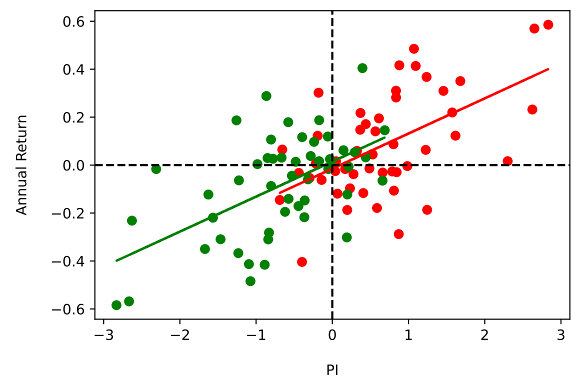
long position, whereas the contrarian-type GAP (GAP_Con) would indicate taking a short position. If the opening price of a stock is lower than the closing price on the previous day,, then a GAP_Mom strategy would indicate taking a short position, whereas the GAP_Con strategy would indicate taking a long position. The GAP_Mom employs a 1% stop-loss mechanism, whereas GAP_Con employs a 1% take-profit mechanism. Note that due to the use of 1% stop-loss and take-profit mechanisms, the closing price on the previous day is multiplied by 1.01 (fewer trading signal will be generated) to get robust trading signals.

The 2nd to 4th approaches are respectively referred to as n-High-n-Low (**nHnL**), where n indicates the duration of the observation used to generate trading signals. This approach is based on the well-known strategy referred to as trading range breakout [40]. On any given trading day, if the opening price is higher than the highest price during the previous n days, then a momentum-type nHnL (nHnL_Mom) strategy would take a long position, whereas a contrarian-type nHnL (nHnL_Con) strategy would indicate taking a short position. If the opening price is higher than the lowest price during the previous n days, then nHnL_Mom would indicate taking a short position, whereas nHnL_Con would indicate taking a long position. In addition, nHnL_Mom employs a 1% stop-loss mechanism, whereas nHnL_Con employs a 1% take-profit mechanism. Note that due to the use of 1% stop-loss and take-profit mechanisms, the highest price and lowest price on the previous n day are multiplied by 1.01 (fewer trading signal will be generated) to get robust trading signals. Finally, n is set to 1, 3 and 5, as follows: **1H1L**, **3H3L**, **5H5L**.

The above-mentioned eight investment schemes (4 strategies $\times$ 2 types) are applied to TW50 stocks and then calculated the annual return (**Ret**) and Sharpe ratio (**Sharpe**) as performance indicators. The correlation coefficient (CC) between PI (in the training data) and trading performance (in the testing data) is also calculated, the results are listed in Table III. Mom_Ret and Mom_Sharp respectively indicate the CC between PI_Mom and **Ret** obtained using a momentum-type strategy as well as between PI_Mom and the **Sharpe** using the same strategy. Con_Ret and Con_Sharpe respectively indicate the CC between PI_Con and the **Ret** obtained using a contrarian-type strategy as well as between PI_Con and the **Sharpe** using the same strategy. The results listed in Table III show positive correlations between PI and the trading performance of **GAP** (average CC of 0.539) and **nHnL** (average CC of 0.148 to 0.469). Generally, the CCs between PIs and **Ret** are about 0.035 higher than the CCs between PIs and **Sharpe**, even though PI is calculated by the idea of Sharpe ratio.

Fig. 10 illustrates the relationship between **PI** and the trading performance of **GAP**. Each red point (dot) in Fig.

(a) The correlations between PI and annual return.



(b) The correlations between PI and Sharp ratio.

