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Post-normal science and mathematics education in uncertain times: Educating future citizens for extended peer communities

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ABSTRACT

In this paper we argue that the academic fields of critical mathematics education and post-normal science are complementary and can provide mutual benefits for the future. Post-normal science promotes the idea of extended peer communities, through which citizens participate with knowledge and insights in urgent and complex societal issues with conflicting stakes. Meanwhile, critical mathematics education is a philosophy of mathematics education that includes attention to the central role of mathematics in a technological society, the effects of this role, and the need for epistemic dialogue in learning and teaching mathematics. We argue that a mathematics education based on these ideas can prepare students to participate in extended peer communities. We focus particularly on the uncertainty that characterises post-normal situations. Post-normal science distinguishes various characteristics of uncertainty and highlights its centrality to post-normal situations. A critical mathematics education should prepare citizens who are able to deal with the different ways in which uncertainty matters in such situations. We illustrate these ideas through brief descriptions of three classroom studies in which students discuss issues of uncertainty and risk.

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1. Introduction

Contemporary society faces a range of complex and urgent challenges characterised by a high degree of uncertainty. Examples include climate change, global epidemics, and new technologies like genetically modified organisms. Funtowicz and Ravetz (1993) argue that such challenges require *post-normal science*. Post-normal science is distinguished from “normal” science, characterised as a kind of puzzle solving activity where scientists choose solvable problems and produce knowledge associated with a high level of certainty. The quality of this work is assured through a peer community that usually consists of other scientists. On issues where uncertainty and stakes are low (e.g. in terms of costs, impacts or risks), experts can feed decision makers with solutions based on a relatively value-free idea of science. Although there is an increased awareness of the limitations of science on complex issues, research suggests that the idea that policy should be informed by objective science is still strong (see, for example, Hauge et al., 2014; Hauge, 2011).

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Problems like climate change, global epidemics or new technologies require a different approach, for which Funtowicz and Ravetz (1993) proposed the term post-normal science. These kinds of problems have several features, including high degrees of complexity, uncertainty and risk. These factors arise in the context of non-linear dynamics, multiple actors and contested values. For example, while the creation of a GMO crop strain in the laboratory may be considered normal science, the question of how to deploy GMO crops in commercial agriculture is much more complex. The release of GMOs into an ecosystem is difficult to model and may have unforeseeable outcomes. The use of GMOs may implicate farmers, agro-industrial companies, governments, aid agencies, local communities, protest movements etc. Each set of actors brings different values to the problem, leading to different ideas about what information is significant, what forms of data should be collected, what impacts should be considered serious, etc. In this sense, quality in knowledge production for policy has a strong connection to relevance; it must consider pluralities of knowledge perspectives and values, and uncertainty and its characteristics must be placed at the centre of debate. In the culture of normal science, by contrast, only expert voices count.

A key feature of post-normal science, therefore, is the idea of *extended peer communities*:

> the evaluation of scientific inputs to decision making requires an 'extended peer community'. This extension of legitimacy to new participants in policy dialogues has important implications both for society and for science. With mutual respect among various perspectives and forms of knowing, there is a possibility for the development of a genuine and effective democratic element in the life of science. (Funtowicz & Ravetz, 1993, p. 740–741)

In effect, Funtowicz and Ravetz are calling for the involvement of a wide range of stakeholders, not only in discussing the results of research, but ultimately in the conduct of science itself:

> The relevant peer community is thus extended beyond the direct producers, sponsors and users of the research, to include all with a stake in the product, the process, and its implications both local and global. This extension of the peer community may include investigative journalists, lawyers and pressure groups. (Funtowicz & Ravetz, 1993, p. 747)

By extending the peer community to include a much wider range of stakeholders, including those directly affected by the situation in question, both science and solutions are likely to be enhanced, through, for example, an extended sense of what information should be taken into account ('extended facts') as well as a more broadly-based treatment of risk and uncertainty.

