CLEVER OR WISE: THE KINDS OF MATHEMATICS WE TEACH ARE PREPARING US FOR WHAT?

THE STORY OF MY WORRY

Yasmine Abtahi

Western Norway University of Applied Sciences

Yasmine.Abtahi@hvl.no

ABSTRACT

It has been years that I ask what it means to be clever and what it means to be wise and that how and in what ways cleverness and wisdom guid my actions? Recently, I asked myself the following question: "The kinds of mathematics that we teach are preparing us for what? To grow clever or to become wise?" In this text, I pursue a deeper exploration of the respective conceptualisations of "cleverness" and "wisdom" and, in particular, I try to understand the role played by mathematics in the growth and development of these two qualities. My mission in this text is to conceptualise the experiences that are offered to students, based on the kinds of mathematics that we teach. I begin by explaining what I mean by "the kinds of mathematics that we teach," then I explain how I view wisdom and cleverness, and, finally, I critique the role played by the institutional context of mathematics in the type of orientation that mathematics instruction does not take and in the type of socio-political dimension that it promotes.

Keywords: the mathematics that we teach, critique, worry, cleverness, wisdom

Cleverness and wisdom

To make sense of cleverness and wisdom, a combination of three views will be my point of departure: the views of Aristotle, Bettelheim (1976) and Durant (1961). In *Nicomachean Ethics*, Aristotle makes a sharp distinction between cleverness and wisdom: a clever man is he who knows how to get what he wants (regardless of whether what he wants is good or not good) but a wise man is he who knows not only this but also what is actually worth wanting. Bettelheim (1976) in his book about fairy tales explains that cleverness is intellect without character while wisdom is the outcome of inner depth – "of meaningful experiences which have enriched one's life, a reflection of a rich and well-integrated personality" (p. 111). He further emphasises that the basis for achieving this well-integrated personality is struggling with one's deep and uncertain attachments in order to better understand the nature of one's dilemmas and concerns. Finally, Will Durant (1961) differentiate between scitentics and philosophers. He explains:

> Science wishes to resolve the whole into parts, the organism into organs, the obscure into the known. It does not inquire into the values and ideal possibilities of things or into their total and final significance. The scientist is as impartial as Nature in Turgenev's poem: He is as interested in the leg of a flea as in the creative throes of a genius. But the

> philosopher is not content to describe the fact. He wishes to ascertain its relation to experience in general and thereby to get at its meaning and its worth. He combines things in interpretive synthesis. He tries to put together, better than before, that great universe-watch which the inquisitive scientist has analytically taken apart" (p.231) .

Without making myself preoccupised with the terms "scitentics" and "philosophers", for me, Will Durant differentiation between science and philosophy signifies the relationship between wisdom and cleverness. Fact, he says, is not complete except in relation to a purpose and a whole. Facts without perspective and valuation, cannot save us from despair. Facts for me are tools of cleverness and perspective and valuation tools of wisdom.

It might look that I sharply dichotomise between cleverness and wisdom and that I ignor the overlap. For the purpose of this paper, I intentionally signify the dichotomy, because this dichotomy has prompted me to ask a number of specific questions about mathematics. Does the mathematics that we teach and value help us question and deal with our very personal dilemmas and concerns? Does acquiring the mathematical knowledges offered by the primary and secondary school systems help us, first, to know what we want (i.e., a kind of awareness associated with cleverness) and, second, to become more well-integrated persons who know what is worth wanting – both for ourselves as individuals and for the communities, individuals and things with which each of us has a relationship (i.e., a kind of awareness associated with wisdom)? Finally, does the mathematics that we teach provide us with a space in which we can mature our ability to respond to and act in accordance with essential social values, including tolerance, fairness and acceptance (i.e., a kind of ability associated with wisdom)?