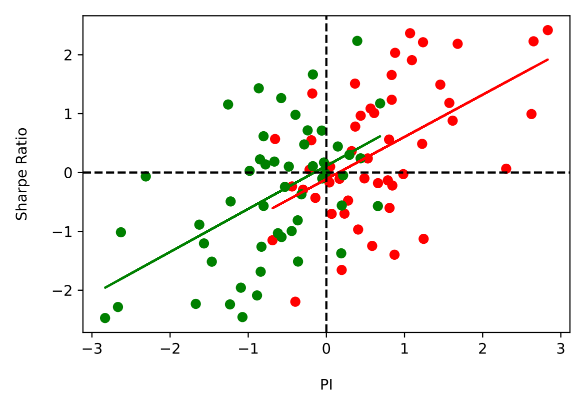


Fig. 10: The correlations between PI and annual return, and between PI and Sharp ratio.

10 represents a stock, and the coordinates of which indicate PI_Mom and the trading performance of GAP_Mom (**Ret** or **Sharpe**), and the red lines indicate the regression of the red points. Each green point (dot) also represents a stock, the coordinates of which represent PI_Con and trading performance of GAP_Con (**Ret** or **Sharpe**), and the green lines indicate the regression of the green points. An obvious trend can be found that the higher the PI, the higher the trading performance can be obtained, especially for **Ret**. The moderate degree of correlation between the **PI** and the trading performance of a stock indicates the effectiveness of the **PI** in identifying stock characteristics as momentum- or contrarian-type.

The proposed FOCUS employs a novel **PI** to facilitate the classification of stock characteristics as momentum- or contrarian-type. Intuitively, stocks with a positive PI_Mom should be classified as momentum-type, whereas stocks with a negative PI_Mom should be classified as contrarian-type. Stocks classified as momentum (contrarian) should have pos-

TABLE IV: Correlation between the type-1 **FQ** degree and trading performance of strategies

Strategy	Mom_Ret		Mom_Sharpe		Con_Ret		Con_Sharpe		Average	
	CC	Accuracy	CC	Accuracy	CC	Accuracy	CC	Accuracy	CC	Accuracy
GAP	0.544	60%	0.514	60%	0.544	60%	0.514	60%	0.529	60%
1H1L	0.469	52%	0.447	52%	0.470	52%	0.448	52%	0.459	52%
3H3L	0.245	58%	0.221	58%	0.244	58%	0.221	58%	0.233	58%
5H5L	0.178	58%	0.139	58%	0.177	58%	0.138	58%	0.158	58%

TABLE V: Correlation between the type-2 **FQ** degree and trading performance of strategies

Strategy	Mom_Ret		Mom_Sharpe		Con_Ret		Con_Sharpe		Average	
	CC	Accuracy	CC	Accuracy	CC	Accuracy	CC	Accuracy	CC	Accuracy
GAP	0.487	82.4%	0.431	82.4%	0.707	86.7%	0.663	86.7%	0.572	84.5%
1H1L	0.706	64.7%	0.665	64.7%	0.785	62.5%	0.733	62.5%	0.722	63.6%
3H3L	0.723	65.2%	0.687	65.2%	0.601	62.5%	0.557	62.5%	0.642	63.9%
5H5L	0.723	65.2%	0.687	65.2%	0.601	62.5%	0.557	62.5%	0.642	63.9%

itive **Ret** and **Sharpe** and be profitable under momentum (contrarian) strategies, as indicated by points in the first and third quadrants in Fig. 10. Therefore, the accuracy of the PI classification for each strategy and both indicators can be calculated, as shown in Table III. For several reasons, **Ret** and **Sharpe** always present the same sign, whereas PI_Mom and PI_Con always present the opposite sign. Our results revealed that the accuracy of **PI** is between 52% and 60%, which is slightly better than a random prediction (i.e., 50%), thereby demonstrating the effectiveness of PI for classification. However, Fig. 10 revealed a high degree of volatility in the trading performance of stocks with a PI_Mom between 0 and 1.6 (PI_Con between -1.6 to 0) as well as a lack of classification accuracy. A PI in this range would be unreliable (i.e., lacking explanatory ability). In this situation, therefore, the type-2 **FQ** with uncertainty factor is designed to solve this limitation in the following section.

