


# Distribution line parameter estimation driven by probabilistic data fusion of D-PMU and AMI

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## Abstract

This paper proposes a novel distribution line parameter estimation method, driven by the probabilistic data fusion of the distributed phasor measurement unit (D-PMU) and the advanced measurement infrastructure. The synchronized and high-precision D-PMU is utilized to tackle the challenge risen by the a-synchronization of smart meters. Correspondingly, a time-alignment algorithm is proposed to obtain the time-synchronous error (TSE) dataset for the up-stream smart meter. The non-parametric estimation method is performed then to evaluate the probabilistic density curve of TSE. Furthermore, TSE data of down-stream smart meters are generated by implementing the acceptance-rejection process based on the obtained probabilistic density curve. Leveraging the generated TSE dataset, a new time-shifted D-PMU curve is probabilistically aligned or fused with the down-stream advanced measurement infrastructure curves. According to the complete voltage drop model, the line parameter estimation of resistance and reactance is formulated as a quadratic programming problem and solved by Optimal Toolbox in MATLAB by conducting multi-run Monte-Carlo simulations under various scenarios. Simulation results demonstrate the effectiveness and robustness of the proposed methodology.

## 1 | INTRODUCTION

### 1.1 | Motivation and incitement

Line parameter estimation is the cornerstone for the upper-level applications in power system, such as state estimation [1–3] and fault analytics. Line parameters inherently change with environment and topology, and sometimes, the recorded line length is not properly documented or updated, especially in the distribution networks. Unlike transmission lines, the length of distribution lines is shorter; consequently, voltage drop and angle differences between two connected nodes are smaller, which makes distribution line parameter estimation more challenging, since small errors have a great impact on the estimation accuracy [4]. Fortunately, the advent of advanced monitoring technologies, such as D-PMU and AMI, provides new possibilities for refined parameter estimation in distribution networks.

Indeed, the D-PMUs have been employed for event detection [5], state estimation [6], topology detection [7], and islanding detection [8] in the distribution networks. Likewise, the extensive deployment of smart meters in the distribution networks offers intelligent monitoring and control for various applications, such as observability enhancement [9], energy management [10], line outage identification [11], topology and parameter estimation [12, 13], and distribution system state estimation [14, 15].

The distribution line parameters estimation can be formulated as a mathematical problem. The transmission line parameters estimation is based on a redundant set of telemetry measurements. However, it is worthwhile to mention that the parameter estimation of distribution lines is challenging due to a lack of parameter records, non-updated parameters, short-length of distribution lines, small bus voltage differences, and a small number of real-time measurements. Under such

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conditions, the parameter estimation techniques for transmission networks cannot be applied directly to distribution networks, which means that a particular method needs to be adopted. A unique feature of smart meters is to monitor the distribution system state; however, the smart meter causes a-synchronization in the distribution network. Fortunately, D-PMU can provide synchronized and high-precision sampling data. However, the installation of D-PMU devices is limited in current distribution networks due to the high cost. Thus, it is necessary to integrate supportive elements such as D-PMU and AMI to achieve enhanced observability in such a situation. The general framework for distribution line parameter estimation using the combined integration of D-PMU and AMI include the following aspects:

- Models of distribution networks.
- Inclusion and time-alignment of D-PMU and AMI.
- Probabilistic Data Fusion of D-PMU and AMI.

## 1.2 | Literature review

Recently, various innovative distribution line parameter estimation approaches have been proposed by leveraging D-PMU or AMI measurements. A graphical model-based approach employing D-PMU is proposed in [16, 17] to estimate distribution line impedance with a linear-coupled power flow (LCPF) model. Exploring the application of D-PMU, the authors in reference [18] proposed a least-squares (LS) algorithm based impedance estimation for dynamic line rating (DLR) monitoring. In [19, 20], a data-driven joint estimation method with the inclusion of D-PMU for parameters and topology is proposed. The estimation of measurement uncertainty and a correction factor of instrument transformers and D-PMU are addressed in [21–24] for line parameter estimation. In another study assuming that the line impedance value deviates from the planning value within 10% [25], convolutional neural networks (CNN) are applied to classify line impedance based on PMU phasor data, which may not work for the deviation up to 30% [26] and changed R/X-ratios.

Having the same note of employing AMI data, the linear regression method is presented in [27] to estimate the parameters between customer pairs iteratively, presuming the approximation error is insignificant with comparison to the joint measurement and model error [28–30]. It is important to mention that the computational efficiency of customer pairs iteratively presuming may decrease with the growing number of customer pairs and frequent changes in system configuration. Moreover, the aforementioned line parameters estimation methods using AMI are assumed that the smart meters to be synchronized.

Nevertheless, according to [20], as a consequence of imperfect synchronism, the time interval deviation is varying from milliseconds to few seconds; yet the zero-mean Gaussian distribution error model due to TSE may not be guaranteed for instant measurements referring to distributed generations (DGs), flexible loads, and electric vehicles with relatively dynamic behaviours. Therefore, it is crucial to investigate and

focus on aligning smart meter data with D-PMU and attempt to determine the statistic characteristic of TSE.