The extended peer community is a persuasive idea, but it also raises questions. As researchers in the field of mathematics education, we are aware that any understanding and interpretation of complex scientific problems requires some degree of mathematical literacy (see an example of this idea in Barwell, 2013a, on climate change). Participation in an extended peer community clearly demands an ability to interpret texts relating to advanced mathematical ideas (think, for example, of public discussion about climate change, or about restarting nuclear power production in Japan). These texts, even if simplified for a 'non-scientific' reader, may include or refer to data, graphs and charts, statistical analysis, probability and risk, use of mathematical models and so on. Moreover, the presentation of these ideas is never neutral; it represents particular positions and interests, and uncertain risks may impact stakeholders differently. Yet to participate in extended peer communities, citizens need to be able to engage with these mathematical texts and to some extent the ideas and techniques to which they refer. The question we explore in this paper, therefore, is: how can citizens be mathematically educated so that they can participate in extended peer communities? (See also the discussion on the emergence and role of citizen science by Wildschut, this issue). In order to keep our discussion focused, we specifically examine the place of uncertainty in post-normal science. Uncertainty is a key feature of post-normal situations. Participation in extended peer communities (and in democratic debates more generally) therefore requires a critical understanding of uncertainty. This theoretical paper advances the limited previous work on post-normal science and mathematics by drawing experiences from previous work together and going more into depth on how uncertainty, as a central feature of post-normal science, can be addressed within mathematics education in a way that supports the development of critical citizens.

Our approach to mathematics education is based on Skovsmose's (1994) *critical mathematics education*, which we discuss in the next section. We go on to show how post-normal science and critical mathematics education are partly complementary perspectives, before going into more depth on the topic of uncertainty. We focus on uncertainty since it is a challenging topic, plays a central role in understanding post-normal situations, and is one of the primary reasons that extended peer communities are necessary in such situations. We include excerpts from classroom discussions conducted in Norway to illustrate our ideas.

2. Critical mathematics education

Both the UN (2006, 2009) and the OECD (2003, 2006) suggest that mathematics and, in particular, mathematical literacy are important for critical reflection and democratic participation in society. The OECD's Programme for International Student Assessment (PISA) defines mathematical literacy as: "[. . .] an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OECD, 2006; p. 72). PISA has been criticized, however, for emphasising the individual in its definition of mathematical literacy, while paying less attention to democratic values. National curricula are in some instances more specific about the democratic value of critical citizenship. For example, the Norwegian National Curriculum states a need for citizens' competence to "understand and critically evaluate information, statistical analyses [. . .] to understand and impact processes in society" (Ministry of Education and Research, 2006).

The appearance of references in curriculum and education policy texts to mathematical literacy linked to critical thinking and society may seem promising for post-normal science and the participation of future citizens in extended peer communities. Mathematics curricula, however, have largely failed to embrace these ideals. Mathematics teaching often appears to foster a view of mathematics as an activity that either produces either a correct or an incorrect answer. This 'traditional', procedurally oriented view is challenged by many mathematics education researchers, professional associations and curricula, who often advocate an alternative approach broadly focused on problem-solving. While this 'reform' approach may encourage students to develop more productive forms of mathematical thinking, it is not necessarily any better at promoting the kind of critical thinking that we suggest is necessary for participation in extended peer communities, or, indeed, in the broader political debates surrounding post-normal situations.

Within the academic field of mathematics education, however, there are more critical perspectives on the role of mathematics in education in wider society. Some question the role of abstract mathematics in education and promote ethnomathematics (see e.g. Khan, 2011; for a discussion), some promote mathematics as a tool for students to achieve insights into (their own) problems of social justice (e.g., Gutstein, 2006, 2013), and others promote mathematics education where students reflect on how mathematics shapes their lives, as well as society, in often invisible ways (see, in particular, Skovsmose, 1994). These areas have come to be known as critical mathematics education. Skovsmose's (1994) work and the theoretical program it originated (Skovsmose, 1998, 2012) have established some key ideas about the role of mathematics and mathematics education in society, as well as useful concepts for tackling these ideas in mathematics classrooms. In this paper, we show how these ideas complement post-normal science. We argue, in fact, that ideas from critical mathematics education can form the basis for preparing students to participate in extended peer communities. Indeed, d'Ambrosio (e.g., 1998), a Brazilian mathematics educator, has argued that the field has a "responsibility for the future"; that is, he argues, faced with the enormous challenges and threats to environmental, social, international and individual peace, mathematics educators have to act. Preparing students to participate in extended peer communities can be part of a program for mathematics education for the future.

