MODELING A KNUCKLE BOOM CRANE CONTROL TO REDUCE PENDULUM MOTION USING THE MOVING FRAME METHOD

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ABSTRACT

A Knuckle Boom Crane is a pedestal-mounted, slew-bearing crane with a joint in the middle of the distal arm; i.e. boom. This distal boom articulates at the 'knuckle (i.e.: joint)' and that allows it to fold back like a finger. This is an ideal configuration for a crane on a ship where storage space is a premium. This project researches the motion of a ship mounted knuckle boom crane to minimize the pendulum motion of a hanging load. To do this, the project leverages the Moving Frame Method (MFM). The MFM draws upon Lie group theory—SO(3) and SE(3)—and Cartan's Moving Frames. This, together with a compact notation from geometrical physics, makes it possible to extract the equations of motion, expeditiously. The work reported here accounts for the masses and geometry of all components, interactive motor couples and prepares for buoyancy forces and added mass on the ship. This research solves the equations numerically using a relatively simple numerical integration scheme. Then, the Cayley-Hamilton theorem and Rodriguez's formula reconstructs the rotation matrix for the ship. This work displays the motion on 3D web pages, viewable on mobile devices.

NOMENCLATURE

$[B]$:	B-Matrix
$[D]$:	Combined angular velocity matrix
$\{F^*\}$:	Generalized force
$\{F\}$:	Force and moment list
H :	Angular momentum
$J_c^{(\alpha)}$:	3x3 Mass moment of inertia matrix
$[M]$:	Mass matrix
$[M^*]$:	Reduced mass matrix
$[N^*]$:	Reduced non-linear velocity matrix
q :	Generalized position
$\{\dot{q}\}$:	Generalized velocity variable list
$\{\ddot{q}\}$:	Generalized acceleration variable list
δW :	Virtual work
$\{\dot{X}\}$:	Velocity list
$\{\tilde{X}\}$:	Virtual displacements
$\delta\omega$:	Variation of frame connection matrix
$\overleftrightarrow{\delta\pi}$:	Virtual rotational displacement
Ω :	Time rate of the frame connection matrix
ω :	Angular velocity vector
$\tilde{\omega}$:	Skew-symmetric angular velocity matrix

INTRODUCTION

Engineering Background

FIGURE 1. Knuckle boom crane on a ship [1]

The knuckle boom crane is a pedestal-mounted, slew-bearing crane with a joint in the middle of the boom. A knuckle boom crane is ideal on big vessels, like Inspection, Maintenance and Repair (IMR) vessels, where storage space is a premium.

During offshore operations harsh weather conditions may occur. IMR vessels are equipped for this kind of weather. Most cranes have active heave compensation (AHC), which adjusts the hanging cable to keep the load stable despite motion of the ship, but AHC is used when the load is beneath the surface of the water. There is a need for more accurate control of suspended objects; and, thus, for the analysis of crane-induced ship motion.

A control system would sense motion in the ship due to waves and wind, and keep the crane tip in a set position.

This could prove beneficial in many scenarios at sea. Examples of scenarios may be: deployment of a load close to platform during high waves or when loading from one vessel to another.

GOALS

As a research paper, this project wants to model and do the calculations, in realistic dimensions, for a knuckle boom crane on a vessel. And with this, eventually make a control system for a knuckle boom crane to reduce pendulum motion. The work in this paper is built upon previous work by Jardim, et. Al. [3] by adding an extra boom to the crane.

As a pedagogical paper, it introduces the MFM. The MFM obviates complexities introduced by an injudicious use of vector algebra in 3D dynamics.

The work will demonstrate how the 3D web can enhance engineering analyses with visualization.

With this in mind, we first introduce the Moving Frame Method.

THE MOVING FRAME METHOD

Élie Cartan (1869-1951) [4] assigned a reference frame to each point of an object under study (a curve, a surface, Euclidean space itself). Then, using an orthonormal expansion, he expressed the rate of change of the frame in terms of the frame. The MFM leverages this by placing a reference frame on every moving link. However, then we need a *method to connect moving frames*. For this, we turn to Sophus Lie.

Marius Sophus Lie (1842-1899) developed the theory of continuous groups and their associated algebras. The MFM adopts the mathematics of rotation groups and their algebras, yet distils them to simple matrix multiplications. However, then we need a simplifying notation. For this, we turn to Frankel.

Ted Frankel [5] developed a compact notation in geometrical physics. The MFM adopts this notation to enable a methodology that is identical for both 2D and 3D analyses. The notation is also identical for single bodies and multi-body linked systems. In turn, this uplifts students' understanding from the conceptual to the pragmatic, enabling them to analyze machines of the 3D world.

The MFM greatly ensures a consistency of notation across sub-disciplines of dynamics, it greatly simplifies the complexity of dynamics. The notation remains consistent from introductory to advanced analysis, from 2D to 3D, and from single-body to multi-body analysis. In this paper, we apply the MFM to a linked multi-body system. Impelluso [6] conducted a pedagogical assessment that also introduces the method. Allow us to summarize the MFM.

THE MODEL

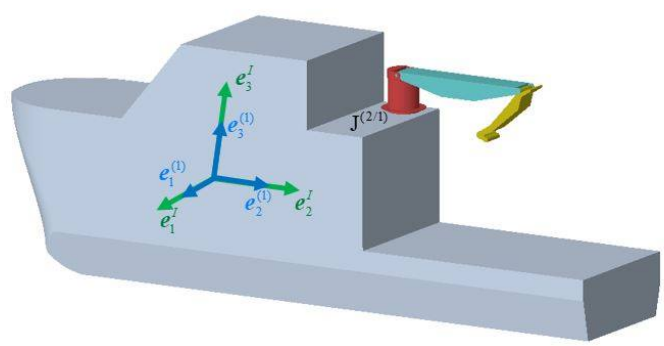

FIGURE 2. Model and frames of the ship

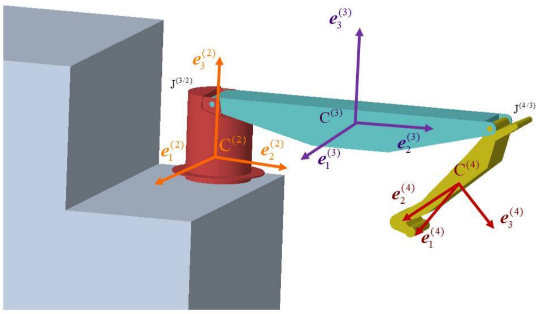

FIGURE 3. Model and frames of the knuckle boom crane

Overview of the Moving Frame Method

The analysis commences with the first body, the ship itself. From the ship, there is a systematic progression to the tower of the crane (red), then to the crane's main boom (blue) and finally to the crane's outer boom (yellow). We supply each body with a Cartesian frame (five frames in total when including the inertial frame). We number the frames in ascending order, starting with the ship and ending at the knuckle part of crane.

The multi-body system consists of four linked bodies. The ship is body 1, the tower of the crane is body 2, the main boom is body 3 and the outer boom is body 4. Each individual body is endowed with its own moving Cartesian coordinate system: $s_C^{(\alpha)} = \left\{ s_1^{(\alpha)} \;\; s_2^{(\alpha)} \;\; s_3^{(\alpha)} \right\}$, where the superscript α = 1, 2, 3 or 4.