Let me give an example. In thinking about mathematics and society, I often play a game with myself and, depending on the kind of class that I am teaching, with my students as well. I regularly think about mathematical concepts that are true within the four walls of a maths classroom but do not apply to other contexts, specifically in relation to human interactions with other humans and with things. An example is the mathematical idea of 1 = 1. In a mathematics classroom, this is a true and indisputable a priori statement. Yet, in real life (at least in mine), no two potatoes are the same, no two stones are the same, no two days are the same and no two people are the same (or are even treated, respected and valued in the same way). I am still looking to see when 1 = 1 in my own life, outside the scope of the consumeristic and mechanistic "stuff on the Walmart shelves" and outside the scope of the formal mathematics that we teach. I wonder about the ways in which and the extent to which teaching students that 1 = 1 gives them the perspective to ask essential questions about human dignity and equality – such as "What does equality actually mean?" – and about whether teaching students that 1 = 1 raises any broader, non-mathematical questions such as: "is everyone really equal?". These questions inevitably lead to my core question, which is as follows: the kinds of mathematics that we teach and value are preparing us for what?

The kinds of mathematics that we teach and value

Critical mathematics education is a domain that for decades has been challenging both conventional mathematics and its various approaches to teaching and learning. For example, Valero (2018) asks "What is mathematics in relation to society, what does mathematics do as part of the school curriculum, and what are the potentials of mathematics education to produce or challenge inequalities in society and among students?" (p. 103). Similarly, Gutstein (2012) looks into ways in which mathematics could be an instrument for promoting social emancipation; Pais

(2013) challenges the purported universality of mathematics; and Valero (2004) focuses on the marginalisation and even oppression perpetrated against non-Eurocentric mathematics. In this paper, building on the work of all these scholars, I seek to challenge the types of experiences that are offered to students as they study and work on mathematics. My rationale for this endeavour is the fact that the body of mathematical knowledges that we teach is our social production and the fact that knowledge construction is ultimately a political project that is built into networks of social and bureaucratic institutions that include schools, ministries of education and other producers of education policy. Hence, this body of knowledges not only exists but also is deeply embedded in society in ways that secure its reproduction. I believe that it is essential to, once in a while, challenge what is regularly reproduced in societies, including the types of things that we teach under the umbrella of "mathematics."The kinds of mathematics that we teach not only are highly valued but their reproduction is made secure by various means such as the contents of international mathematics curricula as well as the cultural, historical and linguistic resources we tap into for mathematics instruction purposes. The production of the kinds of mathematics that we teach is a social and communal process. I believe that, in critically examining the value of a particular form of knowledge (including the internationally recognized approach to mathematics), one of the primary considerations is not its learning and teaching but the ways in which this form of knowledge is securely reproduced.

Several theories of social reproduction, such as those articulated by Bourdieu and Passeron (1977) and by Bernstein (1975, 1996), explain the fact that curricula play essential roles in reproducing particular social qualities and values by constructing and presenting particular types of knowledge as legitimate. Apple (1979) emphasises the idea that the ideologically based contents of many curricula legitimize some kinds of knowledge and make them appear natural and consistent with common sense. Similarly, mathematics curricula (or at least those used in Canada, Norway, the UK and Australia), with their typical forms of outcomes and objectives, validate specific forms of mathematics as being useful for reproducing particular characteristics of their respective societies. For example, in the British national curriculum, pupils need to "recognise, find and name a half as one of two equal parts of an object, shape or quantity" (National Curriculum, year, p. 8). In the province of Ontario, students "divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half, three thirds, two fourths or two quarters) (OMC, 2008). In the Norwegian Education Curriculum (2006), "Using 1/2, 1/4 students learn to 'describe part of a whole.'" These are examples of the kinds of fractional concepts that we teach.

In this paper, I use the distinction and the relationship between what I call "wise" and what I call "clever" to challenge the kinds of mathematics that we teach. In order to challenge the kinds of mathematics that we teach and establish a foundation for thinking about them, I divide this paper into two principal sections, the first being critical and the second being hopeful. In the first section, I specify and closely examine the types of mathematics that we teach, the resources that we draw upon to teach them, and the types of power and values that we associate with the same. Then, in the next section, I suggest a possible framework for striking an appropriate balance between cleverness and wisdom in the context of mathematics education. My hope is that, by elucidating what I mean by "becoming more wise," I will ultimately put forward a vision of a body of mathematical knowledges that not only will help to make us more clever but also will help us to gain wisdom by learning from experiences and by giving us new spaces in which to directly reflect those knowledges. This includes reflection on the experiences through which we gain insight into and understanding of the networks of relationships of which we are a part.