## 1.3 | Contribution and paper organization

This paper proposes a novel data-driven approach by integrating D-PMU and AMI data with a probabilistic model for distribution line parameter estimation. Firstly, the time curve of AMI measurement is aligned with D-PMU data, and hence, averaged TSE data for each timestamp is computed. Secondly, the probabilistic model of TSE is obtained by the Gaussian-kernel-based non-parametric density estimation method. Provided that the TSE of remaining smart meters in the same area conforms with the same probabilistic model, TSE data is generated by the acceptance-rejection sampling method according to the obtained probabilistic density model. Lastly, distribution line resistances and reactances are estimated using an optimal algorithm, that is, LSQ (least square) or QP. By conducting multi-run Monte Carlo simulations in Matlab 2014 @b, the expected values of  $R$  and  $X$  are computed. The numerical tests using typical low-voltage distribution feeders for validation of the proposed approach is facilitated and discussed. The main contributions of this research are three-fold:

- Introducing a new framework using the combined integration of D-PMU and AMI for the distribution line parameter estimation.
- Developing a time-alignment algorithm of D-PMU and upstream AMI curve and a unique non-parametric probabilistic density model for D-PMU and AMI data fusion.
- Implementing line parameter estimation algorithm by applying multi-run Monte Carlo simulation and testing on different TSE distributions to verify the robustness of the proposed methodology.

The rest of this paper is structured as follows: Section 2 formulates the data fusion work into a time alignment and probabilistic model estimation problem. Section 3 explains the methodology proposed for distribution line parameter estimation. Section 4 presents the results of a simulation model and evaluates the proposed approach. Section 5 concludes the paper.

## 2 | MODELLING AND PROBLEM FORMULATION

### 2.1 | Distribution line model

The pi-equivalent model of the distribution line is employed and shown in Figure 1. It comprises the line parameters such as  $V_1$  and  $V_2$  are voltage phasors at two ends of the line,  $R$  and  $X$  represent the equivalent resistance and reactance between two buses, and  $B$  is the line susceptance value. The Ref. [4] pointed out that the character of the overhead distribution line is dominated by the series resistance and inductance value;

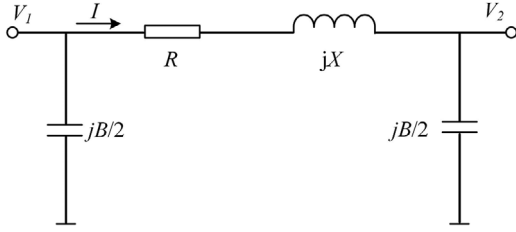


FIGURE 1 π-equivalent model of distribution line

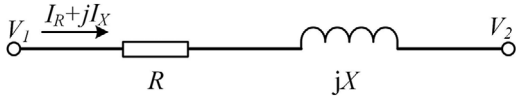


FIGURE 2 The simplified circuit of the overhead line

correspondingly, the shunt impedance's impact is ignorable. The simplified circuit model of the overhead line is displayed in Figure 2.

### 2.2 | Voltage drop model

The diagram of voltage drop phasor is illustrated in Figure 3

As indicated in Figure 3,  $\Delta V_2$  and  $\delta V_2$  denote the vertical and horizontal component of voltage drop phasor, respectively. Thus, voltage difference phasor can be expressed as follows [31]:

$$V_1 = V_2 + \Delta V_2 + j\delta V_2 \quad (1)$$

According to the Ohm's Law, the voltage drop can be rewritten as

$$V_1 - V_2 = (R + jX)(I_R + jI_X), \quad (2)$$

where  $I_R = P_2/V_2$ ,  $I_X = Q_2/V_2$ . Thus, the complete voltage drop model is deployed and presented as follows:

$$V_1^2 - V_2^2 = I^2 R^2 + I^2 X^2 + 2P_2 R + 2Q_2 X, \quad (3)$$

where  $I$  denotes the line current phasor,  $P_2$  is active power consumption (kW) and  $Q_2$  denotes reactive power consumption (kVAR).