Next, a body frame is defined by partial derivatives of the coordinate functions, wherein each basis vector is tangent to the coordinate function $\mathbf{e}_i^{(\alpha)} \equiv \dfrac{\partial}{\partial s_i^{(\alpha)}}$. Thus, we obtain the following moving frame

$$\mathbf{e}^{(\alpha)}(t) = \left(\mathbf{e}_1^{(\alpha)}(t) \quad \mathbf{e}_2^{(\alpha)}(t) \quad \mathbf{e}_3^{(\alpha)}(t) \right) \tag{1}$$

Equation (1) represents a time-dependent moving frame, associated with the moving body.

When necessary, we deposit an inertial frame from the first body, at the start of the analysis $(t = 0)$:

$$\left\{ \mathbf{e}_1^I \quad \mathbf{e}_2^I \quad \mathbf{e}_3^I \right\} = \left\{ \mathbf{e}_1^{(\alpha)}(0) \quad \mathbf{e}_2^{(\alpha)}(0) \quad \mathbf{e}_3^{(\alpha)}(0) \right\} \tag{2}$$

General Introduction to the kinematics of frames.

Consider the translation of a body, stated in an inertial frame. We use "*x*" to represent coordinates assessed from an inertial frame, reserving "*s*" for position coordinates assessed from moving frames.

$$\mathbf{r}_C^{(\alpha)}(t) = \mathbf{e}^I x_C^{(\alpha)}(\mathrm{t}) \tag{3}$$

Equation (3) presents the basis to the left of the components and changes the order of the traditional notation that places the basis on the right. With this notation, we view the rotation matrices as matrix operators on columns of components.

The vector $\mathbf{s}_C^{(\alpha+1/\alpha)}$ represents the distance between the center of mass $C^{(\alpha+1)}(t)$ of a child body and the center of mass $C^{(\alpha)}(t)$ of the parent body. This vector is expressed in the parent frame:

$$\mathbf{s}_C^{(\alpha+1/\alpha)}(t) = \mathbf{e}^{(\alpha)}(t) s_c^{(\alpha+1/\alpha)}(t) \tag{4}$$

To locate the absolute location of a center of mass on a child body, we first proceed to the parent in the inertial frame and then accumulate the distance to the child, in the parent frame:

$$\mathbf{r}_C^{(\alpha+1)}(t) = \mathbf{r}_C^{(\alpha)}(t) + \mathbf{e}^{(\alpha)}(t) s_C^{(\alpha+1/\alpha)}(t) \tag{5}$$

To orient the moving frame, a 3×3 rotation matrix $R^{(\alpha)}(t)$ expresses the rotation of the body-α vector-basis $\mathbf{e}^{(\alpha)}(t)$ from inertial vector-basis $\mathbf{e}^I$:

$$\mathbf{e}^{(\alpha)}(t) = \mathbf{e}^I R^{(\alpha)}(t) \tag{6}$$

The vector-basis $\mathbf{e}^{(\alpha+1)}(t)$ and the relative rotation of a body-(α+1), is given by a relative rotation matrix $R^{(\alpha+1/\alpha)}(t)$ as:

$$\mathbf{e}^{(\alpha+1)}(t) = \mathbf{e}^{(\alpha)}(t) R^{(\alpha+1/\alpha)}(t) \tag{7}$$

By utilizing the group nature of SO(3), this can also be expressed in the inertial frame:

$$\mathbf{e}^{(\alpha+1)}(t) = \mathbf{e}^I R^{(\alpha)}(t) R^{(\alpha+1/\alpha)}(t) = \mathbf{e}^I R^{(\alpha+1)}(t) \tag{8}$$

Frame Connections Matrices and SE(3)

This section combines both the rotation and displacement in one expression. By structuring rotation and translations together, one obtains a homogeneous transformation matrix. Denavit and Hartenberg [7] were the first to use homogenous transformation matrices, but they did not recognize at the time that such transformations were members of the Special Euclidean Group, denoted as SE(3). A more thorough development of the following theory is found in reference [8].

We define a frame connection as a combination of the frame's orientation and position:

$$\left(\mathbf{e}^{(\alpha)}(t) \quad \mathbf{r}_C^{(\alpha)}(t)\right) = \left(\mathbf{e}_1^{(\alpha)}(t) \quad \mathbf{e}_2^{(\alpha)}(t) \quad \mathbf{e}_3^{(\alpha)}(t) \quad \mathbf{r}_C^{(\alpha)}(t)\right) \tag{9}$$

We define a frame connection matrix $E^{(\alpha)}(t)$. Let 0_3 represent a 3×1 column zero vector. Let $x_C^{(\alpha)}(t)$ denote the column coordinates with respect to the inertial frame. Thus:

$$E^{(\alpha)}(t) = \begin{bmatrix} R^{(\alpha)}(t) & x_C^{(\alpha)}(t) \\ 0_3^T & 1 \end{bmatrix} \tag{10}$$

With this frame connection matrix, we relate the moving and inertial frame connections:

$$\left(\mathbf{e}^{(\alpha)}(t)\, \mathbf{r}_C^{(\alpha)}(t)\right) = \left(\mathbf{e}^I \quad \mathbf{0}\right) E^{(\alpha)}(t) \tag{11}$$

The relation between the child (α+1)-frame and the parent body-α-frame is expressed using the *relative* frame connection matrix $E^{(\alpha+1/\alpha)}$:

$$E^{(\alpha+1/\alpha)}(t) = \begin{bmatrix} R^{(\alpha+1/\alpha)}(t) & s_C^{(\alpha+1/\alpha)}(t) \\ 0_3^T & 1 \end{bmatrix} \tag{12}$$

Thus, we can assert:

$$\left(\mathbf{e}^{(\alpha+1)}(t) \quad \mathbf{r}_C^{(\alpha+1)}(t)\right) = \left(\mathbf{e}^{(\alpha)}(t) \quad \mathbf{r}_C^{(\alpha)}(t)\right) E^{(\alpha+1/\alpha)}(t) \tag{13}$$

This recapitulates Eqns. (8) and (5).

Finally, in adherence with group theory, the absolute frame connection matrix of the (α+1)-body is the product of the absolute frame connection matrix of the preceding body-α and the relative frame connection matrix for the two bodies; and is a member of SE(3):

$$E^{(\alpha+1)}(t) = E^{(\alpha)}(t)\, E^{(\alpha+1/\alpha)}(t) \tag{14}$$

With this foundation, we now apply this work to the analysis of the knuckle boom crane.