Finally, I show that the road ahead, in terms of reflection on the kinds of mathematics that give us experiences that help us to become wiser, is bumpy and long but still is worthy of taking.

I start my journey by specifying what I mean by "the kinds of mathematics that we teach and value." Along the way, I provide concrete examples, to make my points as clear as possible.

What I mean by "the kinds of mathematics that we teach and value"

In general, mathematics deals with numbers, structures, underlying logics and changes. Yet, within school systems, there surely must exist another understanding of mathematics that is more global because we, as mathematics educators, have a common understanding of what we mean by mathematics as well as the ability to communicate with one another both at international conferences and through scholarly publications. For the purposes of this paper, I refer to this body of formal knowledges as "the kinds of mathematics that we teach." In the next paragraph I highlight the various kinds of mathematics that we teach, first by explaining what I do not mean by this characterization and then by further clarifying what I actually mean by it. The latter successively focuses on the knowledge-content, the socio-cultural and historical, the linguistic and the political contexts.

In referring to "the kinds of mathematics that we teach and value" I do not endorse the essentialist position put forward by Rowlands and Carson (2002) that mathematics is a universal subject based on the supposed justification that, "although aspects of culture do influence mathematics, nevertheless these cultural aspects do not determine the truth content of mathematics" (p. 98); this position is supported by the further explanation that there is a single essence that runs through all representations of mathematical knowledge – that, regardless of whether a particular approach to mathematics was developed by a Mongolian tribe or in a European university, the underlying concepts of all mathematical traditions are the same. This non-relational position articulated by Rowlands and Carson (2002) is criticised by Adam, Alangui, and Barton (2003) and by Pais (2012) on the basis that it by default includes students whose background is the same as or is similar to that of the original context of the particular form of mathematics in question while excluding students who do not have this kind of privileged connection. I also eschew the latter position. Further, in referring to "the kinds of mathematics that we teach" I do not endorse Bernard Russell's (1992) uncertain, inexact and incomplete view of mathematics; according to this perspective, rather than internal certainty being found within mathematics as an autonomous, self-contained entity, at best only a close approximation to uncertainty can be found through its various applications. Furthermore, according to Skovsmose and Greer (2012), it is the latter that is typically fostered in mathematics classrooms. Now that I have begun to explain what I do not mean by "the kinds of mathematics that we teach," I should state what I am actually referring to.

In using this expression, I point towards the content of an internationally known body of knowledges and discourses that we, as mathematics researchers and educators, collectively and individually refer to as comprising mathematics. Even if we assume that we do not have a focused and unified body of knowledge to designate using this term, the goal of large-scale international conferences like ICME is to unite the world around a common understanding of what is meant by school mathematics and by the effective and comprehensive learning and teaching of the same. For example, as is stated on its website, ICME's purposes include the promotion of international activities and publications that improve the collaborative exchange and dissemination of ideas and information related to all aspects of the theory and practice of contemporary mathematical education, as well as the active support of efforts to improve the quality of mathematics teaching and learning worldwide. ICME explores important topics related to the teaching and learning of mathematics, including arithmetic and number systems, measurement, algebra, geometry, probability, statistics, calculus and discrete mathematics (including logic, game theory and algorithms). These are examples of the conventional topics that I refer to as coming within the scope of “the kinds of mathematics that we teach and value.”

The internationality of the kind of mathematics we teach can also be found in mathematics software packages and applications that are designed in one country and are extensively used and researched in other countries. For example, the GeoGebra geometry software package was designed in Austria and is widely used in many other countries, such as in South Africa (Berger, 2011), Turkey (Mukiri, 2016), Kenya (Aktümen et al., 2011), Israel (Anabousy & Tabach, 2016), and both Canada and Norway based on my personal experiences.

Reproduction of “the kinds of mathematics that we teach and value”

The kinds of mathematics that we teach not only are highly valued but their reproduction is made secure by various means such as the contents of international mathematics curricula as well as the cultural, historical and linguistic resources we tap into for mathematics instruction purposes. The production of the kinds of mathematics that we teach is a social and communal process. I believe that, in critically examining the value of a particular form of knowledge (including the internationally recognized approach to mathematics), one of the primary considerations is not its learning and teaching but the ways in which this form of knowledge is securely reproduced.

