MODELLING BUOY MOTION AT SEA USING MFM

HÅVARD LØKKEBØ

TORD-ESKIL H. TØRRESSEN

THORSTEIN H. RYKKJE

THOMAS J. IMPELLUSO

BACHELOR'S THESIS IN MARINE ENGINEERING

BERGEN, NORWAY 2019

Høgskulen på Vestlandet

Western Norway University of Applied Sciences

Modelling buoy motion at sea using MFM

Håvard Løkkebø
Tord-Eskil H. Tørressen
Thorstein H. Rykkje
Thomas J. Impelluso

Department of Mechanical- and Marine Engineering
Western Norway University of Applied Sciences
NO-5063 Bergen, Norway

IMM [2019]-M[39]

Høgskulen på Vestlandet
Avdeling for ingeniør- og økonomifag
Institutt for maskin- og marinfag
Inndalsveien 28,
NO-5063 Bergen, Norge

Norsk tittel:	Beregning av en havbøyes bevegelse ved hjelp av MFM
Author(s), student number:	Håvard Løkkebø, h181296 Tord-Eskil H. Tørressen, h181307 Thorstein H. Rykkje Thomas J. Impelluso
Study program:	[Marine engineering]
Date:	[June 2019]
Report number:	IMM 2019-M39
Supervisor at HVL:	Thomas J. Impelluso, tjm@hvl.no Thorstein H. Rykkje, trry@hvl.no
Assigned by:	DOF
Contact person:	Helge N. Austbø
Antall filer levert digitalt:	1

Proceedings of the ASME 2019 International Mechanical Engineering & Exposition
IMECE2019
November 8-14, 2019, Salt Lake City, Utah, USA

DRAFT IMECE2019-10437

MODELLING BUOY MOTION AT SEA USING MFM

Tord-Eskil H. Tørressen
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
tord-eskil@hotmail.com

Håvard Løkkebø
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
havardlokkebo@gmail.com

Thorstein R. Rykkje
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
trry@hvl.no

ABSTRACT

This project creates a model to assess the motion induced on a buoy at sea, under wave conditions. We use the Moving Frame Method (MFM) to conduct the analysis. The MFM draws upon concepts and mathematics from Lie group theory—SO(3) and SE(3)—and Cartan's notion of Moving Frames. This, together with a compact notation from geometrical physics, makes it possible to extract the equations of motion, expeditiously. This work accounts for the masses and geometry of all components and for buoyancy forces and added mass. The resulting movement will be displayed on 3D web pages using WebGL. Finally, the theoretical results will be compared with experimental data obtained from a previous project done in the wave tank at HVL.

INTRODUCTION

Knowledge about how the waves and weather affect vessels has been of great interest in ship modeling. Today, some place buoys at sea, loaded with sensor devices to monitor weather conditions at sea. Such a buoy is used to collect data about the state of the sea. This includes wave height, angle of incidence, wave period and speed and even wind speeds. This paper endeavors to research how these items affect the position and orientation of such buoys.

Figure 1 shows the object of interest in this analysis, it's a downscaled Tideland Buoy manufactured and distributed by XYLEM.

FIGURE 1. Tideland Buoy at sea

NOMENCLATURE

e	:	Frame
F	:	Force vector
M	:	Moment vector
g	:	Gravity
H	:	Angular momentum
I_d	:	3x3 identity matrix
J_C	:	3x3 Mass moment of inertia matrix
K	:	Kinetic energy
L	:	Linear momentum

H	:	Angular momentum
R	:	Rotation matrix
$\mathbf{r}_C^{(1)}$	:	Absolute position vector
ω	:	Angular velocity vector
$\ddot{\omega}$	:	Skew symmetric angular velocity matrix
CB	:	Center of Buoyancy
V	:	Displaced Volume

GOALS

The key to solving this lies in calculating displaced volume and orientation of the buoy at every instant in time. These two parameters will determine the force and moments which acts on the buoy.

Furthermore, we intend to compare the theoretical results to past experimental results to verify the mathematical model.

THE MOVING FRAME METHOD

Élie Cartan (1869-1951) [1] assigned a reference frame to each point of an object under study (a curve, a surface, Euclidean space itself). Then, using an orthonormal expansion, he expressed the rate of change of the frame in terms of the frame. The MFM leverages this by placing a reference frame on every moving link. However, then we need a *method to connect moving frames*. For this, we turn to Sophus Lie.

Marius Sophus Lie (1842-1899) developed the theory of continuous groups and their associated algebras. The MFM adopts the mathematics of rotation groups and their algebras, yet distils them to simple matrix multiplications. However, then we need a simplifying notation. For this, we turn to Frankel.