Critical approaches to mathematics education are concerned with how students can learn to participate in a democratic society, through an emphasis on critique. A critical approach to mathematics education includes students learning and using mathematical methods to examine social, environmental or economic problems, as well as students learning about the nature and role of mathematics in modern society. Mathematics is seen as political, both in the sense that mathematics is an essential part of political debate, and in the sense that mathematics is one of the key tools in the construction of a technological-industrial society and is implicated in every kind of problem, from warfare, to computer viruses, to environmental crises (Barwell, 2013b). A link with post-normal science is already apparent. Mathematics is necessary to understand and debate many aspects of post-normal situations; but mathematics is also implicated in the existence of these situations, through its central role in the design and use of modern technology.

While a number of authors have contributed to the development of critical mathematics education, the most elaborated approach comes from the work of Skovsmose (e.g. 1994, 1998, 2011, 2012). In this section, we summarise some key ideas derived from Skovsmose's work, highlighting their particular relevance for the task of preparing students to participate in extended peer communities. The main concepts we discuss are: the role of mathematics in shaping society, forms of mathematical knowing, and dialogue. It is important to understand that critical mathematics education is neither a method for teaching mathematics, nor a curriculum of what to teach. Rather, it is a philosophy of mathematics education, offering concepts and ideas with which to think or rethink the more pragmatic concerns of mathematics teaching.

The role of mathematics in shaping society is strongly mediated by technology (see also Saltelli and Giampietro, this issue). Skovsmose (1994) discusses different perspectives on the nature of technology. One view is that technology consists of tools, through which humans can act on nature (p. 45). This view of technology is based on a separation of humans from nature, indeed, on the idea that humans can 'overcome' the limitations of nature. This separation is arguably implicated in the many environmental problems with which we are faced. A second, more critical view, however, is that technology is a social force:

> Technology concerns all aspects of social life, it becomes an absolute for social organisation Not only the form of production but the whole 'civilisation' undergoes a technological reconstruction. Nature in its original sense disappears, and we become inhabitants in a technological reconstruction of our social reality. (Skovsmose, 1994; p. 45)

This view of technology describes how technology is used to organise labour, production, and human behaviour and changes the way in which these things are understood. From government surveillance to Facebook, our lives are structured by different forms of technology. In fact, the clearest examples are all forms of information technology.

Information technology is driven by mathematics; more specifically, it depends on mathematical models. Internet search engines, to give one example, work through the use of sophisticated algorithms, which encode a mathematical model of some aspects of the internet. There are two important points about this marriage of mathematics and information technology. First, mathematical models are designed by human beings. Mathematical models are not simply neutral representations of the world; they encode choices, values, interests, etc., reflecting the interests and preferences of the human designer, as well as the limitations of mathematical methods. Search engine algorithms, for example, may reflect commercial or political interests, highlighting some sources rather than others. Moreover, technology renders this human dimension invisible. We tend more easily to see search engines as sophisticated tools (which they are), rather than human choices, values and interests. Second, through technology in particular, mathematical models undergo an important shift. Mathematical models are designed to describe or model aspects of the world. As such, they are powerful and useful. Models of weather systems allow us to make reasonable predictions of weather in the short term. When information technology is built into the fabric of society, however, the mathematical models that drive them no longer describe reality, they become part of reality—they become prescriptive (Skovsmose, 1994; p. 55). Search engines do not simply describe what is available online, they influence what we look at (and what we buy). There is, therefore, a significant, but largely invisible role for mathematics in structuring society and influencing human behaviour. Skovsmose calls this role the 'formatting power of mathematics' (p. 43).