Several theories of social reproduction, such as those articulated by Bourdieu and Passeron (1977) and by Bernstein (1975, 1996), explain the fact that curricula play essential roles in reproducing particular social qualities and values by constructing and presenting particular types of knowledge as legitimate. Apple (1979) emphasises the idea that the ideologically based contents of many curricula legitimize some kinds of knowledge and make them appear natural and consistent with common sense. Similarly, mathematics curricula (or at least those used in Canada, Norway, the UK and Australia), with their typical forms of outcomes and objectives, validate specific forms of mathematics as being useful for reproducing particular characteristics of their respective societies. For example, in the British national curriculum, pupils need to “recognise, find and name a half as one of two equal parts of an object, shape or quantity” (National Curriculum, year, p. 8). In the province of Ontario, students “divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half, three thirds, two fourths or two quarters) (OMC, 2008). In the Norwegian Education Curriculum (2006), “Using 1/2, 1/4 students learn to ‘describe part of a whole.’” These are examples of the kinds of fractional concepts that we teach.

The power of the kinds of mathematics that we teach and value

The body of knowledge that we characterize and teach as mathematics is politically valuable because it applies to the rapidly moving and highly complex real worlds of business, economics, science and engineering, as well as to many other fields. This is the kind of mathematics that helps us to understand, engage with, communicate about and criticize issues related to these and to many other worlds (as viewed by Barwell (2013) and by Abtahi et al. (2018)). This mathematics includes a pool of knowledges that has “formatting power” and is able to shed light on complex and dynamic human phenomena (Skovsmose, 2005). In the heavily cited concept of the “formatting power of mathematics,” Skovsmose (2005) identifies three kinds of knowing that are relevant to mathematics teaching and learning. *Mathematical knowing* “refers to the competencies we normally describe as mathematical skills” (p. 100); *technological knowing* “refers to the ability to apply mathematics and formal methods in pursuing technological aims” (pp. 100–101); and, finally, *reflective knowing* “has to do with the evaluation and general discussion of what is identified as a technological aim and the social and ethical consequences of pursuing that aim with selected tools” (p. 101). A point that I would like to raise here is that, even though the focus in Skovsmose’s ideas is on the power of mathematics to do positive things for individuals and for societies, these ideas do not directly consider what kinds of mathematics give us what kinds of tools to deal with what kinds of social and individual problems.

Sometimes, when reflecting on the “formatting power of mathematics,” I imagine that mathematics is a selfish giant that seeks to arrogate and keep for itself all of the good things that give individuals and societies power. Yet, my question and my worry is whether there might exist another form of mathematical knowledge or another form of experience that we owe to the next and subsequent generations of children and whether this has been hidden by this powerful giant. I have good reason to be worried. What if the kinds of mathematics that we teach (as laid out in curricula, for example) do not bring to the foreground the possibility that, by focusing on and dealing with a certain kind of mathematics, we might actually become less capable of dealing with social and individual problems that have a significant intangible component – including issues related to human dignity, a sense of security, tolerance and honesty, among others. Ernest (2018) raises similar kinds of concerns:

> …the mathematical way of thinking promotes a mode of reasoning in which there is a detachment of meaning. Reasoning without meanings provides a training in ethics-free thought. Values neutrality and ethical irrelevance is presupposed because meanings, contexts and their associated purposes and values are stripped away and discounted as irrelevant to the task in hand. […] Such reasoning and perspectives contribute to a dehumanized outlook. For without meanings, values or ethical considerations reasoning can become mechanical and technical and “thing” or object-orientated (p. 194).