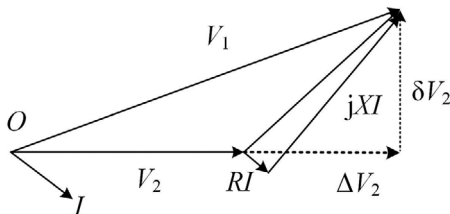


FIGURE 3 The phasor diagram of voltage drop

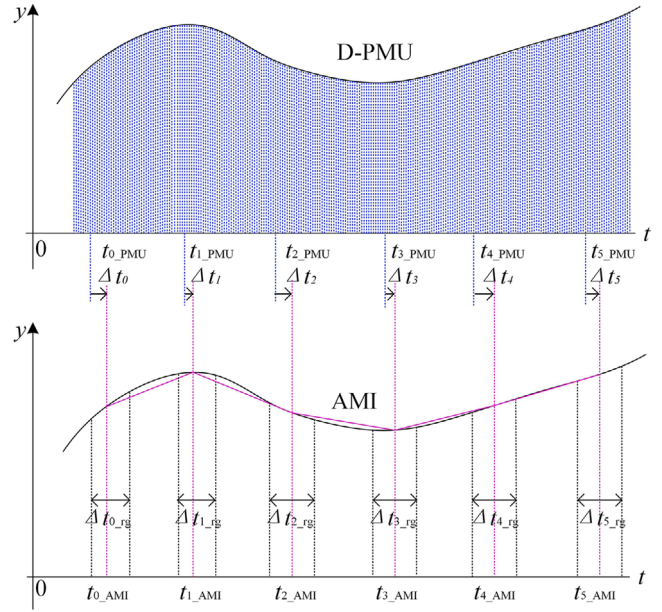


FIGURE 4 Time curves of D-PMU and AMI with TSE

### 2.3 | Time synchronization error

Communication delay and internal clock error are the two main causes of TSE, which are not constant and changes randomly. In principle, AMIs and D-PMU measurements are taken at regular intervals; however, the time de-synchronization will introduce time shifts to the original resolution interval. As a result, errors appear in voltage and current measurements. Additionally, as the TSE increases, measurement errors tend to increase, resulting in worse effects on the parameters estimation process [32].

Generally, smart meters in the same local area are manufactured by the same producer, and their clocks are calibrated by the same time protocol such as NTP (network time protocol) as well; therefore, it is acceptable to assume that TSEs of these smart meters have the same statistic characteristics. Curves of D-PMU and AMI data with TSE are displayed in Figure 4.

$\Delta t_{i\_rg}$  denotes the range of time delay of AMI.  $\Delta t = \{\Delta t_0, \Delta t_1, \Delta t_2, \Delta t_3, \Delta t_4, \Delta t_5\}$  is the representation of TSE at each time stamp for smart meter with reference to D-PMU and expressed as

$$\Delta t_i = t_{i\_AMI} - t_{i\_PMU} (i = 0, 1, 2, 3, \dots). \quad (4)$$

Although typically  $\Delta t$  varies from milliseconds to few seconds [12], it was assumed to be up to 450 s in this paper.

The objective of this work is to infer the statistic characteristic of TSE of smart meters to fuse D-PMU and AMI data, and thus, distribution line parameter estimation can be carried out with the integration of a-synchronized meters.

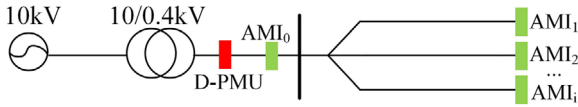


FIGURE 5 The diagram for D-PMU and AMIs in distribution networks

### 3 | METHODOLOGY FOR PARAMETER ESTIMATION

Traditionally, a predefined shape is presumed and used for pattern forecast, that is, Normal (Gaussian), Uniform, Weibull, Rayleigh, Beta, Log-normal and  $t$  Location-scale distribution etc. The determination of the probabilistic distribution model is completed by the Maximum Likelihood Estimation (MLE) Algorithm and evaluated by well-known Kolmogorov–Smirnov (KS) statistics [33]. However, the distribution pattern of time synchronization error  $\Delta t$  is uncertain and may not be guaranteed to express in the mentioned common probabilistic distribution model. Hence, the innovation of new non-parametric probabilistic data fusion of D-PMU and AMI measurements for the distribution line parameter estimation is presented, and a novel algorithm is proposed.

The diagram for D-PMU and AMIs in the distribution networks is illustrated for example in Figure 5.

As indicated in Figure 5, the up-stream smart meter  $AMI_0$  is utilized to obtain TSE distribution with D-PMU at the secondary side of the transformer. For the down-stream smart meters  $AMI_1, AMI_2, \dots, AMI_i$ , there are no PMU allocated at the customer's side due to the cost limits.

Probabilistic data fusion of D-PMU and AMI is proposed in this paper. That is, the probabilistic density curve of TSE of an up-stream smart meter is obtained by the time-alignment algorithm and non-parametric kernel density estimation (KDE). According to the PD curve, TSE datasets for the down-stream smart meters are produced by the acceptance-rejection sampling method. Then, the TSEs of down-stream smart meters are “added” to D-PMU measurements for each time interval, which means that the point on the D-PMU curve is being shifted with a probabilistic time interval for fusion with the AMI curve. This process is carried out for optimal line parameter estimation using a well-known MCS method for multi runs. The flowchart of the proposed parameter estimation procedure is shown in Figure 6.

The main steps of our proposed methodology are divided as follows and illustrated elaborately in the following subsections.

- Calculate the value of TSE of up-stream smart meter  $AMI_0$  for each time interval according to D-PMU measurements using the time-alignment algorithm.
- Non-parametric kernel density estimation approach is applied to obtain the PD curve of TSE of the up-stream smart meter.
- Generate random TSE data for the down-stream smart meters  $AMI_1, AMI_2, \dots, AMI_i$  in the same area utilizing the PD curve obtained in Step 2 via Acceptance–Rejection method.