The idea that mathematics formats our society is in line with the philosophy of post-normal science, where emphasis is put on what is often denoted as *uncertainty in the problem framing* (Walker et al., 2003). The formatting power of mathematics implies the significance of mathematics in both the emergence and the response to post-normal situations. Our understanding of climate change, for example, is almost entirely derived from mathematical analyses and mathematical models, which are associated with uncertainty. There are clearly real social effects of how climate change is modelled and understood, and the question of how mathematics based analyses of climate change should be transformed from descriptive to prescriptive is an important one. The need to pay attention to such things is part of the Funtowicz & Ravetz's (1993) argument for extended peer communities.

So far, we have summarised the idea that mathematics plays a significant role in shaping social reality. The next step is to consider what students need to know about mathematics from the point of view of critical mathematics education. In this regard, Skovsmose (1994, pp. 100–101) proposes three kinds of knowing in mathematics. Mathematical knowing refers to the ability to use various mathematical skills, such as producing mathematical expressions, developing mathematical justifications or proofs, and performing calculations and procedures. Technological knowing refers to the ability to apply mathematics and mathematical methods in the context of technology. Technological knowing includes, for example, the construction and application of the models and algorithms that drive technology. Finally, reflective knowing refers to the ability to consider the impact of mathematical and technological knowing, including consideration of the aims of technology, as well as associated social and ethical issues (see also Saltelli and Giampietro, this issue). Although all three forms of knowing are inter-related, attention to reflective knowing is a distinctive aspect of critical mathematics education.

Skovsmose gives a more extended definition for reflective knowing as:

> the competence needed to be able to take a justified stand in a discussion of technological questions. In this sense we may relate reflective knowledge to the general competence needed to be able to react as a critical citizen in today's societies. The possibility for the public to be not only subjects, i.e. geared only to receive outputs from the 'system', but also to provide inputs to the 'system', presupposes reflective understanding. (p. 101)

Thus a critical mathematics education includes reflective knowing as an important dimension. Learners should not only learn the methods and applications of mathematics; they must learn about its consequences. They must, for example, learn about the formatting role of mathematics in society.

This position has important implications for how the mathematics of post-normal situations might be addressed in educational contexts. It is not sufficient to teach the statistical techniques used to, for example, calculate global temperature increases. Nor is it sufficient to teach how to apply these techniques, using computers, to run analyses of temperature data. Such tasks do not require students to engage with the broader issues or to participate in the related political processes. To teach for reflective knowing would involve discussion of the meaning and consequences of calculations of global temperature changes, discussion of possible actions, and discussion of the role of mathematics, such as the limitations of the statistical methods. Such discussions might lead to political action, whether at a local informal level, or through participation in formal political processes (see Renert, 2011). Fostering reflective knowing, then, is an important aspect of critical mathematics education of direct relevance to education in the context of post-normal science. In particular, for citizens to participate in extended peer communities, they need some degree of mathematical and technological understanding but, we argue, most crucially, they need reflective knowing, in order to be able to critically engage with the information they receive or generate for themselves.

Finally, we need to consider the nature of students' participation in critical mathematics education, since this will also have implications for their potential participation in extended peer communities. The question of participation, however, turns out to be related to epistemology. In essence, Skovsmose argues against a monologic perspective on mathematical knowing. Transmission models of teaching and learning are clearly monologic, in the sense that they assume a fixed body of knowledge that is simply transmitted to learners 'as is'. There is little room for reflective knowing in such a perspective. There are, however, few mathematics educators who explicitly subscribe to a transmission model. A more common approach is the constructivism derived from Piaget's genetic epistemology. Skovsmose (pp. 203–205) argues that this approach is also monologic, though in a more subtle way. In Piagetian-inspired teaching, students are assumed to construct mathematical knowledge and thus the teacher's role is to provide appropriate experiences in order that they may do so. The nature of the mathematics to be constructed is not really open to challenge, however. It is in this sense that Skovmose characterises teaching inspired by constructivism as monologic.