Ted Frankel [2] developed a compact notation in geometrical physics. The MFM adopts this notation to enable a methodology that is identical for both 2D and 3D analyses. The notation is also identical for single bodies and multi-body linked systems. In turn, this uplifts students' understanding from the conceptual to the pragmatic, enabling them to analyze machines of the 3D world. Allow us to introduce the MFM.

Angular Kinematics

The left side of Fig. 2 presents a grey inertial orthogonal coordinate system—longer grey lines—designated by $\{x_1, x_2, x_3\}$. The Cartesian coordinate system is used.

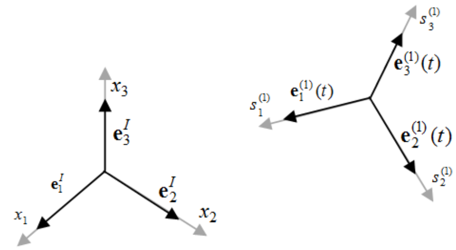

FIGURE 2. Frame relations

The left side of Fig. 2 also presents the associated *inertial frame basis vectors* designated by $\mathbf{e}^I = \begin{pmatrix} e_1^I & e_2^I & e_3^I \end{pmatrix}$ in bold black, with superscript "I." The frame basis vectors derive from the directional derivatives of the coordinates: $e_i^I \equiv \partial / \partial x_i$. Care is taken in this class to distinguish between coordinate systems and frames.

The *right* side of Fig. 2 presents a moving coordinate system designated by $\{s_1^{(1)}, s_2^{(1)}, s_3^{(1)}\}$ which may be embedded on the first body—superscript (1). The right figure also presents the associated time dependent moving frame: $\mathbf{e}^{(1)}(t) = \begin{pmatrix} e_1^{(1)}(t) & e_2^{(1)}(t) & e_3^{(1)}(t) \end{pmatrix}$.

We use a rotation matrix, a member of the Special Orthogonal Group, $R \in SO(3)$, to relate the orientation of a moving frame to an inertial frame:

$$\mathbf{e}^{(1)}(t) = \mathbf{e}^I R^{(1)}(t) \tag{1}$$

Orthogonality —$\left(R^{(\alpha)}(t)\right)^{-1} = \left(R^{(\alpha)}(t)\right)^T$ —implies

$$\mathbf{e}^I(t) = \mathbf{e}^{(1)}\left(R^{(1)}(t)\right)^T \tag{2}$$

The time rate of frame rotation:

$$\dot{\mathbf{e}}^{(\alpha)}(t) = \mathbf{e}^I \dot{R}^{(\alpha)}(t) \tag{3}$$

Recasting (3) in the moving frame, using (2):

$$\dot{\mathbf{e}}^{(1)}(t) = \mathbf{e}^{(1)}(t)\left(R^{(1)}(t)\right)^T \dot{R}^{(1)}(t) \tag{4}$$

We define the skew-symmetric angular velocity matrix. We note that this element is a member of the associated algebra, so(3)

$$\overleftrightarrow{\omega^{(1)}(t)} = \left(R^{(1)}(t)\right)^T \dot{R}^{(1)}(t) = \begin{bmatrix} 0 & -\omega_3^{(1)}(t) & \omega_2^{(1)}(t) \\ \omega_3^{(1)}(t) & 0 & -\omega_1^{(1)}(t) \\ -\omega_2^{(1)}(t) & \omega_1^{(1)}(t) & 0 \end{bmatrix} \tag{5}$$

We may now express (4) as:

$$\dot{\mathbf{e}}^{(1)}(\mathrm{t})=\mathbf{e}^{(1)}(\mathrm{t})\overleftrightarrow{\omega^{(1)}(\mathrm{t})} \tag{6}$$

We have now expressed the time rate of frame rotation in its own frame, in accordance with the general philosophy of Cartan. The skew-symmetric angular velocity matrix is isomorphic to the same frame to the angular velocity vector of that frame:

$$\boldsymbol{\omega}^{(1)}(\mathrm{t})=\mathbf{e}^{(1)}(\mathrm{t})\begin{pmatrix}\omega_1^{(1)}(\mathrm{t})\\ \omega_2^{(1)}(\mathrm{t})\\ \omega_3^{(1)}(\mathrm{t})\end{pmatrix} \tag{7}$$