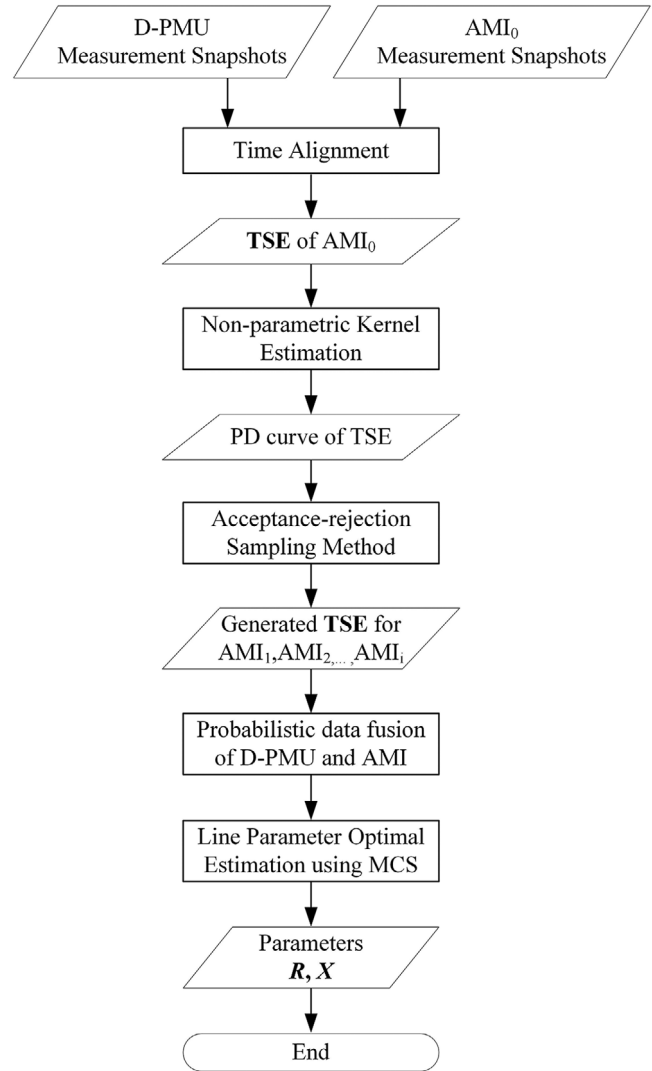


FIGURE 6 Flowchart of the parameter estimation procedure

- Add the generated TSE of down-stream smart meter to the D-PMU measurements for each time interval to formulate a time-shifted D-PMU curve which is regarded to be probabilistically aligned or fused with the down-stream AMI curve. Then Compute the expected value of line parameters  $R$  and  $X$  by performing the optimal parameter estimation algorithm using MCS for multi-runs in Matlab 2014@b.

#### 3.1 | Time alignment of D-PMU and AMI

A time-alignment algorithm is proposed to calculate TSE data of up-stream smart meter  $AMI_0$  taking D-PMU measurements for reference. Here, active power is selected as the same quantity used for AMI-PMU data alignment. D-PMU curve  $Y_{pmu}$  and a-synchronized AMI data  $Y_{ami}$  are aligned by Algorithm 1 as following:

where  $DT2$  represents the up-stream smart meter's TSE data, the “matching coefficients”  $ud$  and  $ld$  are the upper-level and

**ALGORITHM 1** Time alignment for D-PMU and AMI Data

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**Input:** D-PMU and AMI measurement  $Y_{pmu}, Y_{ami}$   
**Output:** TSE data  $DT^2$  and shifted D-PMU curve  $Y_i$

- 1 Generate the Spline-Interpolated gaussian noisy D-PMU data  $(X, Y_{pmu})$  with time interval  $Nc/gT$ :
- 2 Let  $Nc$  be the length of  $X$ ,  $x_q \leftarrow 1:(Nc/gT):Nc$
- 3  $v_q \leftarrow \text{Spline}(X, Y_{pmu}, x_q)$ .
- 4 **for** all  $k \in 1, 2, \dots, Nc$  **do**
- 5      $d \leftarrow 0$ ;  $M \leftarrow \text{zeros}(1)$ ;
- 6     **for** all  $i \in 1, 2, \dots, gT$  **do**
- 7         **if**  $ld * vq(i) \leq Y_{ami}(k) \leq ud * vq(i)$   
            &&  $k - tmax < i/gT * Nc < k - tmin$  **then**
- 8              $d \leftarrow d + 1$ ;
- 9              $M(d) \leftarrow i/gT * Nc$ ;
- 10             $xi \leftarrow i - \text{round}(d/2)$ ;
- 11             $Y_i(k) \leftarrow vq(xi)$ ;
- 12         **end**
- 13          $DT(k) \leftarrow k - \text{mean}(M)$
- 14     **end**
- 15 **end**
- 16  $n \leftarrow 0$ ;  $I \leftarrow \text{zeros}(0)$ ;
- 17 **for** all  $j \in 1, 2, \dots, X$  **do**
- 18     **if**  $\text{abs}(DT(j)) > tmax$  **then**
- 19          $n \leftarrow n + 1$ ;  $I(n) \leftarrow j$ ;
- 20     **end**
- 21 **end**
- 22  $DT^2 \leftarrow$  Remove elements of  $DT$  with the orders in  $I$ ;

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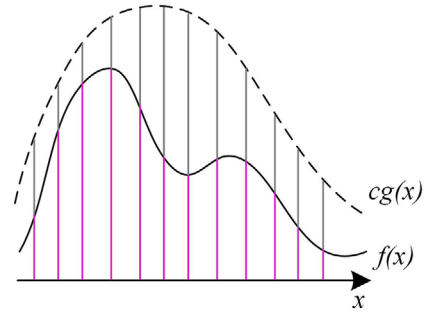
**TABLE 1** Parameter table