C. Parameter Selection and Effectiveness of Fuzzy-Set Quantifying Module

In this sub-section, the process of optimizing the parameters of membership functions (α, β, γ, and δ) is examined. Ideally, the outputted fuzzy degrees would strongly correlate with the characteristics and trading performance of the stock for qualifying. Therefore, the parameters are optimized by training data, and the objective function is set as the average value of the four CCs in Table III (Mom_Ret, Mom_Sharpe, Con_Ret, Con_Sharpe with PI replaced by the fuzzy degrees calculated by type-1 **FQ** or type-2 **FQ**). Note that the fuzzy degrees in type-2 **FQ** are a ranges of type-2 fuzzy-set; therefore, the midpoint of the interval is used to calculate the CC.

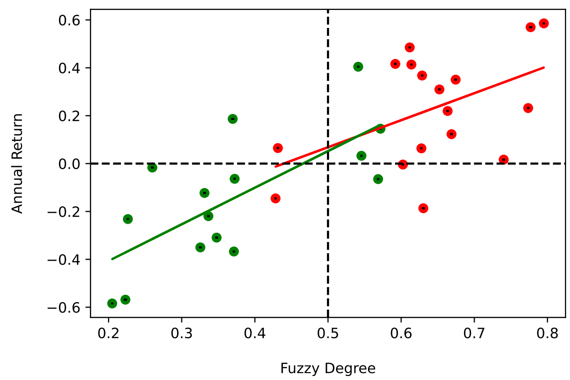
As described in Section III-C, type-1 **MMF** and **CMF** were set to be symmetrical ($\alpha = \beta$), due to the fact that PI_Mom and PI_Con are nearly symmetric to the y-axis. Thus, only one parameter (α) would have to be optimized in type-1 **FQ**, and three parameters (α, γ, and δ) would have to be optimized in type-2 **FQ**. The search space of α and δ was from 0 to 5 in increments of 0.1, and the search space of γ was from -5 to 5 in increments of 0.1.

The strategies (**Gap** and **nHnL**) are applied to the training data (from Sep. 1, 2015 to Sep. 30, 2019, 1,000 trading days) for parameters optimization. The strategies are then applied to testing data (Oct. 1, 2019 to Dec. 20, 2020, 300 trading days) with which to verify the performance of the optimized **FQ** module. Note that in the designed FOCUS, any stock with an uncertain fuzzy degree exceeding the momentum fuzzy (contrarian) degree should be treated as uncertain; these stocks are ignored while calculating CC. In addition, a constraint is set that at least 15 stocks must be available for optimization (to ensure the effectiveness of the system) and any parameters set that fails in this regard are eliminated.

After optimization and the removal of uncertain stocks, the optimal parameter for type-1 **FQ** was α = 2.4 for all strategies (**Gap** and **nHnL**). The optimal parameters (α, γ, δ) for type-2 **FQ** were as follows: **Gap** = (4.8, 0.3, 1.4), **1H1L** = (5.0, 1.1, 2.0), **3H3L** = (2.7, 0.9, 1.5), and **5H5L** = (2.7, 0.9, 1.5). Table IV (Table V) lists the CCs between the type-1 (type-2) **FQ** fuzzy degrees and trading performance of all strategies. Since only linear transformation adopted in type-1 **FQ**, the CCs between type-1 fuzzy degrees and the trading performance (Table III) are similar to CCs between **PI** and trading performance (Table IV). The same phenomenon can also be found in the accuracies.