The buoy 1-frame will rotate (pitch, yaw and roll). We model its rotation with a rotation matrix as follows:

$$\begin{gathered}\begin{pmatrix}\mathbf{e}_1^{(1)}(\mathrm{t}) & \mathbf{e}_2^{(1)}(\mathrm{t}) & \mathbf{e}_3^{(1)}(\mathrm{t})\end{pmatrix}= \\ \begin{pmatrix}\mathbf{e}_1^{\mathrm{I}} & \mathbf{e}_2^{\mathrm{I}} & \mathbf{e}_3^{\mathrm{I}}\end{pmatrix}\begin{bmatrix}\mathrm{R}_{11}(\mathrm{t}) & \mathrm{R}_{12}(\mathrm{t}) & \mathrm{R}_{13}(\mathrm{t})\\ \mathrm{R}_{21}(\mathrm{t}) & \mathrm{R}_{22}(\mathrm{t}) & \mathrm{R}_{23}(\mathrm{t})\\ \mathrm{R}_{31}(\mathrm{t}) & \mathrm{R}_{32}(\mathrm{t}) & \mathrm{R}_{33}(\mathrm{t})\end{bmatrix}\end{gathered} \tag{8}$$

Linear Kinematics

We express the position and translational velocity of the buoy from the inertial frame in three equivalent forms shown in Eqn. (9):

$$\boldsymbol{r}_C^{(1)}(t)=\begin{pmatrix}\mathbf{e}_1^I & \mathbf{e}_2^I & \mathbf{e}_3^I\end{pmatrix}\begin{pmatrix}x_{1C}^{(1)}(t)\\ x_{2C}^{(1)}(t)\\ x_{3C}^{(1)}(t)\end{pmatrix}=\mathbf{e}^I x_C^{(1)}(t) \tag{9}$$

In this analysis, we will reserve $\mathrm{r}^{(\alpha)}$ to denote vectors from an inertial frame $\mathrm{x}^{(\alpha)}$ as their coordinates. The subscript "C" denotes that the frame is located at the center off mass.

From Eqn. (9) we obtain the velocity of the moving buoy as:

$$\boldsymbol{v}_C^{(1)}(t)=\dot{\boldsymbol{r}}_C^{(1)}(t)=\begin{pmatrix}\mathbf{e}_1^I & \mathbf{e}_2^I & \mathbf{e}_3^I\end{pmatrix}\begin{pmatrix}\dot{x}_{1C}^{(1)}(t)\\ \dot{x}_{2C}^{(1)}(t)\\ \dot{x}_{3C}^{(1)}(t)\end{pmatrix}=\mathbf{e}^I\dot{x}_C^{(1)}(t) \tag{10}$$

Incidentally, the placement of the components after the basis is not whimsical. It is in accordance with viewing rotation matrices as operators on components.

Linear Kinetics

With a moving frame placed upon the buoy, one asserts the linear momentum:

$$\mathbf{L}(\mathrm{t})=\mathbf{e}(\mathrm{t})\,\mathrm{m}\,\boldsymbol{v}_{\mathrm{C}}(\mathrm{t}) \tag{11}$$

The rate of change of the linear momentum is:

$$\dot{\mathbf{L}}(\mathrm{t})=\mathbf{e}(\mathrm{t})\,\mathrm{m}\,\dot{\boldsymbol{v}}_{\mathrm{C}}(\mathrm{t})+\dot{\mathbf{e}}(\mathrm{t})\,\mathrm{m}\,\mathbf{v}_{\mathrm{C}}(\mathrm{t}) \tag{12}$$

Using (6) we find

$$\dot{\boldsymbol{L}}(\mathrm{t})=\mathbf{e}(\mathrm{t})\Big(\mathrm{m}\Big(\dot{\boldsymbol{v}}_{\mathrm{C}}(\mathrm{t})+\overleftrightarrow{\omega(\mathrm{t})}\,\boldsymbol{v}_{\mathrm{C}}(\mathrm{t})\Big)\Big) \tag{13}$$

Newton's Law equated the previous to the applied forces:

$$\dot{\mathbf{L}}(\mathrm{t})=\mathbf{F}(\mathrm{t}) \tag{14}$$

Thus, extracting the components, we find:

$$\mathbf{F}(\mathrm{t})=\mathrm{m}\Big(\dot{\mathbf{v}}_{\mathrm{C}}(\mathrm{t})+\overleftrightarrow{\omega(\mathrm{t})}\;\mathbf{v}_{\mathrm{C}}(\mathrm{t})\Big) \tag{15}$$

Angular Kinetics

With J defined as the mass moment of inertia in the moving frame, we assert the angular momentum in which the moving frame is explicitly stated:

$$\mathbf{H}_{\mathrm{C}}(\mathrm{t})=\mathbf{e}(\mathrm{t})\,\mathrm{J}_{\mathrm{C}}\,\omega(\mathrm{t}) \tag{16}$$