Parameters	Value
Sampling interval for AMI	30 min
Number of samples for AMI $Nc$	$2.4 \times 10^3$
Sampling interval for D-PMU	1s
Number of samples for D-PMU $gT$	$4.32 \times 10^6$
minimum value of TSE $tmin$	1.8 ms
maximum value of TSE $tmax$	450 s
minimum value of matching coefficient $ld$	0.9998
maximum value of matching coefficient $ud$	1.0002

lower-level constants, respectively. Since the D-PMU dataset is discrete, the matching coefficients are required to be close to 1 but not equal to 1. By conducting the algorithm with different setting parameters for the matching coefficients, it is found that the algorithm has better performance when  $ud$  is 1.0002 and  $ld$  equals 0.9998.  $Nc$  denotes the number of samples for AMI,  $gT$  means the number of samples for D-PMU.  $tmin$  and  $tmax$  are the minimum and maximum values of TSE, respectively. The value of the parameters is displayed in Table 1. Additional remarks are presented below.

**Line 1-3:** Data Preparation for sampled D-PMU data by conducting the interpolation function *Spline*, a specific Matlab function.

**Line 4-15:** Data Point Matching of  $Y_{ami}$  by scanned the local D-PMU curve partitioned by the TSE bounds from  $ld * vq(i)$  to

**FIGURE 7** Diagram of acceptance–rejection method

$ud * vq(i)$ . It is worthwhile noting that more than one matching point would be found; it is the middle one in the series that is selected as a matching point. TSE is supposed to range from  $tmin$  to  $tmax$  at each time stamp  $k$ . Moreover, it is anticipated that all the sampled data points on the AMI curve are on the time-shifted D-PMU curve represented as  $Y_i$ .

**Line 16-22:** Remove the abnormal point with TSE greater than  $tmax$ . Thus, the TSE dataset of the up-stream smart meter is obtained and denoted as  $DT^2$ .

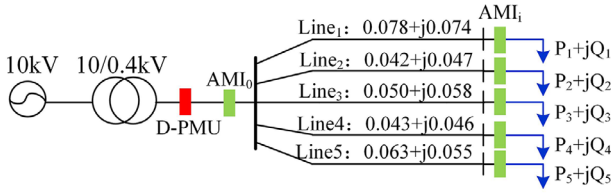
By implementing the time alignment algorithm, the TSE dataset of the up-stream smart meter is calculated and further utilized for probabilistic density (PD) estimation in Section 3.2.

### 3.2 | PD estimation and TSE data resampling

As mentioned before, the previous distribution function of the time synchronous error is unavailable. Therefore, The non-parametric Kernel density estimation (KDE) approach is introduced to obtain the PD curve of TSE of the up-stream smart meter. Histogram and kernel [34] concepts are deployed to estimate the non-parametric probabilistic density model [35]. The kernels, like the Gaussian Kernel adopted by us or the Laplacian Kernel used in [3], with enhanced mathematical properties are selected to perform the nonparametric kernel density estimation of TSE. It should be mentioned that all the above PD Estimation are included in a specific Matlab function *Ksdensity*. The smoothly shaped curve of the TSE produced by the KDE method is employed to generate the random TSE dataset of down-stream smart meters subsequently.

Herein, the PD curve  $f(x)$  of TSE for up-stream smart meter is obtained by integrating D-PMU and AMI data. According to the aforementioned assumption, TSE data samples for the down-stream smart meters are generated following its distribution  $f(x)$ . The acceptance–rejection method is used due to its efficiency and intelligence, as described in [36] and shown in Figure 7.

Given a probability distribution  $G$ , with density function  $g(x)$ , and constant  $c > 0$ ,  $\sup_x f(x)/g(x) \leq c$ , where  $c$  is expected be as close to 1 as possible for the sake of sampling efficiency; the random variable  $X$  complies with  $G$ , and  $Y \sim U$  independent



**FIGURE 8** The equivalent single-line diagram for the tested distribution feeders

from  $X$ ,  $X$  is accepted if

$$Y \leq \frac{f(X)}{cg(X)}. \quad (5)$$

Here, uniform distribution function is selected as the envelope function. Two independent uniform random variables  $x$  and  $y$  are generated. According to the above theory and the sampling rule denoted in Equation (5), it can be inferred that  $x$  complies with  $f(x)$  if the random point  $(x, y)$  falls on or below the PD curve  $f(x)$ . The process of TSE data generation is summarized as follows:

*Step a)* : Generate random  $x \sim U(tmin, tmax)$  and random  $y \sim U(0, y_m)$ , where  $tmin$  and  $tmax$  are the lower and upper limits of TSE for smart meters, respectively; and  $y_m$  is the maximum value of  $f(x)$  on  $(tmin, tmax)$ .  $x$  and  $y$  are two independent variables.

*Step b)* : If  $y < f(x)$ , then  $(x, y)$  is accepted; otherwise, repeat *Step a* until  $x, y$  meet the conditions.