KINEMATICS OF SHIP/CRANE SYSTEM

Kinematics of Body-1: the ship

The ship is the first body in this system. Its frame connection matrix $E^{(1)}(t)$, which includes the rotation matrix $R^{(1)}(t)$ and the inertial-frame coordinates of the position $x_C^{(1)}(t)$, is:

$$E^{(1)}(t) = \begin{bmatrix} R^{(1)}(t) & x_C^{(1)}(t) \\ 0_3^T & 1 \end{bmatrix} \tag{15}$$

This frame connection matrix relates the position and orientation of the ship frame to the inertial frame connection in the following way:

$$\left(\mathbf{e}^{(1)}(t) \quad \mathbf{r}_C^{(1)}(t)\right) = \left(\mathbf{e}^I \quad \mathbf{0}\right) E^{(1)}(t) \tag{16}$$

Inserting $E^{(1)}(t)$ into Eq. (16) yields:

$$\begin{aligned} \left(\mathbf{e}^{(1)}(t) \quad \mathbf{r}_C^{(1)}(t)\right) &= \left(\mathbf{e}^I \quad \mathbf{0}\right) \begin{bmatrix} R^{(1)}(t) & x_C^{(1)}(t) \\ 0_3^T & 1 \end{bmatrix} \\ &= \left(\mathbf{e}^I R^{(1)}(t) \quad \mathbf{e}^I x_C^{(1)}(t)\right) \end{aligned} \tag{17}$$

Equation (17) recapitulates Eq. (3) and (6).

The inverse of the frame connection matrix is known:

$$\begin{bmatrix} R^{(1)}(t) & x_C^{(1)}(t) \\ 0_3^T & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \left(R^{(1)}(t)\right)^T & -\left(R^{(1)}(t)\right)^T x_C^{(1)}(t) \\ 0_3^T & 1 \end{bmatrix} \tag{18}$$

Next, we obtain the time derivative of the frame connection matrix by taking the time derivative of each term:

$$\dot{E}^{(1)}(t) = \begin{bmatrix} \dot{R}^{(1)}(t) & \dot{x}_c^{(1)}(t) \\ 0_3^T & 0 \end{bmatrix} \tag{19}$$

Thus, by taking the time derivative of Eq. (16), we find:

$$\left(\dot{\mathbf{e}}^{(1)}(t) \quad \dot{\mathbf{r}}_C^{(1)}(t)\right) = \left(\mathbf{e}^I \quad \mathbf{0}\right) \dot{E}^{(1)}(t) \tag{20}$$

We next desire to express the rate of change of the frame and location in terms of the same frame connection structure.

$$\left(\dot{\mathbf{e}}^{(1)}(t) \quad \dot{\mathbf{r}}_C^{(1)}(t)\right) = \left(\mathbf{e}^{(1)}(t) \quad \mathbf{r}_C^{(1)}(t)\right)\left(E^{(1)}(t)\right)^{-1} \dot{E}^{(1)}(t) \tag{21}$$

For notational purposes, the product of $\left(E^{(1)}(t)\right)^{-1}$ and $\dot{E}^{(1)}(t)$ is defined as the time rate of the frame connection matrix $\Omega^{(1)}$:

$$\Omega^{(1)} \equiv \left(E^{(1)}(t)\right)^{-1} \dot{E}^{(1)}(t) \tag{22}$$

Thus:

$$\begin{pmatrix}\dot{\mathbf{e}}^{(1)}(t) & \dot{\mathbf{r}}_C^{(1)}(t)\end{pmatrix} = \begin{pmatrix}\mathbf{e}^{(1)}(t) & \mathbf{r}_C^{(1)}(t)\end{pmatrix}\Omega^{(1)}(t) \tag{23}$$

We obtain, upon inserting Eq. (18) and (19) in (22):

$$\Omega^{(1)} = \begin{bmatrix} \left(R^{(1)}(t)\right)^T \dot{R}^{(1)}(t) & \left(R^{(1)}(t)\right)^T \dot{x}_c^{(1)}(t) \\ 0_3^T & 0 \end{bmatrix} \tag{24}$$

This matrix provides information about the linear and angular velocities of the coordinate frame attached to the ship. The sub-matrix in the upper left corner of $\Omega^{(1)}$ is a skew-symmetric angular velocity of the frame:

$$\overleftrightarrow{\omega^{(1)}(t)} = \left(R^{(1)}(t)\right)^T \dot{R}^{(1)}(t) \tag{25}$$

This matrix represents the time rate of the coordinate frame in its own frame:

$$\dot{\mathbf{e}}^{(1)}(t) = \mathbf{e}^{(1)}(t)\overleftrightarrow{\omega^{(1)}(t)} = \mathbf{e}^{(1)}(t)\begin{bmatrix} 0 & -\omega_3^{(1)}(t) & \omega_2^{(1)}(t) \\ \omega_3^{(1)}(t) & 0 & -\omega_1^{(1)}(t) \\ -\omega_2^{(1)}(t) & \omega_1^{(1)}(t) & 0 \end{bmatrix} \tag{26}$$

By un-skewing the angular velocity matrix into a column and associating the components as coordinates of the same frame, one obtains the angular velocity vector of that frame:

$$\boldsymbol{\omega}^{(1)}(t) = \mathbf{e}^{(1)}(t)\begin{pmatrix}\omega_1^{(1)} \\ \omega_2^{(1)} \\ \omega_3^{(1)}\end{pmatrix} \tag{27}$$

We express the linear velocity for the ship in the inertial frame:

$$\begin{aligned} \dot{\mathbf{r}}_C^{(1)}(t) &= \mathbf{e}^{(1)}(t)\left(R^1(t)\right)^T \dot{x}_C^{(1)}(t) = \mathbf{e}^{(1)}(t)\, v_C^{(1)}(t) \\ \dot{\mathbf{r}}_C^{(1)}(t) &= \mathbf{e}^I \dot{x}_c^{(1)}(t) \end{aligned} \tag{28}$$

Kinematics of Body 2 – Tower

We attach a coordinate frame $\mathbf{e}^{(2)}(t)$ to the center of mass $C^{(2)}$ of the tower:

$$\mathbf{e}^{(2)}(t) = \begin{pmatrix}\mathbf{e}_1^{(2)}(t) & \mathbf{e}_2^{(2)}(t) & \mathbf{e}_3^{(2)}(t)\end{pmatrix} \tag{29}$$

We find the relative position from the origin of $\mathbf{e}^{(1)}(t)$ to the origin of $\mathbf{e}^{(1)}(t)$ by first translating from the center of mass $C^{(1)}$ of the ship, to the joint where the rotation happens, then rotating to obtain the new orientation, and finally translating from the joint to the center of mass $C^{(2)}$ of the link.