The rate of change of the angular momentum is:

$$\dot{\mathbf{H}}_{\mathrm{C}}(\mathrm{t})=\mathbf{e}(\mathrm{t})\,\mathrm{J}_{\mathrm{C}}\,\dot{\omega}(\mathrm{t})+\dot{\mathbf{e}}(\mathrm{t})\,\mathrm{J}_{\mathrm{C}}\,\omega(\mathrm{t}) \tag{17}$$

Using (6) we find

$$\dot{\mathbf{H}}_{\mathrm{C}}(\mathrm{t})=\mathbf{e}(\mathrm{t})\Big(\mathrm{J}_{\mathrm{C}}\,\dot{\omega}(\mathrm{t})+\overleftrightarrow{\omega(\mathrm{t})}\,\mathrm{J}_{\mathrm{C}}\,\omega(\mathrm{t})\Big) \tag{18}$$

Euler's Law gives the previous equated to the applied torques:

$$\dot{\mathbf{H}}_{\mathrm{C}}(\mathrm{t})=\mathbf{M}_{\mathrm{C}}(\mathrm{t}) \tag{19}$$

Thus, extracting the components, we find:

$$\mathbf{M}_{\mathrm{C}}(\mathrm{t})=\mathbf{e}(\mathrm{t})\Big(\mathrm{J}_{\mathrm{C}}\,\dot{\omega}(\mathrm{t})+\overleftrightarrow{\omega(\mathrm{t})}\,\mathrm{J}_{\mathrm{C}}\,\omega(\mathrm{t})\Big) \tag{20}$$

THE MODEL AND APPLIED LOADS

Figure 3 shows the model used in the calculations of moment of inertia, with all frames attached.

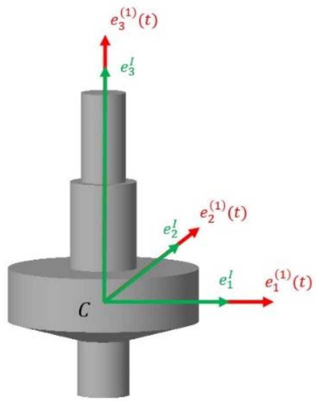

FIGURE 3. Buoy made in Creo Parametric [7] with frames attached.

In equation(15) and (20), we assert the following applied forces and moments respectively.

$$\mathrm{F(t)} = \begin{pmatrix} 0 \\ 0 \\ \mathrm{F_{Buoyancy}} - \mathrm{mg} \end{pmatrix} \tag{21}$$

$$\mathrm{M_C(t)} = \begin{pmatrix} 0 \\ \mathrm{M_{wave}} \\ 0 \end{pmatrix} \tag{22}$$

There are two main forces acting on the buoy at all time, gravity and buoyancy. These are the main forces acting to resist the motion induced by a wave. The wave will cause disturbance and force the buoy to move up and down as well as inducing a rotation.

SUBMERGED VOLUME CALCULATION

The submerged volume of the buoy is calculated at every timestep using numerical integration. We solve for the volume by dividing the cylinder into many small tall rectangles. We first split the cylinder into many small slices in the $\mathbf{e}_1$ direction, then we split these slices into small squares in the $\mathbf{e}_2$ direction. Thus, producing numerous small cubes approximating the volume of the buoy.

Figure 4 shows how these slices have been split up to create all the small cubes that are used in the approximation of displaced mass. By using these values we find the displaced mass at each timestep for an arbitrary orientation. The displaced mass is directly related to the magnitude of the buoyancy, as the

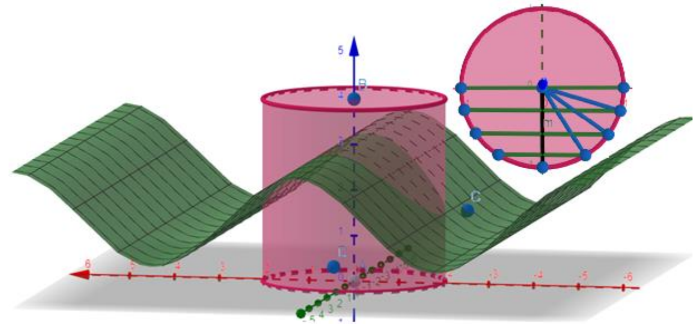

FIGURE 4. Image from GeoGebra showing the splitting of the circle.

buoyancy is simply the displaced mass multiplied by the density of said mass, in this case water.