### 3.3 | Optimal estimation of line parameters

The generated TSE of the down-stream smart meter is added to the D-PMU measurements for each time interval; thus, a new time-shifted D-PMU curve probabilistically aligned to the down-stream AMI curve is formulated and leveraged for optimal line parameter estimation. According to the voltage drop complete model in Equation (3), the distribution line parameter estimation problem is formulated as:

$$\min_{R, X} V_1^2 - V_2^2 - (I^2 R^2 + I^2 X^2 + 2P_2 R + 2Q_2 X) \quad (6)$$

Considering the constraints that both  $R$  and  $X$  should be positive, the problem is updated to a quadratic programming model as following:

$$\begin{aligned} \min_{R_m, X_m} \quad & \sum_{i=1}^N (V_{1i}^2 - V_{2i}^2 - (I_i^2 R_m^2 + I_i^2 X_m^2 \\ & + 2P_{2i} R_m + 2Q_{2i} X_m)) \quad (7) \\ \text{s.t.} \quad & R_m, X_m > 0 \end{aligned}$$

where  $N$  is the number of samples,  $V_{1i}, V_{2i}, I_i, P_{2i}, Q_{2i}$  denote the  $i$ th measurements,  $m = 1, 2, 3, \dots, M$  is the number of Monte Carlo Simulation.

The expected value of estimated  $R$  and  $X$  is calculated as

$$\begin{aligned} \bar{R}_{es} &= (1/M) \sum_{m=1}^M R_m \\ \bar{X}_{es} &= (1/M) \sum_{m=1}^M X_m \end{aligned} \quad (8)$$

## 4 | NUMERICAL TEST

Numerical tests were performed to evaluate and show the effectiveness of the proposed methodology for the distribution line parameter estimation. All use cases are conducted on Matlab 2014@b on a Core i7-6700K CPU processor @4.00 GHz. The equivalent single-line diagram of the low-voltage distribution network is shown in Figure 8.

### 4.1 | Simulation scenarios

#### 4.1.1 | TSE distribution

To validate the proposed algorithm, three tested distribution (Uniform, Rayleigh, and Gaussian Mixture Model (GMM)) patterns of TSE data are used. Whereas it is mentioned in the Introduction Section that TSE does not follow any predefined distribution, standard TSE distributions are chosen here for conducting test only and demonstrating the robustness to different TSE distribution patterns:

- *Uniform Distribution*:  $\Delta t \sim U(tmin, tmax)$
- *Rayleigh Distribution*:  $\Delta t \sim \text{raylpdf}(tmin : tmax / Nc : tmax, 0.05)$
- *GMM Distribution*:  $\Delta t \sim \text{GMM}(\mu_1, \delta_1, \mu_2, \delta_2, r) \sim \text{GMM}(0.075, 0.02, 0.175, 0.012, 0.75)$  where  $r$  is the ratio of two gaussian distributions.

#### 4.1.2 | Random error in meter readings

The voltage magnitude error of D-PMU is set as 0.2% to mimic measurement errors. For a 0.2-class metering device, the magnitude error of CT (current transformer) and VT (voltage transformer) is 0.2%, while the phase error of CT and VT is about 0.3crad (1crad =  $1e^{-2}$ rad) [10]. Likewise, the measurement error of the 0.2-class smart meter can be modeled as Gaussian variance and added to the measurements [32].

#### 4.1.3 | Loading conditions

To mimic the realistic scenarios, the loading data are set as Gaussian distributions with standard deviation = 0.2 and with stochastic power factors (PF) ranging from 0.75 to 0.95.

The proposed D-PMU and AMI based methodology for line parameters estimation is implemented in the MATLAB 2014@b programming environment considering the various stochastic TSEs.

#### 4.1.4 | TSE distribution obtaining and generation

The probabilistic density curve of TSE estimates obtained by time alignment of D-PMU and AMI, as well as newly generated sampling TSE data for the down-stream smart meters, are depicted in Figure 9.

As indicated in Figure 9, the proposed time alignment algorithm is feasible to obtain the TSE distribution's PD curves. Further, it is demonstrated that non-parametric kernel density estimation and the acceptance–rejection sampling method are effective.

#### 4.1.5 | Estimated parameters and comparison

The proposed methodology is validated by implementing 5000-run Monte Carlo Simulations and compared with the state-of-the-art in [28–30]:

- (a).Standard Situation: Investigate the line parameter estimation results assuming that D-PMU and AMI curves are strictly aligned without TSE.
- (b).Method in [28–30]: Investigate the parameter estimation results considering TSE but with no data fusion of D-PMU and AMI data.
- (c).Proposed Method: Investigate the parameter estimation results considering TSE and using the proposed probabilistic data-fusion driven methodology.

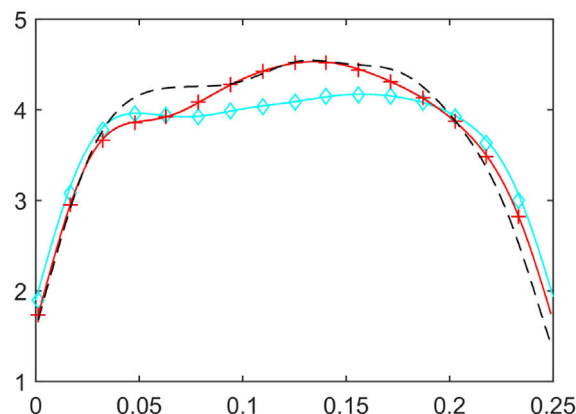
To verify the effectiveness of the proposed approach, (a) is termed as the standard situation. Correspondingly, the estimated results of  $R$  and  $X$  in (a) are denoted as  $R_{std}$  and  $X_{std}$ , respectively. The absolute value of Relative Error (RE) is deployed to estimate the obtained results and expressed as

$$\begin{aligned}
 RE_l\%(r) &= \text{abs}((\bar{R}_{est} - R_{std})/R_{std}) * 100 \\
 RE_l\%(x) &= \text{abs}((\bar{X}_{est} - X_{std})/X_{std}) * 100
 \end{aligned}
 \tag{9}$$