The first translation from $C^{(1)}$ to the joint $\mathrm{J}^{(2/1)}$ is obtained by moving in all three directions.

$$\mathbf{s}_J^{(2/1)} = \mathbf{e}^{(1)}(t) s_J^{(2/1)} = \mathbf{e}^{(1)}(t)\begin{pmatrix} b^{(2)} \\ l_1^{(2)} \\ h_1^{(2)} \end{pmatrix} \tag{30}$$

At the first joint, the rotation happens about the third axis. The following frame rotation relation describes this motion:

$$\mathbf{e}^{(2)}(t) = \mathbf{e}^{(1)}(t) R^{(2/1)}(t) = \mathbf{e}^{(1)}(t)\begin{bmatrix} \cos\theta^{(2)}(t) & -\sin\theta^{(2)}(t) & 0 \\ \sin\theta^{(2)}(t) & \cos\theta^{(2)}(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{31}$$

The last translation from the joint $\mathrm{J}^{(2/1)}$ to the center of mass of the second body is obtained by moving in the 2- and 3-direction up to center of mass of the tower. This translation is expressed using the $\mathbf{e}^{(2)}$-frame:

$$\mathbf{s}_C^{(2/1)} = \mathbf{e}^{(2)}(t) s_C^{(2/1)} = \mathbf{e}^{(2)}(t)\begin{pmatrix} 0 \\ l_2^{(2)} \\ h_2^{(2)} \end{pmatrix} \tag{32}$$

We express the relation between the first and second frame connections using the relative frame connection matrix $E^{(2/1)}$:

$$\begin{pmatrix}\mathbf{e}^{(2)}(t) & \mathbf{r}_C^{(2)}(t)\end{pmatrix} = \begin{pmatrix}\mathbf{e}^{(1)}(t) & \mathbf{r}_C^{(1)}(t)\end{pmatrix} E^{(2/1)}(t) \tag{33}$$

We obtain this frame connection matrix by taking each of the steps described above: translating all three distances up to the joint without rotation, rotating at the joint without translation, and finally translating up the center of mass of the tower without rotation:

$$E^{(2/1)}(t)=\begin{bmatrix} I_3 & s_J^{(2/1)} \\ 0_3^T & 1 \end{bmatrix}\begin{bmatrix} R^{(2/1)}(t) & 0_3 \\ 0_3^T & 1 \end{bmatrix}\begin{bmatrix} I_3 & s_C^{(2/1)} \\ 0_3^T & 1 \end{bmatrix}$$
$$E^{(2/1)}(t)=\begin{bmatrix} R^{(2/1)}(t) & R^{(2/1)}(t)s_C^{(2/1)}+s_J^{(2/1)} \\ 0_3^T & 1 \end{bmatrix} \quad (34)$$

The second frame is related to the inertial through the absolute connection matrix $E^{(2)}(t)$:

$$\left(\mathbf{e}^{(2)}(t) \quad \mathbf{r}_C^{(2)}(t)\right)=\left(\mathbf{e}^I \quad \mathbf{0}\right)E^{(1)}(t)E^{(2/1)}(t)=\left(\mathbf{e}^I \quad \mathbf{0}\right)E^{(2)}(t) \quad (35)$$

$$E^{(2)}(t)=\begin{bmatrix} R^{(2)}(t) & x_C^{(2)}(t) \\ 0_3^T & 1 \end{bmatrix} \quad (36)$$

When multiplying the above matrices, we obtain:

$$R^{(2)}(t)=R^{(1)}(t)R^{(2/1)}(t) \quad (37)$$

$$x_C^{(2)}=R^{(2)}(t)s_C^{(2/1)}+R^{(1)}(t)s_J^{(2/1)}+x_C^{(1)} \quad (38)$$

The frame connection matrix, its inverse and derivative are used to calculate the time rate of the frame connection matrix, $\Omega^{(2)}$. From this we extract the angular velocity vector of the tower:

$$\omega^{(2)}(t)=\left(R^{(2/1)}(t)\right)^T\omega^{(1)}(t)+\omega^{(2/1)}(t) \quad (39)$$

The tower rotates about the shared common axis of the ship: $\mathbf{e}_3^{(1)}(t)\equiv\mathbf{e}_3^{(2)}(t)$. Thus, the *relative* angular velocity vector $\omega^{(2/1)}$ can be expressed by $\dot{\theta}^{(2)}e_3$, where $e_3=\begin{pmatrix}0 & 0 & 1\end{pmatrix}^T$.

$$\omega^{(2)}(t)=\left(R^{(2/1)}(t)\right)^T\omega^{(1)}(t)+\dot{\theta}^{(2)}e_3 \quad (40)$$

The linear velocity vector of the second body from the inertial frame is extracted from the expression for the time derivative of the frame:

$$\left(\dot{\mathbf{e}}^{(2)}(t) \quad \dot{\mathbf{r}}_c^{(2)}(t)\right)=\left(\mathbf{e}^{(2)}(t) \quad \mathbf{r}_C^{(2)}(t)\right)\Omega^{(2)}(t) \quad (41)$$

$$\begin{aligned}\dot{x}_c^{(2)}(t)&=R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_C^{(2/1)}}\right)^T\omega^{(2)}(t)\\&+R^{(1)}(t)\left(\overleftrightarrow{s_J^{(2/1)}}\right)^T\omega^{(1)}(t)+\dot{x}_c^{(1)}(t)\end{aligned} \quad (42)$$

Kinematics of Body 3 – Main Boom

The second link of the crane is the third body of the system. The frame $\mathbf{e}^{(3)}$ is attached to the center of mass $C^{(3)}$:

$$\mathbf{e}^{(3)}(t)=\left(\mathbf{e}_1^{(3)}(t) \quad \mathbf{e}_2^{(3)}(t) \quad \mathbf{e}_3^{(3)}(t)\right) \quad (43)$$

The third frame is obtained from the second frame, by translating to the joint between the tower and the main boom, then rotating at the joint, and finally translating to the center of mass $C^{(3)}$ of the main boom. The frame connection matrix which relates the two frames becomes:

$$E^{(3/2)}(t)=\begin{bmatrix} I_3 & s_J^{(3/2)} \\ 0_3^T & 1 \end{bmatrix}\begin{bmatrix} R^{(3/2)}(t) & 0 \\ 0_3^T & 1 \end{bmatrix}\begin{bmatrix} I_3 & s_C^{(3/2)} \\ 0_3^T & 1 \end{bmatrix} \quad (44)$$

$$E^{(3/2)}(t)=\begin{bmatrix} R^{(3/2)}(t) & R^{(3/2)}(t)s_C^{(3/2)}+s_J^{(3/2)} \\ 0_3^T & 1 \end{bmatrix} \quad (45)$$

Here $R^{(3/2)}$ is the rotation matrix for the relative rotation of the third frame, from the second frame, which happens about the 1-axis only:

$$R^{(3/2)}(t)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta^{(3)}(t) & -\sin\theta^{(3)}(t) \\ 0 & \sin\theta^{(3)}(t) & \cos\theta^{(3)}(t) \end{bmatrix} \quad (46)$$

The translation from the center of mass $C^{(2)}$ of the second body to the joint connecting the second and third body is $\mathbf{s}_J^{(3/2)}$, and the translation from the joint to the center of mass of the third body is $\mathbf{s}_C^{(3/2)}$:

$$\mathbf{s}_J^{(3/2)}=e^{(2)}(t)s_J^{(3/2)}=\mathbf{e}^{(2)}(t)\begin{pmatrix} 0 \\ l_1^{(3)} \\ h_1^{(3)} \end{pmatrix} \quad (47)$$

$$\mathbf{s}_C^{(3/2)}=\mathbf{e}^{(3)}(t)s_C^{(3/2)}=\mathbf{e}^{(3)}(t)\begin{pmatrix} 0 \\ l_2^{(3)} \\ h_2^{(3)} \end{pmatrix} \quad (48)$$