For type-2 **FQ**, the optimized parameters retain 17 momentum-type stocks and 15 contrarian-type stocks. On the other hand, 33 stocks are ignored since the degree of uncertainty is greater than the degree of momentum, which are TW.1216, TW.1301, TW.1303, TW.1326, TW.1402, TW.2002, TW.2105, TW.2207, TW.2301, TW.2303, TW.2308, TW.2330, TW.2357, TW.2382, TW.2395, TW.2412, TW.2801, TW.2880, TW.2881, TW.2882, TW.2883, TW.2884, TW.2885, TW.2886, TW.2887, TW.2890, TW.2891, TW.2892, TW.3045, TW.3711, TW.4938, TW.5876, and TW.9910. The midpoint of their momentum degrees are between 0.454 and 0.590, which is less informative and more uncertain; therefore, they are ignored in the developed system. 35 stocks are ignored since the degree of uncertainty is greater than the degree of contrarian, which are TW.1101, TW.1216, TW.1301, TW.1303, TW.1326, TW.1402, TW.2002, TW.2105, TW.2207, TW.2301, TW.2303, TW.2308, TW.2330, TW.2357, TW.2382, TW.2412, TW.2801, TW.2823, TW.2880, TW.2881, TW.2882, TW.2883, TW.2884, TW.2885, TW.2886, TW.2890, TW.2891, TW.2892, TW.3045, TW.3711, TW.4938, TW.5871, TW.5876, TW.9904, and TW.9910. The midpoint of their contrarian degrees are between 0.386 and 0.531, which is also less informative and more uncertain; therefore, they are ignored in the developed system. The type-2

(a) The correlations between stocks' fuzzy degree and annual return



(b) The correlations between stocks' fuzzy degree and Sharp ratio

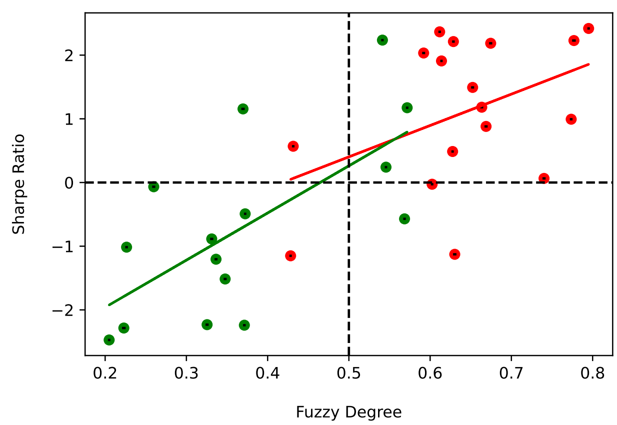


Fig. 11: The correlations between stocks' fuzzy degree and annual return of **Gap** strategy, and between stocks' fuzzy degree and Sharp ratio of **Gap** strategy.

FQ improved the average CC to 0.645 (from 0.345 of type-1 **FQ**) and average accuracy to 69.0% (from 57.0% of type-1 **FQ**), thereby demonstrating the efficacy of the uncertain characteristic and the type-2 **FQ** module.

Fig. 11 illustrates the relationship between type-2 fuzzy degrees and the trading performance of **GAP**. The 17 red points indicate the momentum stocks retained after filtering, the coordinates of which indicate the momentum fuzzy degree and performance of GAP_Mom (**Ret** or **Sharpe**) of a stock, and the red lines indicate the regression of the red points. The 15 green points indicate the contrarian stocks retained after filtering, the coordinates of which indicate the contrarian fuzzy degree and performance of GAP_Mom (**Ret** or **Sharpe**) of a stock, and the green lines indicate the regression of the green points. Fig. 11 also illustrates the accuracy of the type-2 **FQ**. If the momentum (contrarian) fuzzy degree of each remaining red (green) point exceeds 0.5, then it is classified as a momentum (contrarian) stock; otherwise, it is classified as a contrarian (momentum) stock. The x-axis represents the results of classification obtained using the proposed FOCUS, whereas the y-axis represents the ground truth trading performance of the strategy in question. The points in the first and third quadrants were classified correctly. After removing the uncertain stocks by the uncertain characteristic, an obvious trend with less uncertainty can be found that the higher the fuzzy degree, the higher the trading performance can be obtained. From Tables IV and V, the improved correlation and accuracy indicate the effectiveness and necessity of the type-2 **FQ** and the uncertain characteristic in the developed FOCUS.