Let me elucidate using a concrete example drawn from a recent and ongoing world situation, specifically in Chile. Milton Friedman, trained as a mathematician and statistician, was one of the best-known economists of the twentieth century and was largely responsible for developing the so-called “formalist revolution” in economics (Hands, 2003). The formalist revolution involved a substantial increase in the use of mathematics, abstraction and deductive modes of reasoning in economic theory and in applications of the same. In the formalist view, “Theorems and proofs replaced systematic argumentation; success came to be measured almost exclusively in terms of new techniques; and the form of a theoretical argument came to take precedent over its content” (Hands, 2003, p. 509); in this regard it is important to acknowledge that “the formalist converses much of the time in the language of mathematics” (Mayer, 1995, p. 73). Part of this economic-thought revolution included not just the heavy use of mathematical and deductive theoretical models, but also an intensified emphasis on statistical measures that operate exclusively or almost exclusively on the aggregate level (measures like the level of public debt, the level of money supply, the level of inflation, interest rates, exchange rates, flows of international investment, the balance of payments and rates of economic growth). Notwithstanding the importance of these metrics for any economy, this type of hyper-emphasis came at the expense of other, much more granular measures that in many cases focus on the social impact of economic activity, trends and especially policy (measures like levels of economic inequality, unemployment, homelessness, access to basic necessities, and access to education and health care as well as levels of economic diversification including the ability of a country to be self-sufficient in terms of food production).

Friedman was the mastermind behind Chile's economic transformation, which was largely executed by a group of Chileans who were trained by this individual and who soon became known as the "Chicago Boys" (a designation that was based on the fact that Friedman for many years taught at and was closely associated with the University of Chicago). During military dictator Augusto Pinochet's time in power, the Chicago Boys explicitly drew and built on Friedman's theories in the process of leading the Chilean economy along an extreme neoliberal path that included radical deregulation, hyper-aggressive privatization and other unfettered free-market policies – a policy package whose success was measured solely in terms of the above-mentioned macroeconomic metrics and in terms of similar measures.

Five decades later, the Chicago Boys' legacy lives on as health care remains privatised and as pension funds, natural resources, bodies of water, rivers, fruit farms, the entire avocado production, high-quality schools and many universities remain lawfully owned by small group of wealthy people; as a result, the majority of the population has for many years been structured into accepting the low wages and other meagre economic and social outcomes offered by the elite, a situation that has recently resulted in ongoing public protests (specifically from October 2019 up to the present). I believe that Chile's economic transformation under the direction of the Chicago Boys is a prime example of how the use of superficially well-reasoned, abstracted and decontextualised mathematics can devastate ordinary people's social and economic status, sense of dignity and emotional security. Friedman and his followers were seemingly able to mathematically prove what was best for Chile's economy. The Chicago Boys got clever, but did they get any wiser? Freidman, on the other hand, could claim the 1976 Nobel prize in economics. Was this a reflection of authentic achievement – or just a loud but empty accolade, "signifying nothing"? (I am referring to end of quote from Macbeth, by Shakespeare)

Now comes my concern.

An even better question: What did not happen ?

Here I ask, what did happen with and through mathematics, in the Chicago Boys actions to advance Chile's economy, in hope to make a stronger nation? Or an even better question is *what did not happen?* To think about this question, I go back to the dichotomy that I created between cleverness and wisdom, at the beginning of this paper. With and through mathematics, Chicago Boys resolve the whole idea of "the well-being of Chilean" into parts without inquiring into the values and ideal or into the total and final significance of their actions on the society as a whole. The fact was that mathematical ideas could explain economic growth. Yet, the Freidman mathematical economical model, could not put into perspective the social needs of multiple classes of the society and "Facts without perspective and valuation, cannot save us from despair".

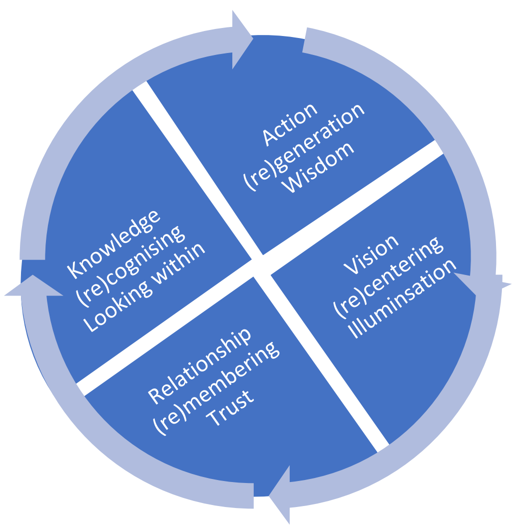
What I am concern about and what I like to strongly highlight is if and in what ways the kinds of mathematics that we teach help our students resolve wholes into parts, the organisms into organs, the obscures into the knowns at the expense of hindering their abilities to "inquire into the values and ideal possibilities of things" and "into their total and final significance"? if remotely the case, that would be an expensive price to pay.