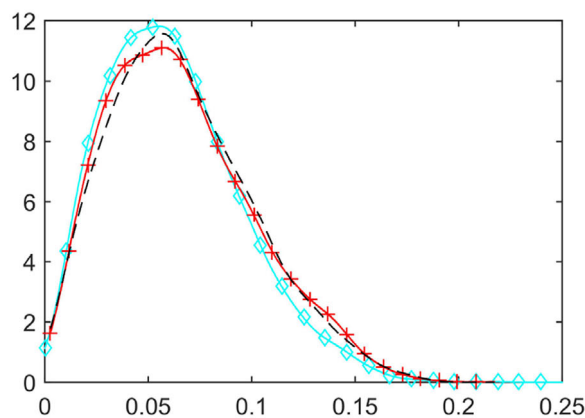
where  $l$  means the  $l_{ij}$  branch line in the distribution network.

## 4.2 | Results and discussion

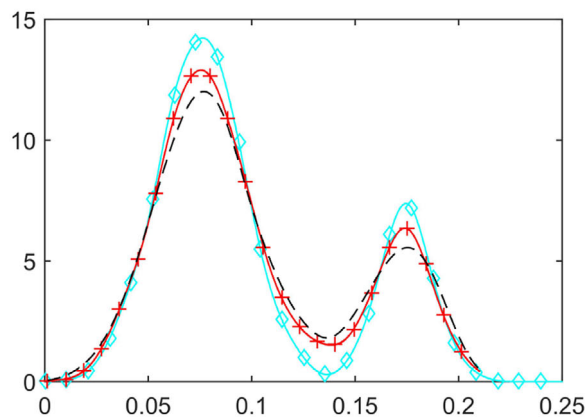
The comparison of obtained values of estimated  $R$  and  $X$  and absolute corresponding REs for the method in [28–30] and the proposed approach are presented in Tables 2 and 3, respectively. The corresponding graphical comparison of REs under different tested TSE patterns is depicted in Figure 10.



(a) PD curves of Uniform distributed TSE



(b) PD curves of Rayleigh distributed TSE



(c) PD curves of GMM distributed TSE

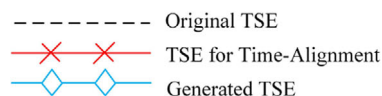


FIGURE 9 PD curves of three tested distributed TSE patterns

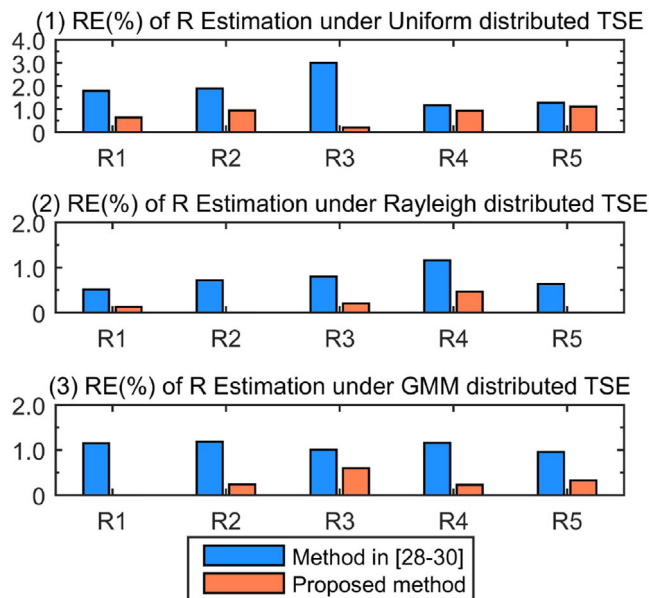
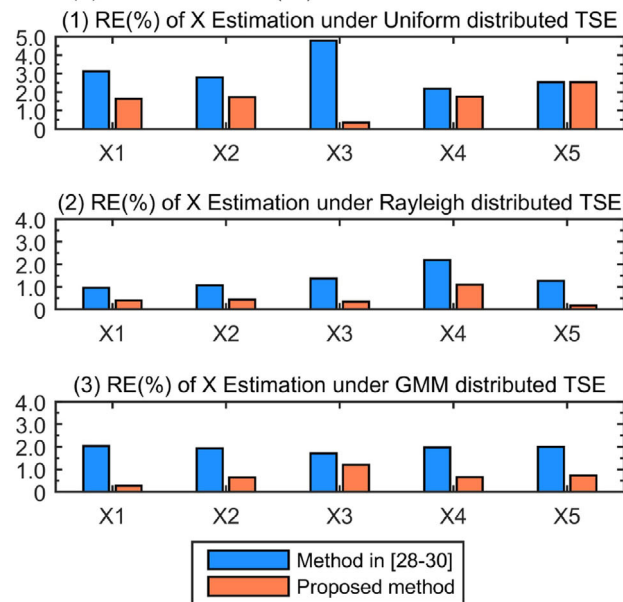
**TABLE 2** Results for resistance parameter estimation