D. Robustness and Effectiveness of the Proposed System

In this section, the robustness and effectiveness of the proposed FOCUS are evaluated through another dataset and implementation methods in the real-world applications. Table VI compares the performance (CC and accuracy) of the proposed systems on the TW50 and Mid-Cap 100 (MC100) data sets, where MC100 are the 100 stocks with the largest capital value excluding stocks in TW50 [41]. There compared systems include random selection (50/50 guess, as benchmark), PI/type-1 **FQ** (the average performance of the PI and type-1 **FQ** since they similarly performed), and type-2 **FQ** (with uncertain characteristic. Note that the performance listed in Table VI is the result on the testing dataset.

For the TW50 dataset in Table VI, it can be found that the CCs increase from 0 (random) to 0.153–0.534 (PI/type-1) and to 0.572–0.722 (type-2); the accuracies increase from 50.0% (random) to 52.0%–60.0% (PI/type-1) and to 63.6%–84.5% (type-2). The proposed system significantly improves the CC and accuracy, especially for type-2 **FQ**. For the MC100 dataset in Table VI, the CCs increase from 0 (random) to 0.013–0.320 (PI/type-1), and to 0.413–0.454 (type-2); the accuracies change from 50.0% (random) to 62.6%–81.3% (PI/type-1) and to 68.4%–71.1% (type-2). The proposed system also significantly improves the CC, but the accuracy of type-2 **FQ** is slightly lower than type-1 **FQ**. These results demonstrate the efficiency of the involved type-2 **FQ** and the uncertain characteristic in the developed module that improves the CC and accuracy regardless the employed strategies and stocks (different liquidity and size of capital value). The improved and high accuracy are also an indication for the efficiency of the developed FOCUS in quantifying and classifying stocks as contrarian- or momentum-type.

TABLE VI: Performance comparison on Taiwan 50 and Mid-Cap 100 (MC100)

TW50	Random		PI/Type-1 **FQ**		Type-2 **FQ**	
	CC	Accuracy	CC	Accuracy	CC	Accuracy
GAP	0.000	50.0%	0.534	60.0%	0.572	84.5%
1H1L	0.000	50.0%	0.464	52.0%	0.722	63.6%
3H3L	0.000	50.0%	0.235	58.0%	0.642	63.9%
5H5L	0.000	50.0%	0.153	58.0%	0.642	63.9%
MC100	Randomness		PI/Type-1 **FQ**		Type-2 **FQ**	
	CC	Accuracy	CC	Accuracy	CC	Accuracy
GAP	0.000	50.0%	0.320	81.3%	0.445	70.9%
1H1L	0.000	50.0%	0.162	74.2%	0.454	70.9%
3H3L	0.000	50.0%	0.013	67.2%	0.413	71.1%
5H5L	0.000	50.0%	0.083	62.6%	0.443	68.4%

In addition, the proposed FOCUS is applied to the real-world applications. The results of FOCUS classification are used as the stock selection (FOCUS-selected), that is, the

TABLE VII: Trading performance of FOCUS-selected stocks on TW50 and MC100

TW50-**GAP**	Annual Returns	Sharpe Ratio	Win Rates
All stocks	9.6%	1.687	47.2%
FOCUS	24.9%	4.095	52.8%
TW50-**1H1L**			
All stocks	2.6%	0.631	42.8%
FOCUS	5.9%	1.831	54.8%
TW50-**3H3L**			
All stocks	1.6%	0.461	39.4%
FOCUS	2.2%	0.885	48.5%
TW50-**5H5L**			
All stocks	1.4%	0.444	40.7%
FOCUS	2.0%	0.890	43.4%
MC100-**GAP**	Annual Returns	Sharpe Ratio	Win Rates
All stocks	17.1%	3.418	54.2%
FOCUS	18.8%	4.037	55.9%
MC100-**1H1L**			
All stocks	6.3%	1.464	47.8%
FOCUS	7.0%	1.724	48.2%
MC100-**3H3L**			
All stocks	3.6%	1.141	42.8%
FOCUS	3.6%	1.196	47.8%
MC100-**5H5L**			
All stocks	3.0%	1.038	43.1%
FOCUS	3.2%	1.186	47.8%