The frame $\mathbf{e}^{(3)}$ can be related to the preceding frame $\mathbf{e}^{(2)}$ by the frame connection matrix $E^{(3/2)}$. It can also be related to the inertial frame by using all the preceding frame connection matrices:

$$\left(\mathbf{e}^{(3)}(t) \quad \mathbf{r}_C^{(3)}(t)\right)=\left(\mathbf{e}^{(2)}(t) \quad \mathbf{r}_C^{(2)}(t)\right)E^{(3/2)}(t) \quad (49)$$

$$\left(\mathbf{e}^{(3)}(t) \quad \mathbf{r}_C^{(3)}(t)\right)=\left(\mathbf{e}^I \quad \mathbf{0}\right)E^{(1)}(t)E^{(2/1)}(t)E^{(3/2)}(t) \quad (50)$$

The absolute frame connection matrix of the third frame from the inertial:

$$E^{(3)}(t) = \begin{bmatrix} R^{(3)}(t) & x_C^{(3)}(t) \\ 0_3^T & `1 \end{bmatrix} \tag{51}$$

By multiplying out the matrices in the above equation, one can obtain the absolute rotation $R^{(3)}$ and position $x_C^{(3)}$ of the third frame from the inertia:

$$R^{(3)}(t) = R^{(1)}(t) R^{(2/1)}(t) R^{(3/2)}(t) \tag{52}$$

$$\begin{aligned} x_C^{(3)}(t) &= R^{(1)}(t) R^{(2/1)}(t) \left(R^{(3/2)}(t) s_C^{(3/2)} + s_J^{(3/2)} \right) + \\ & R^{(1)}(t) R^{(2/1)}(t) s_C^{(2/1)} + R^{(1)}(t) s_J^{(2/1)} + x_c^{(1)}(t) \end{aligned} \tag{53}$$

Through further calculations similar to the previous links, the rates for the arm is obtained. The angular velocity vector of the third frame is:

$$\omega^{(3)}(t) = \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}(t)\right)^T \omega^{(1)}(t) + \left(R^{(3/2)}(t)\right)^T \dot{\theta}^{(2)} e_3 + \dot{\theta}^{(3)} e_1 \tag{54}$$

The linear velocity vector of the third frame from the inertia:

$$\dot{x}_c^{(3)}(t) = R^{(1)}(t) \begin{pmatrix} R^{(2/1)}(t) R^{(3/2)}(t) \left(\overrightarrow{s_C^{(3/2)}}\right)^T \omega^{(3)}(t) + \\ R^{(2/1)}(t) \left(\overrightarrow{s_J^{(3/2)}}\right)^T \omega^{(2)}(t) + \\ R^{(2/1)}(t) \left(\overrightarrow{s_C^{(2/1)}}\right)^T \omega^{(2)}(t) + \\ \left(\overrightarrow{s_J^{(2/1)}}\right)^T \omega^{(1)}(t) \end{pmatrix} + \dot{x}_c^{(1)}(t) \tag{55}$$

Kinematics of Body 4 – Outer Boom

The third link of the crane is the fourth body of the system. The frame $\mathbf{e}^{(4)}$ is attached to the center of mass $C^{(4)}$:

$$\mathbf{e}^{(4)}(t) = \left(\mathbf{e}_1^{(4)}(t) \quad \mathbf{e}_2^{(4)}(t) \quad \mathbf{e}_3^{(4)}(t) \right) \tag{56}$$

We obtain the fourth frame from the third frame, by translating to the joint between the main boom and the outer boom, then rotating at the joint, and finally translating to the center of mass $C^{(4)}$ of the outer boom. The frame connection matrix which relates the two frames becomes:

$$E^{(4/3)}(t) = \begin{bmatrix} I_3 & s_J^{(4/3)} \\ 0_3^T & 1 \end{bmatrix} \begin{bmatrix} R^{(4/3)}(t) & 0 \\ 0_3^T & 1 \end{bmatrix} \begin{bmatrix} I_3 & s_C^{(4/3)} \\ 0_3^T & 1 \end{bmatrix} \tag{57}$$

$$E^{(4/3)}(t) = \begin{bmatrix} R^{(4/3)}(t) & R^{(4/3)}(t) s_C^{(4/3)} + s_J^{(4/3)} \\ 0_3^T & 1 \end{bmatrix} \tag{58}$$

Here $R^{(4/3)}$ is the rotation matrix for the relative rotation of the third frame, from the second frame, which happens about the 1-axis only:

$$R^{(4/3)}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta^{(4)}(t) & -\sin\theta^{(4)}(t) \\ 0 & \sin\theta^{(4)}(t) & \cos\theta^{(4)}(t) \end{bmatrix} \tag{59}$$

The translation from the center of mass $C^{(3)}$ of the third body to the joint connecting the third and fourth body is $\mathbf{s}_J^{(4/3)}$, and the translation from the joint to the center of mass of the fourth body is $\mathbf{s}_C^{(4/3)}$:

$$\mathbf{s}_J^{(4/3)} = \mathbf{e}^{(3)}(t) s_J^{(4/3)} = \mathbf{e}^{(3)}(t) \begin{pmatrix} 0 \\ l_1^{(4)} \\ h_1^{(4)} \end{pmatrix} \tag{60}$$

$$\mathbf{s}_C^{(4/3)} = \mathbf{e}^{(4)}(t) s_C^{(4/3)} = \mathbf{e}^{(4)}(t) \begin{pmatrix} 0 \\ l_2^{(4)} \\ h_2^{(4)} \end{pmatrix} \tag{61}$$

The frame $\mathbf{e}^{(4)}$ can be related to the preceding frame $\mathbf{e}^{(3)}$ by the frame connection matrix $E^{(4/3)}$. It can also be related to the inertial frame by using all the preceding frame connection matrices:

$$\left(\mathbf{e}^{(4)}(t) \quad \mathbf{r}_C^{(4)}(t) \right) = \left(\mathbf{e}^{(3)}(t) \quad \mathbf{r}_C^{(3)}(t) \right) E^{(4/3)}(t) \tag{62}$$

$$\begin{aligned} &\left(\mathbf{e}^{(4)}(t) \quad \mathbf{r}_C^{(4)}(t) \right) = \\ &\left(\mathbf{e}^{I} \quad \mathbf{0} \right) E^{(1)}(t) E^{(2/1)}(t) E^{(3/2)}(t) E^{(4/3)}(t) \end{aligned} \tag{63}$$

The absolute frame connection matrix of the fourth frame from the inertial:

$$E^{(4)}(t) = \begin{bmatrix} R^{(4)}(t) & x_C^{(4)}(t) \\ 0_3^T & 1 \end{bmatrix} \tag{64}$$

By multiplying out the matrices in the above equation, one can obtain the absolute rotation $R^{(4)}$ and position $x_C^{(4)}$ of the third frame from the inertia:

$$R^{(4)}(t) = R^{(1)}(t) R^{(2/1)}(t) R^{(3/2)}(t) R^{(4/3)}(t) \tag{65}$$

$$
\begin{aligned}
&x_C^{(4)}(t) = \\
&R^{(1)}(t)R^{(2/1)}(t)R^{(3/2)}(t)\left(R^{(4/3)}(t)s_C^{(4/3)} + s_J^{(4/3)}\right) + \\
&R^{(1)}(t)R^{(2/1)}(t)\left(R^{(3/2)}(t)s_C^{(3/2)} + s_J^{(3/2)}\right) + \\
&R^{(1)}(t)R^{(2/1)}(t)s_C^{(2/1)} + R^{(1)}(t)s_J^{(2/1)} + x_c^{(1)}(t)
\end{aligned}
\tag{66}
$$