After this we find the center of the displaced mass by using equation (23). This will determine the point where the buoyancy acts on the buoy.

$$CB = \frac{\sum V_i x_i}{\sum V} \tag{23}$$

The wave only comes from the x-direction, e_1, thereby giving us a uniform wave point of interactions in the y direction. This means that the wave height at each x position will be equal at every y value for that specific x. Knowing this, we can locate the height for each x value, and then use this number to form a volume under the wave. This volume is limited by the small cubes that describes the form of the cylinder. This will give us the displaced volume. Since we have all the wave heights throughout the cylinder, we can find the volume for each cube and there by the volume of each column at every x value, multiply this value with its x distance for center, and with this we get the center of mass for the displaced mass of the water.

SIMPLIFICATION OF THE MODEL

The key differences in the wave tank experiments and our calculations are the fact that we don't account for drag forces, drifting of the buoy and the mooring link to the sea bottom.

RECONSTRUCTION

After we solve for the angular velocity, we solve for the orientation of the buoy.

We must know the rotation matrix of the buoy for several reasons. First, it is required for computing the acceleration and angular acceleration for the buoy, second it is required to apply added mass forces. Third, we need it for visualization.

We reconstruct the rotation matrix of the buoy from the angular velocity. We compute the rotation matrix $R^{(1)}(t)$ by solving the following equation:

$$\dot{R}^{(1)}(t) = R^{(1)}(t)\overleftrightarrow{\omega^{(1)}(t)} \tag{24}$$

Let us assume for a moment that $\overleftrightarrow{\omega^{(1)}(t)}$ is constant and is designated as $\overleftrightarrow{\omega_0}$. Then, with initial value $R(0)$, the solution is:

$$R^{(1)}(t) = R(0)\exp(t\overleftrightarrow{\omega_0}) \tag{25}$$

There does exist a known analytical, closed form solution to equation (25), but only for cases in which $\overleftrightarrow{\omega_0}$ is constant. It derives from the Cayley Hamilton Theorem and is known as the Rodrigues' rotation formula to obtain a series expansion of the exponential of a matrix.

However, we note that those results held for a constant angular velocity matrix. We will not have a constant angular velocity matrix, so we must improvise.

We will assume that during the numerical integration of the equations of motion t to $t+\Delta t$, that the angular velocity is constant. While this is not the case here, it can be applied to each individual time step of the Runge-Kutta integration. Therefore, we adopt the *mid-point integration method* using the mean value of the angular velocity, $\omega(t+\Delta t/2)$

$$\omega(t+\Delta t/2) \equiv \left(\omega(t)+\omega(t+\Delta t)\right)/2 \tag{26}$$

Essentially, after coming out of each time step, we will have a new omega. We use that newfound expression and compute an assumed constant angular velocity matrix by averaging. We then use that constant value to reconstruct the rotation matrix

$$R(t+\Delta t) = R(t)\left(I_d + \frac{\overleftrightarrow{\omega_0(t+\Delta t)}}{\|\omega(t+\Delta t)\|}\sin\left(t\|\omega_0(t+\Delta t)\|\right) + \left(\frac{\overleftrightarrow{\omega_0(t+\Delta t)}}{\|\omega(t+\Delta t)\|}\right)^2 \left(1-\cos\left(t\|\omega_0(t+\Delta t)\|\right)\right)\right) \tag{27}$$

The system of equations together with the submerged volume was solved in Matlab using ode45.

EXPERIMENTAL ANALYSIS

Parameters used

According to an internal HVL report [4], the experiment at HVL were conducted on two different downscaled buoys. One which is 1/4 and one which is 1/8 of the original Tideland Buoy. We are using data from the 1/4 experiments as we wish to be as close geometrically to the original model as possible. The buoy is tested with several different waves heights and frequencies. We have chosen one.

Mass of buoy:	$m_{buoy} = 10$ kg
Mass of chains:	$m_{chain} = 3.5$ kg
Wave angular frequency	$\omega_{wave} = 2.03$ rad/s
Height of buoy:	$h = 0.766$ m
Radius of buoy:	$r = 0.462$ m
Draft of buoy in equilibrium:	$T = 0.209$ m
Distance from keel to COG	$COG = 0.205$m

Moment of inertia $J_C = \begin{bmatrix} 0.867 & 0 & 0 \\ 0 & 0.867 & 0 \\ 0 & 0 & 0.2796 \end{bmatrix}$ kgm^2

Wavefunction

$$z(x_{1C}^{(1)}, t) = \left(\frac{1}{10}\right)\sin\left(x_{1C}^{(1)} + t\omega_{wave} + \frac{\pi}{2}\right)$$

Figure 5 shows the real sized buoy that was used during this experiment and downscaled with by a factor of 1/4.