(a) On TW50 (b) On MC100

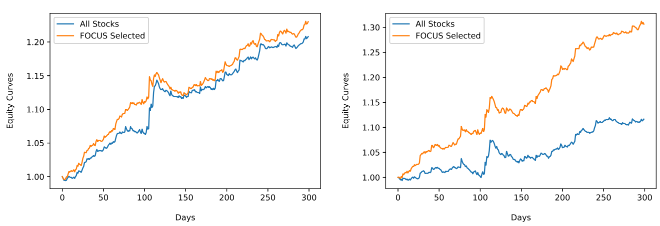


Fig. 12: Equity curves of **Gap** on FOCUS-selected and all stocks.

momentum (contrarian) stocks with the momentum type (contrarian type) strategies is traded. The benchmark strategy is to trade all stocks without selection (All Stocks). Note that all transaction fees and obstacles are ignored in the experiments.

Table VII compares the trading performance on FOCUS-selected stocks and All Stocks. Experimental results show that FOCUS-selected outperforms the All Stocks among all strategies and datasets. Especially when the **GAP** strategy is preformed on TW50 dataset, the annual return (Sharpe ratio) is significantly enhanced from 9.6% to 24.9% (from 1.687 to 4.095), which increased the profitability by 1.5 times. The similar result can be found in Fig. 12, which presents the equity curves of **Gap** on FOCUS-selected and All Stocks. The curves of FOCUS-selected are more stable and always higher than curves of All Stocks. In summary, removing uncertain stocks through type-2 **FQ** can reduce the probability of investing in unsuitable stocks (smaller risk of loss and higher win rate), resulting in more stable and profitable trading performance. These results demonstrate the effectiveness of the proposed FOCUS in the real-world applications.

V. Conclusions

Most of the thousands of existing trading strategies can be classified as momentum- or contrarian-type; however, there is at present no standard approach to the classification of stocks. This represents a serious impediment to investors seeking to match trading strategies with suitable stocks. In this paper, a random trading algorithm using stop-loss and take-profit mechanisms for the extraction of stock characteristics is employed. A profitability index is then used to quantify the characteristics in conjunction with a type-2 fuzzy-set to describe the characteristics into fuzzy degrees. Experiments on the proposed Fuzzy mOmentun Contrarian Uncertain characteristic System (FOCUS) revealed that 41 of the stocks in the TW.50 dataset would perform better under momentum-type strategies, whereas 9 stocks would benefit from contrarian-type strategies. A correlation coefficients of 0.148-0.539 is obtained between PI and trading performance with classification accuracy of 52.0%-60.0%. The proposed FOCUS greatly improved classification performance, resulting in correlation coefficients of 0.572-0.722 with accuracy of 63.6%-84.5%. These results clearly demonstrate the efficacy of FOCUS in the quantification and classification of stocks suited to momentum- and contrarian-type trading strategies. In addition, the proposed FOCUS is applied to the real-world applications, and the FOCUS-selected outperforms the benchmark among all datasets, and increased the profitability by 1.5 times on TW50 dataset. These results demonstrate the effectiveness of the proposed FOCUS in the quantifying and classifying stocks as contrarian- or momentum-type and the real-world applications.