I am not attempting to address this question, in this text. I am trying to raise a worry, to provide a base to think about possible attachments and to better understand the nature of some possible dilemmas and concerns with the kind of mathematics that we teach.

I am aware that so far, I have portrayed a rather gloomy picture regarding the kinds of mathematics that we teach. But please bear with me. As I mentioned at the start of this paper, after my critique comes an invitation to hope. With the spirit and intent to pursue answers to my question in an orderly and systematic manner, I draw upon philosophical perspectives related to the ideas of cleverness and wisdom. More specifically, I revisit Styer's (2017) circular conceptualization of human action. I employ this circular framework in another work – specifically in Abtahi (2019) – which is mostly concerned with the teaching and learning of mathematics. In this text, I am using the circular framework to think and talk about mathematics itself.

(*Re*)view*ing* the kinds of mathematics that we teach and value

As I explain in Abtahi (2019), for many Canadian indigenous communities, circularity is a framework that represents the wholeness that brings all living entities together in a circle of codependent connections. This framework has four major elements: (re)centring, (re)membering, (re)cognising and (re)generating (see Figure 1).



(Re)centring is the process of renewing the centre of one's vision and imagining what is possible or what might be possible. (Re)cognizing opens up possibilities for relating to each other's collaborative, collective and individual knowings as well as to each other's ways of coming to know. (Re)membering is the action of bringing together different ways of knowing, experiences and cultures. (Re)generating focuses on the action and, in particular, on the actualization of the vision and associated values. I use this framework as a possible new orientation to the kinds of mathematics that we teach.

Drawing on the elements of the circular framework set out above, I suggest (re)viewing the kinds of mathematics that we teach as part of an attempt to become more clever or to become more wise. By (*re*)view*ing*, I do not refer simply to a re-examination of the mathematical concepts and to the need to ensure that they make sense to us, based on what we were taught by the education system. The (*re*) in (re)view-ing points to the act of examining our vision towards the kinds of mathematics that we teach. The -ing (i.e., the present-participle ending) indicates movement and progress in action. Thus, (re)viewing means trying once again to view beyond some possible fixed understandings and definitions of what we mean by "mathematics" – (re)centring a newer vision of the kinds of mathematics that can help us to become wiser, to break through some legacies and to reshape (not reform) some of "the kinds of mathematics that we teach." My hope here is that, by (re)viewing our mathematics, we might be able to solve some problems that we cannot solve at present – seeming impasses that can be attributed to the fact that we are using the same methods that created these problems in the first place. This is a phenomenon of which the story of the Chicago Boys is a prime example. Hence, I propose that we (re)centre a kind of mathematics that will help to provide us with the kinds of experiences that will enable us to see ourselves related to and implicated in the lives of all those around us. This vision opens up opportunities to address how mathematics could illuminate the various ways in which one exists in relationships. We should pursue a kind of mathematics that (re)cognizes a diversity of collaborative, collective and individual knowledge systems as well as the various ways in which we come to know. This body of knowledges should (re)cognize the fact that the knowing of mathematics and of life experiences evolves through fluidity and relationality in equal measure and the fact that this type of knowing is accumulated through interaction. Moreover, we should pursue a kind of mathematics that extends this (re)cognition to re(membering) – to provide a basis for trust in order to actively reconceptualize and reconcile a diverse range of traditions and ways of knowing and thus address the complexity of the relationships that we share with other individuals and groups and the ethical responsibilities that we bear towards the same. Finally, we should pursue the (re)generation of a mathematical focus on the types of dynamic action, relationships, experiences and world views that lead to wisdom. Students should be offered experiences to (re)member and (re)cognize the fact that all human beings are formed and revived within webs of relationships to which they have ethical duties.