Skovmose's argues instead for a dialogic approach, by which he means something more than simply allowing students to engage in discussion during mathematics class. Skovsmose's explanation of a dialogic epistemology in mathematics education is, in fact, an important basis for our proposed link between critical mathematics education and post-normal science. This link highlights the treatment of competing knowledge claims, which Skovsmose refers to as 'knowledge conflict':

> my use of the word 'dialogue' has much in common with the term 'negotiation'. The establishing of 'dialogue' as an epistemic concept is implied by giving up the thesis of the homogeneity of knowledge, and accepting that contradictory knowledge claims can rightly be made with the consequence that knowledge conflict becomes a reality. [. . .]
> Knowledge conflict is a sensitive epistemic phenomenon, and cannot be solved by adding new information, collecting more observations or by performing more careful calculations. Knowledge conflict has to be handled in a different way. Critique and reflection are needed. From knowledge conflict, we may hope to develop new concepts and to be able to reflect upon knowledge already held. If knowledge conflict is to enter into a dynamic process, its critical and dialogical nature has to be emphasised [. . .] The upheaval of a knowledge conflict cannot be the result of pure reasoning or of some carefully carried out experiment. The only way forward is negotiation. (pp. 205–206)

The notion of dialogue as a feature of critical mathematics education means that teaching mathematical facts and procedures is insufficient. Mathematical knowledge itself must be open to challenge. In the case of climate change, for example, there is no absolute method with which to measure the temperature of the planet. Temperature measurements are made in different places and at different times and then combined using statistical methods. The choice of locations, of statistical methods, of corrections and so on have become popular in public debates about climate change, illustrating how knowledge conflict can arise and have important consequences for public understanding and policy. Mathematical knowledge, then, is in relation to human activity and, as such, open to uncertainty, values and power (Skovsmose, 1994; p. 206).

There is a clear link between the notion of dialogue in critical mathematics education, post-normal science and the idea of extended peer communities. Post-normal science emphasises precisely issues of uncertainty and the need for negotiation through extended peer communities. Post-normal situations are precisely those in which more information, observations or calculations will not in themselves lead directly to solutions. Rather, post-normal situations require dialogue, and with it, the creation of new concepts and ideas. In the next section, we develop some ideas through a focus on uncertainty.

3. Uncertainty

Post-normal science includes a particular attention to uncertainty and its role in knowledge production and in policy making. The theory of post-normal science includes distinctions between different sorts of uncertainty:

- Technical uncertainty: inexactness arising in particular methods and techniques, which can be dealt with through standard methods, such as statistical measures of error or the use of probability.
- Methodological uncertainty: unreliability arising from the choice of methods, data, etc., where, for example, judgment and academic traditions play a role. For instance, connections between system components are basically known, but cannot be accurately quantified.
- Epistemic uncertainty: the "border with ignorance", arising from lack of knowledge, information or suitable methods, or the lack of awareness of some features of the situation. This sort of uncertainty is in particular related to complex issues with conflicting stakes (based on Funtowicz & Ravetz, 1993, p. 743–744).

These categories are conceptual, rather than material, and in many situations are likely to co-occur. A central idea here is that post-normal situations are characterised by epistemic uncertainty and should be dealt with by post-normal science, where an extended peer community contributes to the knowledge base, to values perspectives and to evaluate the quality of expert knowledge. A specific uncertainty can be characterised as a mix between the three sorts. While the size of a technical uncertainty can be assessed, it is not possible to know whether an uncertainty of the other sorts is large or small. The dimension of the sort of uncertainty is rather characterised by the level of control through quantification. An epistemic uncertainty may be small, but there is not sufficient knowledge to determine that this is the case. An essential feature with these sorts of uncertainty is that there is no objective way of deciding which sort is dominant. On the contrary, the sort of uncertainty and the level of conflict and stakes are interdependent. Uncertainty does not matter much if it does not have implications for values and interests.