Through further calculations similar to the previous links, the rates for the arm is obtained. The angular velocity vector of the third frame is:

$$
\begin{aligned}
&\omega^{(4)}(t) = R^{(4/3)T}(t)R^{(3/2)T}(t)R^{(2/1)T}(t)\omega^{(1)}(t) + \\
&R^{(4/3)T}(t)R^{(3/2)T}(t)\dot{\theta}^{(2)}(t)e_3 + \\
&R^{(4/3)T}(t)\dot{\theta}^{(3)}(t)e_1 + \dot{\theta}^{(4)}(t)e_1
\end{aligned}
\tag{67}
$$

The linear velocity vector of the third frame from the inertia:

$$
\dot{x}_c^{(4)}(t) = R^{(1)}(t)\begin{pmatrix}
R^{(2/1)}(t)R^{(3/2)}(t)R^{(4/3)}(t)\left(\overleftrightarrow{s_C^{(4/3)}}\right)^T \omega^{(4)}(t) + \\
R^{(2/1)}(t)R^{(3/2)}(t)\left(\overleftrightarrow{s_J^{(4/3)}}\right)^T \omega^{(3)}(t) + \\
R^{(2/1)}(t)R^{(3/2)}(t)\left(\overleftrightarrow{s_C^{(3/2)}}\right)^T \omega^{(3)}(t) + \\
R^{(2/1)}(t)\left(\overleftrightarrow{s_J^{(3/2)}}\right)^T \omega^{(2)}(t) + \\
R^{(2/1)}(t)\left(\overleftrightarrow{s_C^{(2/1)}}\right)^T \omega^{(2)}(t) + \\
\left(\overleftrightarrow{s_J^{(2/1)}}\right)^T \omega^{(1)}(t)
\end{pmatrix} + \dot{x}_c^{(1)}(t)
\tag{68}
$$

All equations needed for the kinematics analysis are now obtained: Eq. (27), (28), (40), (42), (54), (55), (67) and (68).

GENERALIZED COORDINATES

Despite the fact that we formulated our results for a translation of the boat, we will ignore translations in this analysis.

The velocities and angular velocities for all three bodies are gathered in a 21 x 1 matrix $\{\dot{X}(t)\}$. These are referred to as Cartesian velocities on the left in Eqn. (69):

$$
\{\dot{X}(t)\} \equiv \begin{pmatrix}
\omega^{(1)}(t) \\
\dot{x}_C^{(2)}(t) \\
\omega^{(2)}(t) \\
\dot{x}_C^{(3)}(t) \\
\omega^{(3)}(t) \\
\dot{x}_C^{(4)}(t) \\
\omega^{(4)}(t)
\end{pmatrix}
\qquad
\{\dot{q}(t)\} \equiv \begin{pmatrix}
\omega^{(1)}(t) \\
\dot{\theta}^{(2)}(t) \\
\dot{\theta}^{(3)}(t) \\
\dot{\theta}^{(4)}(t)
\end{pmatrix}
\tag{69}
$$

The constrained generalized coordinates can be expressed, through the degrees of freedom, by an independent set of essential generalized coordinates in the 6×1 matrix: $\{\dot{q}(t)\}$.

The Cartesian velocities and essential generalized velocities are related linearly through the 21 x 6 matrix $[B(t)]$:

$$
\{\dot{X}(t)\} = [B(t)]\{\dot{q}(t)\}
\tag{70}
$$

In the following, note that:

$$
I_3 \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\qquad
\underset{3\times 3}{0} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\qquad
\underset{3\times 1}{0} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\tag{71}
$$

Thus, the cells of matrix $[B(t)]$ are constructed from the velocities and angular velocities from all three elements:

$$
[B(t)] = \begin{bmatrix}
I_3 & \underset{3x1}{0} & \underset{3x1}{0} & \underset{3x1}{0} \\
B_{21} & B_{22} & \underset{3x1}{0} & \underset{3x1}{0} \\
(R^{(2/1)}(t))^T & e_3 & \underset{3x1}{0} & \underset{3x1}{0} \\
B_{41} & B_{42} & B_{43} & \underset{3x1}{0} \\
R^{(3/2)T}(t)R^{(2/1)T}(t) & R^{(3/2)T}(t)e_3 & e_1 & \underset{3x1}{0} \\
B_{61} & B_{62} & B_{63} & B_{64} \\
B_{71} & B_{72} & R^{(4/3)T}(t)e_1 & e_1
\end{bmatrix}
\tag{72}
$$

The full equations for the B-matrix are listed in the appendix.

KINETICS OF SHIP/CRANE SYSTEM

At this point, we turn to kinetics. Rather than redevelop the foundational theory, we must summarize, due to space limitations.

The critical point is that we define a Virtual Rotation Displacement as follows

$$\overleftrightarrow{\delta\pi^{(\alpha)}(t)} = \left(R^{(\alpha)}(t)\right)^T \delta R^{(\alpha)}(t) \qquad (73)$$

$\delta x_C^{(\alpha)}(t)$ and $\delta R^{(\alpha)}(t)$ are variations from the existing terms $x_C^{(\alpha)}(t)$ and $R^{(\alpha)}(t)$. But $\delta\pi^{(\alpha)}(t)$ is only a definition, as $\pi^{(\alpha)}(t)$ does not exist. The virtual generalized displacement $\left\{\delta\tilde{X}(t)\right\}$ is a $6n\,x1$ matrix:

$$\left\{\delta\tilde{X}(t)\right\} = \begin{pmatrix} \delta x_C^{(1)}(t) \\ \delta\pi^{(1)}(t) \\ \delta x_C^{(2)}(t) \\ \delta\pi^{(2)}(t) \\ \delta x_C^{(3)}(t) \\ \delta\pi^{(3)}(t) \\ \delta x_C^{(4)}(t) \\ \delta\pi^{(4)}(t) \end{pmatrix} \qquad (74)$$

The commutativity of the mixed partials produces the following.

$$\frac{d}{dt}\delta x_C^{(\alpha)}(t) = \delta\dot{x}_C^{(\alpha)}(t) \qquad (75)$$

The variation of the angular velocity $\delta\omega^{(\alpha)}(t)$ is restricted. Previous work by Murakami [7] and D. Holm [8] provide the following form for the restriction on the variation of the angular velocity.

$$\delta\omega^{(\alpha)}(t) = \frac{d}{dt}\delta\pi^{(\alpha)}(t) + \overleftrightarrow{\omega^{(\alpha)}(t)}\delta\pi^{(\alpha)}(t) \qquad (76)$$

Equation (75) and (76) can be written in a compact matrix form and is expressed as follows:

$$\left\{\delta\dot{X}\right\} = \left\{\delta\dot{\tilde{X}}\right\} + \left[D\right]\left\{\delta\tilde{X}\right\} \qquad (77)$$

The skew-symmetric matrix [D] is expressed as:

$$[D] = \begin{bmatrix} \overleftrightarrow{\omega}^{(1)}(t) & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & \overleftrightarrow{\omega}^{(2)}(t) & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & \overleftrightarrow{\omega}^{(3)}(t) & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & \overleftrightarrow{\omega}^{(4)}(t) \end{bmatrix} \qquad (78)$$