Let me give you another example. I assume that we remember the steps involved in the addition of fractions. As a learner of the concept of addition of fractions, I never understood the point of what I was doing. I simply memorised facts and procedures. It was the pursuit of a doctoral study entirely focused on the learning of the addition of fractions that enabled me to understand why we as students had followed the steps that we had followed. I finally understood why and how knowledge of the concept of "unit fractions" is crucial. I also understood how the identification of a common denominator makes the units of the two fractional numbers the same and how this in turn makes the addition of two fractional numbers feasible (and a fair addition of "similar" units). Still, with my doctorate degree, I was unable to determine how knowledge of the concepts of the "common denominator" or of "unit fractions" might help me to think any differently about the world around me or about my experiences within it.

Now I have a story of how I gained a new sense of finding "common denominators" – not in numbers, but in life. My sense comes from reflection on my years and years of conversations, initially with a person I have highest regards for and then with a number of other people as a form of follow-up. This person that regard is a social activist who has lived through a coup and a revolution and who has seen the fall and rise of a long succession of regime changes along with various paradigm-changing ideologies. She has read daily for 78 years. She has stories. I grew up with her. For many conversations, questions, thoughts and ideas, she is able to draw on relevant and often simple stories based on her own experiences, the experiences of others whom she knows (or once knew), and/or various books and articles that she has read. In my conversations with her, I have always wondered how it is possible for her to have so many stories to tell. As I started to reflect on these conversations in greater depth, my question change to how her stories are so consistently relevant to the situation that I am explaining, or to the thoughts and questions that we are discussing. How does she interrelate issues and contexts? How does she make connections? How does she find the commonalities? If mathematics really is as useful as we think, does she use any kind of mathematics? I believe that, in the process of reflecting on her experiences, she has succeeded in weaving a web of interconnected insights. From my grandmother's perspective, life experiences are intrinsically interdependent. Events hardly ever take place in isolation. The connections that she has forged are iterative and circular, developed through attentive listening, reflecting, finding the underlying issue, logic and/or idea, connecting and then attentively listening again and again. In the process of connecting events, new patterns are found for interrelated and interdependent news and stories. She uses what she identifies as "common" in multiple life scenarios, and searches for the roots and units (the underlying logic) that they share. Over the course of many years, my grandmother has found "common units in social contexts" connecting a wide diversity of events. The concept of the "common denominator" now makes sense to me in a way that it never did in the past, including during my years as a student. This is the kind of mathematics that made my grandmother wiser (and not more clever) and that gave me a space in which to reflect on the wholeness and interconnectedness of life.

All this raises the questions of: now what?, how the transition from cleverness to wisdom should be conceptualized, in order to ensure a foundation of mathematical experiences upon which wisdom can be constructed. These mathematical experiences encompass wholeness and interconnectedness and thus promote interdependent relationships that help to clarify individuals' lives, their temporal, spatial and cultural location, and the local elements with which their local needs are connected.

The bumpy road from cleverness to wisdom

Now I return to the dichotomy between cleverness and wisdom, asking once again whether the mathematics that we teach provides us with a space in which we can mature our abilities to respond to and act in accordance with key social values, such as tolerance, fairness and acceptance, and in which we can differentiate between the elements that make us wise and the elements that make us clever.

For a change, we could follow Fasheh's (2015) view and assume that the mathematics that we teach makes us cleverer but not wiser. In the paragraphs that follow, I draw on various indigenous Eastern and Western perspectives to think about some possible ways of achieving a more well-integrated form of personhood, with mathematics serving as a mediating tool under our conscious control.