Funtowicz and Ravetz divide science based problem-solving strategies into three ideal zones: applied science (technical uncertainty and low conflict level), professional consultancy (methodological uncertainty and medium conflict level) and post-normal science (epistemic uncertainty and high conflict level). It is noticeable that these zones can be linked to Skovsmose's types of knowing in mathematics. Both theories make distinctions between (1) a basic level, or zone, involving fairly routine methods, tools, and techniques; (2) a second level involving decision-making within the mathematical/scientific process; and (3) a *meta*-level, requiring critical consideration of the choices and effects of these processes. Technical uncertainty can be handled sufficiently through mathematical knowing, as quantifications of the uncertainty is an appropriate approach. Methodological uncertainty requires an evaluation of the methodology and how the knowledge is applied, which resembles technological knowing. Epistemic uncertainty in post-normal situations calls for critical citizenship and reflective knowing as the societal problem framing.

The underlying tripartite distinction implies, therefore, that within mathematics education, the three sorts of uncertainty can be considered through attention to mathematical, technological and reflective knowing. Mathematical knowing can include learning about sources of inexactness in mathematics, as well as the mathematical methods used to deal with such inexactness. Technological knowing can include learning about how such methods are selected and deployed and how uncertainty is embedded in various kinds of mathematical application. Finally reflective knowing can involve learning about how uncertainty arises in post-normal situations, as well as the social and societal effects of different possible ways of handling uncertainty.

The issue of climate change provides a good example of how mathematics education can address and distinguish between the sorts of uncertainty. To learn about inexactness and mathematical knowing, students may apply statistical concepts on empirical time series, e.g. emission levels or temperatures measured at a specific place. The mathematical content knowledge can be basic (e.g. presentation through graphs or calculation of averages) or more advanced. Methodological uncertainty can be addressed through problem solving or discussions where students engage in methodological aspects, e.g. discussing assumptions of linearity or choosing how to mathematically represent an area's annual temperature. The public debate on climate change, with its conflicting views and disagreements on facts illustrate epistemic uncertainty and the formatting power of mathematics. These discussions could include insights into why experts disagree on temperature changes, what stakes are at risk in climate change, the political consequences of temperature predictions and the implications of public and political demand for more precise predictions. Mathematics education can thereby contribute to tackling mathematically expressed information, not with the purpose of requiring students to believe any particular idea, but through giving them the opportunity to learn about the inherent sorts of uncertainties in the issue of climate change.

4. Uncertainty in mathematics classrooms

In this section we present excerpts from three mathematics classroom activities to illustrate the preceding theory and to highlight several characteristics that need to be present in mathematics education to help prepare students for critical citizenship. We will not address mathematical content knowledge, but rather discuss the role and application of mathematics. The characteristics we discuss are *conflicts of interest*,*values*, *complexity* and *uncertainty*.

4.1. Students discussing traffic safety

A mathematics teacher at a school in a rural area in Norway developed a student project on traffic safety for her 12–13 years old students (for more details see Hauge and Herheim, 2015; Herheim and Rangnes, 2015). The local community had experienced several incidents where cars had driven off the road and into the sea, and the school bus passed this stretch of road every day. In the project, the students carried out a traffic survey and measured the heights of the barriers in five exposed bends in the road. The students participated in deciding how many measurements they should take in order to evaluate the height of the barriers and whether they were safe. Spreadsheets were used to process and present the data.

The teacher had decided the vehicle categories for the traffic count sheet, but the students came up with two suggestions for changes. One was in class when they presented their charts on the traffic counts:

Per: When I now look at what we have counted, I think we should have had a separate column for buses.

Teacher: Yeah, OK? Why would you do that?

Per: Because there are so many people in buses. There are more than for example in a private car.

When Per argues there are more people in buses than in cars, he is considering the much greater impact of a bus running into the sea than a car running into the sea. Regarding the chart as a way to communicate risk, a change in vehicle categories to include the buses as a separate category could make the risk look more severe.