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References

[1] T. Worstall. (2016) Economics is scarce resources allocation - what resource constraint does urban farming solve? [Online]. Available: https://www.forbes.com/sites/timworstall/2016/08/15/economics-is-scarce-resources-allocation-what-resource-constraint-does-urban-farming-solve/#e507ed92a553

[2] Y. Yao, "Momentum, contrarian, and the january seasonality," *Journal of Banking & Finance*, vol. 36, no. 10, pp. 2757 – 2769, 2012.

[3] K. G. Rouwenhorst, "International momentum strategies," *The Journal of Finance*, vol. 53, no. 1, pp. 267–284, 1998.

[4] K. C. Chan, "On the contrarian investment strategy," *The Journal of Business*, vol. 61, no. 2, pp. 147–163, 1988.

[5] T. H. McInish, D. K. Ding, C. S. Pyun, and U. Wongchoti, "Short-horizon contrarian and momentum strategies in asian markets: An integrated analysis," *International Review of Financial Analysis*, vol. 17, no. 2, pp. 312–329, 2008.

[6] M. J. Douglas, *Trading in the zone: master the market with confidence, discipline and a winning attitude*. Penguin, 2000.

[7] W. Hu, G. Liu, W. Zhang, and T. Wu, "Study on random trading behavior, herd behavior and asset price volatility," in *Chinese Control and Decision Conference*. IEEE, 2016, pp. 3157–3163.

[8] J. Dhaene and M. J. Goovaerts, "Dependency of risks and stop-loss order 1," *ASTIN Bulletin: The Journal of the IAA*, vol. 26, no. 2, pp. 201–212, 1996.

[9] A. Warburton and Z. G. Zhang, "A simple computational model for analyzing the properties of stop-loss, take-profit, and price breakout trading strategies," *Computers & Operations Research*, vol. 33, no. 1, pp. 32–42, 2006.

[10] M. E. Wu, C. H. Wang, and W. H. Chung, "Using trading mechanisms to investigate large futures data and their implications to market trends," *Soft Computing*, vol. 21, no. 11, pp. 2821–2834, 2017.

[11] M. E. Wu and W. H. Chung, "Empirical evaluations on momentum effects of taiwan index futures via stop-loss and stop-profit mechanisms," *International Journal of Information Technology & Decision Making*, vol. 18, no. 02, pp. 629–648, 2019.

[12] L. A. Zadeh, "Fuzzy logic," *Computer*, vol. 21, no. 4, pp. 83–93, 1988.

[13] N. N. Karnik, J. M. Mendel, and Q. Liang, "Type-2 fuzzy logic systems," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 6, pp. 643–658, 1999.

[14] A. Polly. Gap trading strategies. [Online]. Available: https://school.stockcharts.com/doku.php?id=trading_strategies:gap_trading_strategies

[15] L. K. C. Chan, N. Jegadeesh, and J. Lakonishok, "Momentum strategies," *The Journal of Finance*, vol. 51, no. 5, pp. 1681–1713, 1996.

[16] U. Holmberg, C. Lönnbark, and C. Lundström, "Assessing the profitability of intraday opening range breakout strategies," *Finance Research Letters*, vol. 10, pp. 27–33, 2013.

[17] Y. C. Tsai, M. E. Wu, J. H. Syu, C. L. Lei, C. S. Wu, J. M. Ho, and C. J. Wang, "Assessing the profitability of timely ppening range breakout on index futures markets," *IEEE Access*, vol. 7, pp. 32 061–32 071, 2019.

[18] J. H. Syu, M. E. Wu, S. H. Lee, and J. M. Ho, "Modified orb strategies with threshold adjusting on taiwan futures market," in *IEEE Conference on Computational Intelligence for Financial Engineering and Economics*, 2019, pp. 1–7.

[19] M. E. Wu, J. H. Syu, J. C. W. Lin, and J. M. Ho, "Evolutionary orb-based model with protective closing strategies," *Knowledge-Based Systems*, vol. 216, p. 106769, 2021.