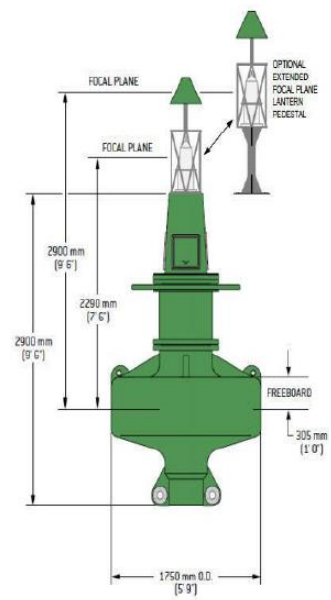

FIGURE 5. Picture of the buoy which is to be placed at sea

Buoy Mooring

In the experiment, the buoy is moored with chain to the "sea-bead." As the buoy heaves it will lift more chain up from the seabed, and in turn this would add mass to the whole system.

As a first pass we solve this issue by adding the mass of the suspended chain at calm sea to the total mass of the buoy. The mass accumulated by all the links is calculated to be 3,5kg.

Added Mass

As a body immersed in water experiences increased moment of inertia from the added mass we must take this into account.

According to the study done by A. H. Techet and B. P. Epps [5] the added mass of a free-floating cylinder is its displaced volume multiplied by its instantaneous velocity.

$$m_a = \rho \pi r^2 \boldsymbol{v}_c^{(1)} \tag{28}$$

There are multiple ways of calculating added mass, we will only use this simple formula in this first pass approach.

Damping

We also took into account a dampening force induced by the friction the water makes on the buoy. In the experiments [4] there is no friction coefficient explicitly stated. As such, we approximated a friction coefficient in the code to meet a satisfying motion agreeing with our mathematical calculations, to use with viscous dampening.

COMPARISON TO PREVIOUS EXPERIMENTAL TESTS

As previously mentioned, we are comparing our theoretical results with data obtained from experiments in the wave tank at HVL. The results we're comparing with were done by a previous bachelor group in autumn 2017 [4].

There were conducted 110 experiments. When one is being conducted in the wave tank one can input a number of different waves, from JONSWAP to ordinary sinuous waves and much more. As we have done calculations on sinuous waves, they are the ones of interest in this comparison and we will shortly present a few graphs for comparison.

There are a few important differences top point out between our calculations and the experiments.

One is the fact that the experiments were conducted while the buoy were moored to the bottom of the wave tank. There were used two methods of mooring, one chained and one with an elastic rubber band. In the analysis of the rubber band it is obvious that it will add another force into the system as it is under constant tension form the buoyancy. In contrast to this stands the analysis of the chained buoy. The chain will also add another force into the system. But we can for simplicity add this extra mass into the system and thus approximating its effect on the buoy.

The last difference is that once the wave passes the buoy it will experience some drifting due to currents in the tank along with a horizontal motion induced by the wave.

As mentioned in "simplification of the model" we ignore the mooring and do not account for any effects this would have on the buoy. In addition to this we also ignore the drifting and horizontal movement. Our only concern is the heave motion and the buoys rotation.

Wave tank data

The heave and rotation of the buoy in the wave tank is measured using very precise cameras from Qualisys measuring motion in 6 DOF, so any errors here will be negligible [6].

First, we check for the heave of the buoy. If our calculations match up with the measured heave, we have a good chance we're close with the theory. In figure 6 we have graphed the motion of the buoy under the effect of a sinus wave with amplitude 0,1 m and a period of 3.083 s.

The heave analysis is important because one of the main concerns when constructing buoys and boats is exactly the heave.

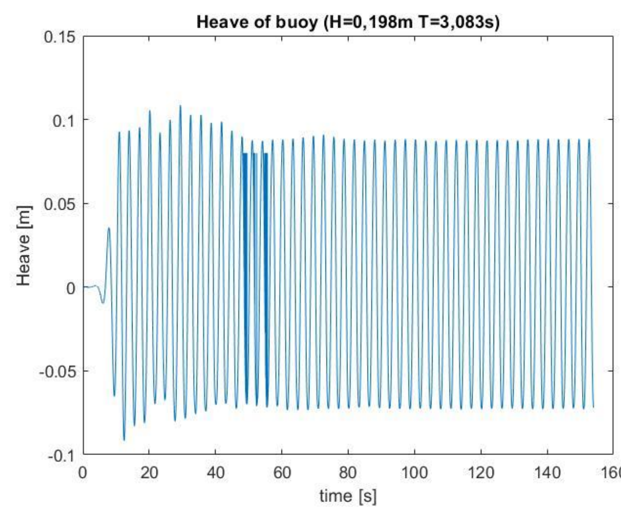

FIGURE 6. Heave data from wave tank.