Finally, the variation of kinetic energy:

$$\delta K = \left\{\delta\dot{X}(t)\right\}^T \left[M\right]\left\{\dot{X}(t)\right\} \qquad (79)$$

Continuing, the ship is exposed to forces and moments due to waves, buoyancy, and gravity. The second, third and fourth body, the tower, main boom and outer boom, are subjected to gravity and motor moments. Reverse moments from the motors which turn the links are applied to the ship and crane, due to Newton's third law of action and reaction. The forces $\left\{Q(t)\right\}$ are summarized as follows:

$$\left\{Q(t)\right\} = \begin{pmatrix} F_C^{(1)I}(t) \\ M_C^{(1)}(t) \\ F_C^{(2)I}(t) \\ M_C^{(2)}(t) \\ F_C^{(3)I}(t) \\ M_C^{(3)}(t) \\ F_C^{(4)I}(t) \\ M_C^{(4)}(t) \end{pmatrix} = \begin{pmatrix} F^{(w)I}(t) + F_b^I - m^{(1)}ge_3 \\ M^{(w)}(t) - M_m^{(t)}(t)e_3 \\ -m^{(2)}ge_3 \\ M_m^{(t)}(t)e_3 - M_m^{(mb)}(t)e_1 \\ -m^{(3)}ge_3 \\ M_m^{(mb)}(t)e_1 - M_m^{(ob)}(t)e_1 \\ -m^{(4)}ge_3 \\ M_m^{(ob)}(t)e_1 \end{pmatrix} \qquad (80)$$

Allow us to interpret the terms:

$F^{(w)I}$ = Force from waves
F_b^I = *Force from buoyance*
$m^{(1)}ge_3$ = *Force from gravity on the ship*
$M^{(w)}(t)$ = *Moment from waves*
$-M_m^{(t)}(t)e_3$ = Re*verse moment on ship from tower*
$-m^{(2)}ge_3$ = *Force from gravity on the tower*
$M_m^{(t)}(t)e_3$ = *Moment that turns the tower*
$-M_m^{(mb)}(t)e_1$ = Re*verse moment on tower from the main boom*
$-m^{(3)}ge_3$ = *Force from gravity on the main boom*
$M_m^{(mb)}(t)e_1$ = *Moment on crane that turns the main boom*
$-M_m^{(ob)}(t)e_1$ = Re*verse moment on the main boom from the outer boom*
$-m^{(3)}ge_3$ = *Force from gravity on the outer boom*
$M_m^{(ob)}(t)e_1$ = *Moment on crane that turns the outer boom*

The superscript I in $F^{(w)I}$ and F_b^I, indicates that the components of the forces are expressed in the inertial frame.

Continuing, the coefficient matrix $\left[B(t)\right]$ which relates the generalized velocities in Eq.(70), also relates the generalized displacements $\left\{\delta\tilde{X}(t)\right\}$ and the essential virtual displacements $\left\{\delta q(t)\right\}$ [6]:

$$\left\{\delta\tilde{X}(t)\right\}=\left[B(t)\right]\left\{\delta q(t)\right\} \tag{81}$$

Proceeding with the virtual work done by the physical forces, where moments and $\overleftrightarrow{\delta\pi^{(\alpha)}(t)}$ are the conjugate pair:

$$\delta W=\left\{\delta\tilde{X}(t)\right\}^T\left\{Q(t)\right\} \tag{82}$$

$$\begin{aligned}&\left[B(t)\right]^T\left[M\right]\left[B(t)\right]\left\{\ddot{q}(t)\right\}+\\&\left[B(t)\right]^T\left(\left[M\right]\left[\dot{B}(t)\right]+\left[D(t)\right]\left[M\right]\left[B(t)\right]\right)\left\{\dot{q}(t)\right\}-\\&\left[B(t)\right]^T\left\{Q(t)\right\}=\{0\}\end{aligned} \tag{83}$$

Defining $M^*(t)$ and $N^*(t)$:

$$\left[M^*(t)\right]=\left[B(t)\right]^T\left[M\right]\left[B(t)\right] \tag{84}$$

$$\left[N^*(t)\right]\equiv\left[B(t)\right]^T\left(\left[M\right]\left[\dot{B}(t)\right]+\left[D(t)\right]\left[M\right]\left[B(t)\right]\right) \tag{85}$$

We obtain:

$$\left[M^*(t)\right]\{\ddot{q}\}+\left[N^*(t)\right]\{\dot{q}\}-\left\{F^*\right\}=0 \tag{86}$$

By using the essential generalized velocities, the above equation sets the mathematical model for deriving the equations of motion for the crane and ship. The equation will be solved using numeric integration methods.

SIMPLIFICATION OF MODEL

To simplify the calculations, wave forces are neglected and buoyancy is ignored. The ship is assumed to be stationary and will only rotate about its mass center.

RECONSTRUCTION OF ROTATION MATRIX

The rotation matrix of the ship at each time step must be known since the terms appear explicitly in the B-matrix. Thus, the rotation matrix $R^{(1)}(t)$ must be computed, and the following equation solved:

$$\dot{R}^{(1)}(t)=R^{(1)}(t)\overleftrightarrow{\omega^{(1)}(t)} \tag{87}$$

Analytically, with initial value $R(0)$ the solution is:

$$R^{(1)}(t)=R(0)\exp(t\overleftrightarrow{\omega_0}) \tag{88}$$

The Caley-Hamilton theorem is used in the derivation of the Rodrigues' rotation formula to reduces a series expansion of the exponential of a matrix to a close analytical form. The Rodrigues' rotation formula admits for a closed expression for the matrix expansion. This is in turn used to reconstruct the unknown rotation matrix of the ship. A derivation of this can be found in [9].

This will only valid for constant angular velocities. In principle, one only need to average this over two time-steps using a central difference approximation. However, for ease of first pass coding, this rule will be violated by assuming a constant value at the start of each time step. The goal here is a qualitative and visualized result.

RESULTS

Parameters Used

Mass of ship:	$m^{(1)}=9000kg$
Mass of tower:	$m^{(2)}=160kg$
Mass of main boom:	$m^{(3)}=150kg$
Mass of outer boom:	$m^{(4)}=150kg$
$C^{(1)}$ to $J^{(2/1)}$ in x-axis:	$b^{(2)}=-7.5m$
$C^{(1)}$ to $J^{(2/1)}$ in y-axis:	$l_1^{(2)}=11.9m$
$C^{(1)}$ to $J^{(2/1)}$ in z-axis:	$h_1^{(2)}=13.5m$
$J^{(2/1)}$ to $C^{(2)}$ in y-axis:	$l_2^{(2)}=-0.14m$
$J^{(2/1)}$ to $C^{(2)}$ in z-axis:	$h_2^{(2)}=2.6m$
$C^{(2)}$ to $J^{(3/2)}$ in y-axis:	$l_1^{(2)}=0.14m$
$C^{(2)}$ to $J^{(3/2)}$ in z-axis:	$h_1^{(3)}=3.5m$
$J^{(3/2)}$ to $C^{(3)}$ in y-axis:	$l_2^{(3)}=10.7m$
$J^{(3/2)}$ to $C^{(3)}$ in z-axis:	$h_2^{(3)}=-0.2m$