[20] J. H. Syu and M. E. Wu, "Modifying orb trading strategies using particle swarm optimization and multi-objective optimization," *Journal of Ambient Intelligence and Humanized Computing*, pp. 1–13, 2021.

[21] J. M. Poterba and L. H. Summers, "Mean reversion in stock prices: Evidence and implications," *Journal of Financial Economics*, vol. 22, no. 1, pp. 27–59, 1988.

[22] N. Jegadeesh and S. Titman, "Overreaction, delayed reaction, and contrarian profits," *The Review of Financial Studies*, vol. 8, no. 4, pp. 973–993, 1995.

[23] T. Bollerslev, R. Y. Chou, and K. F. Kroner, "Arch modeling in finance: A review of the theory and empirical evidence," *Journal of Econometrics*, vol. 52, no. 1-2, pp. 5–59, 1992.

[24] C. Lento, N. Gradojevic, and C. S. Wright, "Investment information content in bollinger bands?" *Applied Financial Economics Letters*, vol. 3, no. 4, pp. 263–267, 2007.

[25] D. G. Uyemura, C. C. Kantor, and J. M. Pettit, "Eva® for banks: Value creation, risk management, and profitability measurement," *Journal of Applied Corporate Finance*, vol. 9, no. 2, pp. 94–109, 1996.

[26] J. Chen. (2019) Annual return. [Online]. Available: https://www.investopedia.com/terms/a/annual-return.asp

[27] Z. Huang and F. Martin, "Pairs trading strategies in a cointegration framework: back-tested on cfd and optimized by profit factor," *Applied Economics*, vol. 51, no. 22, pp. 2436–2452, 2019.

[28] W. F. Sharpe, "The sharpe ratio," *Journal of Portfolio Management*, vol. 21, no. 1, pp. 49–58, 1994.

[29] H. J. Solberg, "The profit factor in fire insurance rates," *The Journal of Insurance*, vol. 24, no. 2, pp. 24–33, 1957.

[30] A. W. Lo, "The statistics of sharpe ratios," *Financial Analysts Journal*, vol. 58, no. 4, pp. 36–52, 2002.

[31] J. J. Buckley, "The fuzzy mathematics of finance," *Fuzzy Sets and Systems*, vol. 21, no. 3, pp. 257–273, 1987.

[32] A. Kandel, *Fuzzy expert systems*. CRC press, 1991.

[33] H. Yu, J. Lu, and G. Zhang, "Topology learning-based fuzzy random neural network for streaming data regression," *IEEE Transactions on Fuzzy Systems*, 2020.

[34] H. Mo, F. Y. Wang, M. Zhou, R. Li, and Z. Xiao, "Footprint of uncertainty for type-2 fuzzy sets," *Information Sciences*, vol. 272, pp. 96–110, 2014.

[35] J. M. Mendel, R. I. John, and F. Liu, "Interval type-2 fuzzy logic systems made simple," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 6, pp. 808–821, 2006.

[36] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[37] J.Dombi, "Membership function as an evaluation," *Fuzzy Sets and Systems*, vol. 35, no. 1, pp. 1–21, 1990.

[38] W. Pedrycz, "Why triangular membership functions?" *Fuzzy Sets and Systems*, vol. 64, no. 1, pp. 21–30, 1994.

[39] J. Balter, "Quarticity estimation on ohlc data," *Journal of Financial Econometrics*, vol. 13, no. 2, pp. 505–519, 2015.

[40] M. M. Lai and S. H. Lau, "The profitability of the simple moving averages and trading range breakout in the asian stock markets," *Journal of Asian Economics*, vol. 17, no. 1, pp. 144–170, 2006.

[41] J. A. Yang, S. R. Chou, H. C. Cheng, and C. H. Lee, "The effects of capital structure on firm performance in the taiwan 50 and taiwan midcap 100," *Journal of Statistics and Management Systems*, vol. 13, no. 5, pp. 1069–1078, 2010.

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