TSE patterns	(a/b/c)	R1( $\Omega$ )	R2( $\Omega$ )	R3( $\Omega$ )	R4( $\Omega$ )	R5( $\Omega$ )
Without TSE	(a)	0.0782	0.0422	0.0499	0.0431	0.0628
Unif	(b)	0.0796	0.0430	0.0514	0.0426	0.0636
	(c)	0.0777	0.0418	0.0498	0.0427	0.0621
Rayleigh	(b)	0.0786	0.0425	0.0503	0.0426	0.0632
	(c)	0.0781	0.0422	0.0498	0.0429	0.0628
GMM	(b)	0.0791	0.0427	0.0504	0.0426	0.0634
	(c)	0.0782	0.0421	0.0496	0.0430	0.0626
TSE patterns	(a/b/c)	RE <sub>1</sub> (%)	RE <sub>2</sub> (%)	RE <sub>3</sub> (%)	RE <sub>4</sub> (%)	RE <sub>5</sub> (%)
Unif	(b)	1.7903	1.8957	3.0060	1.1601	1.2739
	(c)	0.6394	0.9479	0.2004	0.9281	1.1146
Rayleigh	(b)	0.5115	0.7109	0.8016	1.1601	0.6369
	(c)	0.1279	0.0000	0.2004	0.4640	0.0000
GMM	(b)	1.1509	1.1848	1.0020	1.1601	0.9554
	(c)	0.0000	0.2370	0.6012	0.2320	0.3185

**TABLE 3** Results for reactance parameter estimation

TSE patterns	(a/b/c)	X1( $\Omega$ )	X2( $\Omega$ )	X3( $\Omega$ )	X4( $\Omega$ )	X5( $\Omega$ )
Without TSE	(a)	0.0735	0.0464	0.0583	0.0457	0.0553
Unif	(b)	0.0712	0.0451	0.0555	0.0467	0.0539
	(c)	0.0747	0.0472	0.0585	0.0465	0.0567
Rayleigh	(b)	0.0728	0.0459	0.0575	0.0467	0.0546
	(c)	0.0738	0.0466	0.0585	0.0462	0.0554
GMM	(b)	0.0720	0.0455	0.0573	0.0466	0.0542
	(c)	0.0737	0.0467	0.0590	0.0460	0.0557
TSE patterns	(a/b/c)	RE <sub>1</sub> (%)	RE <sub>2</sub> (%)	RE <sub>3</sub> (%)	RE <sub>4</sub> (%)	RE <sub>5</sub> (%)
Unif	(b)	3.1293	2.8017	4.8027	2.1882	2.5316
	(c)	1.6327	1.7241	0.3431	1.7505	2.5316
Rayleigh	(b)	0.9524	1.0776	1.3722	2.1882	1.2658
	(c)	0.4082	0.4310	0.3431	1.0941	0.1808
GMM	(b)	2.0408	1.9397	1.7153	1.9694	1.9892
	(c)	0.2721	0.6466	1.2007	0.6565	0.7233

It is worth mentioning that, for the standard situation, the parameters are estimated without time synchronization error; however, the errors appear in the D-PMU and AMI measurements. Further, it should be highlighted that the proposed methodology is focused on the data-fusion of D-PMU and AMI and obtained better results in the proposed method than the method in [28–30], which indicates a compromised solution and relevance of our proposed method. The presented tables and figures interpret that the proposed data-fusion of D-PMU and AMI for parameters estimation outperforms in all considered situations and indicates the robustness of the proposed methodology to various TSE distributions.

**(a) Relative error(%) of R Estimates****(b) Relative error(%) of X Estimates****FIGURE 10** Comparison of the current method and the proposed method under different tested TSE distribution patterns

## 5 | CONCLUSION

This paper proposes a novel methodology for distribution line parameter estimation. The proposed method is based on combined configuration and probabilistic data-fusion of D-PMU and AMI measurements considering the a-synchronization state of smart meters. The TSE dataset of the up-stream smart meter is acquired by the proposed time-alignment algorithm of D-PMU and AMI data. Then, the PD curve of TSE of the up-stream smart meter is obtained via the non-parametric KDE approach. Furthermore, the acceptance–rejection sampling



technique is introduced and employed to generate TSE data for the down-stream smart meters in the same local area. Consequently, a probabilistic time-shifted D-PMU curve is formed to align or fuse with the down-stream AMI curve. Leveraging the fused D-PMU and AMI measurement data, the distribution line parameter estimation problem is formulated as a QP optimal algorithm according to the voltage drop model. We investigated three different cases to evaluate the performance of the proposed method and applied Monte Carlo simulations for 5000 runs for each case in MATLAB 2014®. Moreover, three tested TSE distribution patterns (Uniform, Rayleigh, and GMM) are utilized to demonstrate the robustness of the proposed method. The obtained results and comparison of REs have verified the effectiveness of the proposed probabilistic data-driven line parameter estimation methodology and its robustness to different TSE distributions. For future work, the proposed approach will be tested on practical data if possible.

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