Buoy Rotations

After we investigated the heave, we focused on the orientation of the buoy. In our experiment waves only come in from one direction. This implies a rotation about the $\mathbf{e}_2$ direction as seen in Figure 3. However, in real life the buoy will most likely start rotating in all degrees of freedom. Thus, we expect some error here.

Figure 7 shows the rotations about the y axis of the buoy obtained from the wave tank experiment.

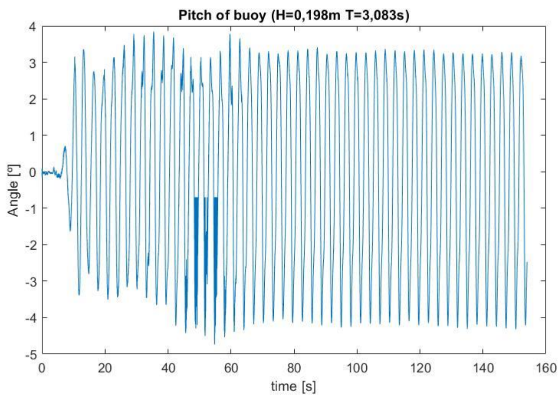

FIGURE 7. Pitch data from wave tank.

Results

Figure 8 shows the movement of the buoy along the z axis as its impacted by a sinuous wave. After 70 seconds the buoy in their experiment stabilized with a heave at maximum 0.078 in the positive z direction and -0.069 in the negative z direction. Comparing this to our results that gave a positive heave of 0.075 and a negative at -0.058, there is a qualitative match between our results and theirs, but it also shows that somewhere we have used imprecise values, that has affected the precision of our results. Another reason this could have occurred is due to the chain that we are not using in our experiment, their chain would affect the buoys ability to move in heave.

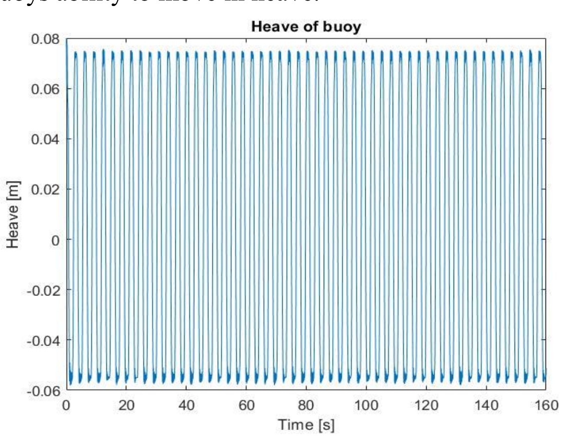

FIGURE 8. Heave from Matlab and MFM.

As we are conducting our experiment a little different it will not be equal in the beginning as they decided to start the experiment with no waves. Then after a given time t, they turned on the waves and the buoy were gradually exposed to force of the wave. We on the other hand exposed the buoy to the full wave right away. This means that our initial heave values are equal to the values that they got after a time t. Thus, our experiment is based on their results after 70 seconds to 160 seconds.

This can cause the buoy to not go in phase with the waves, and thereby make it work against it instead, which would impact our results to some extent.

Figure 9 shows the angle at which the buoy rotated about the y axis. Our buoy rotated approximately 4.1-4.2 degrees back and forth and was very stable. The main difference is the -1 degrees less that their buoy rotated. This is due to the extra friction that the chain they used would give in the direction of the wave, and since the positive direction of rotation is the same as the wave is moving, we will experience a reduction of rotation in that direction. This is clearly shown in the results as we get a higher pitch of about 1 degree in the positive direction compared to their results.

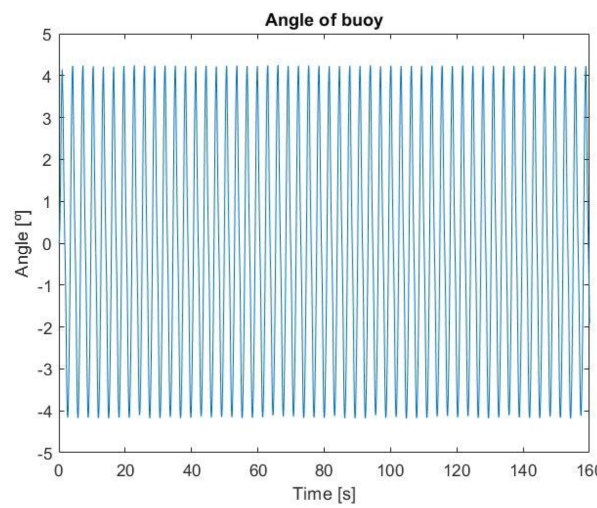

FIGURE 9. Pitch data from Matlab and MFM.