$C^{(3)}$ to $J^{(4/3)}$ in y-axis:	$l_1^{(4)} = 11.3m$
$C^{(3)}$ to $J^{(4/3)}$ in z-axis:	$h_1^{(4)} = 0.2m$
$J^{(4/3)}$ to $C^{(4)}$ in y-axis:	$l_2^{(4)} = 4.1m$
$J^{(4/3)}$ to $C^{(4)}$ in z-axis:	$h_2^{(4)} = -2.4m$
Width of ship:	$Lps = 23\ m$
Length of ship:	$Lsb = 110\ m$
Max. height of ship:	$Lh = 39\ m$
Torque applied to tower:	$\mathrm{M}^{(1)} = 100000$ Nm
Torque applied to M.B.:	$\mathrm{M}^{(2)} = 70000$ Nm
Torque applied to the O.B.:	$\mathrm{M}^{(3)} = 70000$ Nm
Dampening factor:	$df = 1000$ Ns/m

Damping

Wave and buoyancy forces are a part of the equation of motion. However, as a simplification of this first pass, we neglected them. Instead, we applied simple damping to reduce the pitch, roll and the yaw of the ship.

Numerical Integration

We numerically integrate using the Runge-Kutta method to solve the differential equations for the position of the arms and the body.

3D VISUALIZATION AND WEBGL

WebGL (Web Graphics Library) is a JavaScript interface for rendering interactive 2D and 3D computer graphics. WebGL is compatible with most of the major web browsers such as Chrome, Firefox, Safari, and Opera. In addition, it is free of cost and can be used without the need for plugins. It does so by introducing an API which closely conforms to OpenGL ES 2.0, thus being compatible with HTML5.

The webpage was designed with checkboxes for Motor 1, Motor 2, Motor 3 and damping. The ship has full 3D-rotation, hence a movement made by the crane will affect the behavior of the ship in all axes.

Each motor has been checked to observe qualitative responses. The qualitative responses were in accordance with what would be expected from a physical model, thus the 3D-simulation seems realistic.

It is critical to note that the authors are not pushing a software system. WebGL is easy to code and is free. The MFM makes dynamics easy to code. The computations run on cell phones. The reader may proceed to this link on a laptop or mobile device and experiment:

http://home.hib.no/prosjekter/dynamics/2019/knuckle/

CONCLUSION AND FUTURE WORK

This paper presented a qualitative confirmation of behavior and accuracy through a 3D simulation using the MFM. The simulation showed the dynamics of the ship and knuckle boom crane in motion. This is required when the control system is to be developed, which will be part of the future of this project.

Viscosity, drag, and buoyancy were substituted with a generic dampening factor. We included wave moments and added mass.

The structure of the moving frame-method enables an easier and more compact way to adapt linked dynamics problems into code.

As the two lead authors are undergraduates, an underlying goal of this work has also been to demonstrate that the MFM empowers student understanding of dynamics.

Finally, Artificial Intelligence is the study of "intelligent agents": the study of any device that perceives its environment and takes actions that maximize its chance of success at some goal. At this time, "device" is restricted to mean "computers." However, today's mechanical machines think (with onboard CPUs) and communicate (with network cards). Soon, with biologically inspired neural networks, machines will learn. In anticipation of this, simulations of mechanical systems (dynamics) will enhance adaptive machine learning. The Moving Frame Method is unique in that it is eminently programmable and rapidly deployed in new settings, obviating the need for legacy implementations of multi-body dynamics codes extant today.
Thus, the next step in this work is to add learning modules to the evolving software so that ships with onboard sensors can take action depending on conditions or expected conditions from learned behavior.

APPENDIX

$$B_{21} = R^{(2)}(t)\left(\overleftrightarrow{s_C^{(2/1)}}\right)^T \left(R^{(2/1)}(t)\right)^T + R^{(1)}(t)\left(\overleftrightarrow{s_J^{(2/1)}}\right)^T$$

$$B_{22} = R^{(2)}(t)\left(\overleftrightarrow{s_C^{(2/1)}}\right)^T e_3$$

$$B_{41} =$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_C^{(3/2)}\right)^T \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(2)}(t)\left(\overleftrightarrow{s}_J^{(3/2)}\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(2)}(t)\left(\overleftrightarrow{s}_C^{(2/1)}\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(1)}(t)\left(\overleftrightarrow{s}_J^{(2/1)}\right)^T$$

$$B_{42} = \left(R^{(3)}(t)\left(\overleftrightarrow{s}_C^{(3/2)}\right)^T \left(R^{(3/2)}(t)\right)^T + R^{(2)}(t)\left(\overleftrightarrow{s}_J^{(3/2)}\right)^T + R^{(2)}(t)\left(\overleftrightarrow{s}_C^{(2/1)}\right)^T e_3 \right)$$

$$B_{43} = R^{(3)}(t)\left(\overleftrightarrow{s}_C^{(3/2)}\right)^T e_1$$

$$B_{61} = R^{(4)}(t)\left(\overleftrightarrow{s}_C^{(4/3)}\right)^T \left(R^{(4/3)}(t)\right)^T \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}\right)^T (t) +$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_J^{(4/3)}\right)^T \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_C^{(3/2)}\right)^T \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(2)}(t)\left(\overleftrightarrow{s}_J^{(3/2)}\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(2)}(t)\left(\overleftrightarrow{s}_C^{(2/1)}\right)^T \left(R^{(2/1)}(t)\right)^T +$$
$$R^{(1)}(t)\left(\overleftrightarrow{s}_J^{(2/1)}\right)^T$$

$$B_{62} = R^{(4)}(t)\left(\overleftrightarrow{s}_C^{(4/3)}\right)^T \left(R^{(4/3)T}(t)\right)^T \left(R^{(3/2)}(t)\right)^T e_3 +$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_J^{(4/3)}\right)^T \left(R^{(3/2)}(t)\right)^T e_3 +$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_C^{(3/2)}\right)^T \left(R^{(3/2)}(t)\right)^T e_3 +$$
$$R^{(2)}(t)\left(\overleftrightarrow{s}_J^{(3/2)}\right)^T e_3 +$$
$$R^{(2)}(t)\left(\overleftrightarrow{s}_C^{(2/1)}\right)^T e_3$$

$$B_{63} =$$
$$R^{(4)}(t)\left(\overleftrightarrow{s}_C^{(4/3)}\right)^T \left(R^{(4/3)}(t)\right)^T e_1 +$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_J^{(4/3)}\right)^T e_1 +$$
$$R^{(3)}(t)\left(\overleftrightarrow{s}_C^{(3/2)}\right)^T e_1$$

$$B_{64} = R^{(4)}(t)\left(\overleftrightarrow{s}_C^{(4/3)}\right)^T e_1$$

$$B_{71} = \left(R^{(4/3)}(t)\right)^T \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}(t)\right)^T$$

$$B_{72} = \left(R^{(4/3)}(t)\right)^T \left(R^{(3/2)}(t)\right)^T e_3$$

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