But first I have to emphasize that the road from cleverness to wisdom is long and bumpy. This journey is not an easy one, just as was the case 700 years ago. Let me tell a story…

> Once upon a time, a well-known clever man visited a well-known wise man to learn from him. "People say you know good things. Teach me your wisdom," said the clever man to the wise man. The wise man replied, "You are too heavy to learn anything. First, you

> need to empty yourself from the learnings that you have acquired, from your pride, your self-righteousness and your concerns. You shall go and become a beggar." After the clever man asked "A beggar?" the wise man replied "Yes, you shall go and beg for food and shelter for one year, and then you will come back and we will talk again." The clever man agreed to the demand, departed from his home city and went far away to become a beggar. After one year, the clever man returned to speak with the wise man and explained "I did what you said. I became a beggar. I started with begging, but then I began to notice and know the people, the children, the weaving of the city.… I am ready. Teach me your wisdom." "Where did you beg?" asked the wise man. The clever man answered "In a city far away from my own." This response prompted the wise man to ask a further question: "How could you have become humble, in such a place? People there don't know who you are, they don't know the social status (and the attached ego) that you have in your own city. For what they care, you are only a beggar. You shall become a beggar in your own city, where everyone knows you. The first step in becoming wise is to de-learn, reflect upon and re-learn what made you clever. And the first step in de-learning is to become humble and detached. Otherwise, you cannot grow to appreciate anything more significant and deeper than yourself" (an old Persian story, from Yasmine's memory).

This is a story of learning to unlearn in order to relearn. I tell this story because learning to become wise requires de-learning. More specifically, in order to become wise, one needs to dismantle interfering learnt things in order to make way for newer views. The road to wisdom starts with the self-determination to change. But de-learning in itself is not an easy process. De-learning requires a fundamental change from the egocentric discourse of self-legitimation to a new discourse of self-reflection. Unlearning creates a void and hence a context in which to discover things about oneself and about the elements that underlie one's beliefs, opinions and prejudices – elements that include one's assumptions and perceptions. Departing from this view, I once again ask the following question: are the kinds of mathematics that we teach preparing us for self-legitimation or for self-reflection? Now the question shifts to what we should de-learn and what we should re-learn in order to ensure a foundation of mathematical experiences upon which wisdom can be fostered. These mathematical experiences are those that encompass wholeness and interconnectedness and thus promote interdependent relationships that clarify individuals' lives, where they are located and with what local elements their local needs are connected.

Discussion

In this paper, I aim to challenge the body of knowledges that we teach as mathematics, and to do so in relation to the experiences that it provides us, in order to deal with our individual and collective concerns and dilemmas. In order to clarify what I mean by the "mathematics that we teach," I point to this body of knowledges from three perspectives – the mathematics of international mathematics education conferences, the mathematics of the almost-global mathematics curriculum and, finally, the body of knowledges to which we compare "others' ways of living" as mathematical activities. I explain that we value this type of mathematics – as can be seen, for example, in the ideas that underlie Skosmovs's "formatting power of mathematics" – and that we utilise a wide diversity of resources and sources to become better at teaching and learning it.

Beginning with a preoccupation with the foundational and long-established differences between cleverness and wisdom, I asked myself what exactly we are being prepared for by “the kind of mathematics that we teach and value.” In this paper, I build on my sense of what it means to be wise (as opposed to clever) by highlighting essential dimensions of the human experience including relationships, interconnectedness, empathy and caring while contrasting these dimensions with internally rational, context-less and abstracted mathematics.

I suggest a path for thinking about the kinds of mathematics that can provide us with experiences that can help us to become wiser. I believe that an aspect of becoming wise is fulfilment of the ethical duty to acknowledge and appreciate the significance of the relationships that we have with others and to understand how our histories and experiences are layered and thus position us in relation to humans and non-humans (Abtahi, 2019). Hence it is an ethical imperative of those who offer mathematical knowledge and experiences to students to ensure that the latter are exposed to humankind’s vast diversity of place-based cultures and knowledge systems, are encouraged to live in harmonious relationships with others in their own community and around the world, and are instructed (in accordance with our duty to one another) to constantly think and act with reference to these relationships. I envision a path that makes it obvious that the mathematical knowledges that we acquire interweave us more closely with the relationships without which we cannot live. Without engaging in dispassionate reasoning, compartmentalization of values, and rigid and contrived rules, I promote the kinds of mathematics that not only endorse but also prioritise the relationships, connections, empathy and intuition that are required to see and understand other humans and respond to their needs and concerns. I finish by citing T.S. Eliot, who asked in Choruses from “The Rock”:

> Where is the life we have lost in living?
>
> Where is the wisdom we have lost in knowledge?
>
> Where is the knowledge we have lost in information? (p. 96).

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