We are not using viscosity, but instead a factor. This factor will not match the true dissipation. Their viscosity is already taken care of, as the buoy is immersed in water. In our case on the other hand, we will need to implement this force in form of a friction factor acting against the rotation, just like a dampening factor. This uncertainty/inaccuracy between our factor and their real viscosity, will impact the results and thereby the angle at which the buoy rotates.

Again, we are only interested in how the buoy is acting after a give time *t*, as we only want to see how its acting after it has stabilized. After 70 seconds the buoy stabilizes at 3.2 degrees in positive x direction, and a total of -4.2 in the opposite direction. This means that in our results the buoy should rotate about 1 degree more in the negative direction.

CONCLUSION

The calculated heave agrees with the data from the wave tank, but with inaccuracy that we attribute to the integration method.

The angle of rotation is not as accurate as we expected. We believe the origin of this is due to the lack of accuracy in friction and drag forces along with small errors in the moment of inertia matrix J_C.While our frictions are a factor and thereby not as accurate as a real experiment, making the time it used to stop, somewhat off point.

If we can get a better and more accurate way of collecting raw data, these problems will decrease and the full potential of the MFM can be seen. For then, our theoretical results will accumulate with the real physical results gotten from the experiment. If we had used another way of numerical integration that used more precise mathematical methods, we would be able to get more precise peaks as seen in the wave tank experiment conducted last year [4].

Despite these inaccuracies mentioned above the result are still satisfying and clearly resemble the real data. Given the same dimensions of the buoy and the same frequency and amplitude of the wave, we are able to recreate both mathematically and visually the movement of the buoy once it hits water or is placed under water. This clearly shows the MFM works but that some improvements can be made if raw data is improved. Our methods of extracting the data needed to use the MFM are slightly inaccurate. We are clearly getting good resemblance and we see that the method works, but we observe some noise in the results due to inaccuracies in the numerical integration.

FUTURE WORK

This analysis of the buoy motion would be more complete in the future with the addition of waves from the sides in form of a sin(y) component. The inclusion of the horizontal motion of the buoy in the waves along with any motion caused by currents would also make the analysis more complete. But most important, we have to induce the hydrodynamics and not only the dynamics, when dealing with objects floating in water.
If the dynamics of the mooring is included into the calculations, the theoretical result may correspond more with the experiments in the wave tank.

Friction due to the hydrodynamics is also simplified by adding a constant into the calculations. For future work we suggest doing a better experiment where they also calculating the friction coefficient from the results, so that can be used instead of an approximation.

Dealing with the mooring line could also have been done better, instead of adding an extra mass at the bottom of the buoy, more accurate methods could be developed to consider the full effect of the chain.

REFERENCES

[1] Cartan, É, 1986, On Manifolds with an Affine Connection and the Theory of General Relativity, translated by A. Magnon and A. Ashtekar, Napoli, Italy, Bibiliopolis.

[2] Frankel T, (2012) The Geometry of Physics, an Introduction, third edition, Cambridge University Press, New York; First edition published in 1997.

[3] Impelluso, T., 2017, "*The moving frame method in dynamics: Reforming a curriculum and assessment*," International Journal of Mechanical Engineering Education. https://doi.org/10.1177/0306419017730633

[4] "Wave measuring miniature sensor – the effects of waves and anchoring on buoys"

[5] A. H. Techet and B. P. Epps, (2016), "Derivation of added mass around a cylinder", Available: http://web.mit.edu/2.016/www/handouts/Added_Mass_Derivation_050916.pdf

[6] Qualisys owus https://www.qualisys.com/hardware/5-6-7/?fbclid=IwAR0Vqj06mk5FUhAwIr64e5mmeCM_ExXRlNbfLa61CVnhW_vJin9c9pC2Ocw

[7] "Creo Parametric," PTC, [Online]. Available: https://www.ptc.com/en/products/cad/creo/parametric.

FLOW CHART

Here we present a flow chart made and used by the authors of this report. The purpose of this is to help and guide us through the different stages of the report. It is more or less consistent with the actual progress of the project. However, as with most other projects, some tasks demanded more time while others went more smoothly than anticipated.

Yellow indicates deadlines while blue indicates finalization.