The second suggestion the students made concerned sheep. Some sheep had been in the road during traffic count, and these had been counted and placed under the "other" category on their sheet. A student had argued that it was relevant to show explicitly that there had been sheep there, however, because they constituted a risk factor on the road (Hauge and Herheim, 2015; Herheim and Rangnes, 2015).

Within the constraints of a fairly conventional educational setting, the project gave the students some opportunity to experience the formatting power of mathematics in that they could see that choices on how to approach the problem of traffic safety through mathematics influence the perception of risk (for more examples see Hauge and Herheim, 2015) and hence, potentially, the resulting decisions. The teacher allowed for a dialogic approach to learning (Skovsmose, 1994), as the categories for the traffic count were negotiated between the students and the teacher. The project was on a topic that mattered to the students and the teacher let the students have a say in developing the project. Projects like this can be useful preparation for critical citizenship, or for participation in extended peer communities, because the students were actual stakeholders and the project was participatory.

Hauge and Herheim (2015) studied how the three sorts of uncertainty from post-normal science (Funtowizc & Ravetz, 1993) were present in the traffic project. For example, uncertainty related to the students' charts on the traffic count was labelled as *unreliability,* possibly overlapping with *ignorance,* because the students indicated that the communicated risk level would change with different choices of categories. Students were therefore engaging with both methodological and epistemic uncertainty. The excerpt illustrates a similar overlap between technological and reflective knowing. Questioning the vehicle categories can be characterised as a reflection on the application of mathematics to increase its relevance, but the reasoning behind such choices, associated with the number of people in buses, is associated with reflective knowing and the formatting power of mathematics.

Although the school project was considered a success, there was still potential for further learning. The students could, for example, have been challenged to express more explicitly how the choices of categories make a difference. The excerpt above illustrates how the importance of choice was only implicit in the dialogue between Per and the teacher; explicit argumentation might have facilitated further learning for critical citizenship. The students could also have discussed the issue of choices in more general terms: how do experts make such choices? What implications do the students see for their reflections? The students and the teacher might also have benefitted from being offered vocabulary on uncertainty to make their discussion more explicit.

4.2. Students discussing climate change

During a master's course on mathematics education, the students and the lecturer (Hauge) discussed a graph produced by the Intergovernmental Panel on Climate Change (IPCC, 2013) to represent projections of global temperature change. The lecture was an introduction to critical mathematics education, and the aim was to link key concepts to the discussion about the graph and about climate change more generally (for more details, see Hauge and Barwell, 2015; Hauge et al., 2015). These concepts included the notions of formatting and reflective knowing.

The IPCC graph starts at year 1850 and shows the estimated and predicted change in global temperature until 2300 with estimated 95% confidence intervals. The estimates and predictions are based on averages of results produced by a number of different computer-based climate models. The graph presents five predicted courses for average global temperature change, given different future scenarios of global greenhouse emission levels. The trajectories from each of the emission scenarios are accompanied by a number that indicates how many models have contributed. There is a break in the trajectories in year 2100, since fewer models have contributed to the continuation beyond that point.

In the following excerpt, Kjellrun draws the students' attention to this feature:

Kjellrun: But if we look at year 2100, what is happening there?

[pause 16 s.]

Elisabeth: [inaudible] is a break?

Kjellrun: Yes, why is that?

Elisabeth: At least the red one.

Kjellrun: Yes, at least the red one, that's very distinct.

Elisabeth: There are fewer models, you know.

[. . .]

Tor Inge: I'm thinking that the most critical until 2100 do not continue further in the models.

Kjellrun: Yeah, well that's true.

Tor Inge So that the curve isn't as steep when it continues.

The red trajectory to which Elisabeth refers represents the status quo: emissions continue at more or less the same rate as today. Elisabeth recognises that the break is caused by the use of fewer models. Tor Inge's utterances indicate that he imagines a more critical future prediction if all the models had contributed after 2100. Unlike the examples from the traffic safety project, there is no apparent negotiation between the students and the teacher in the above excerpt. Yet, because of Kjellrun's semi-open questions, the excerpt can be argued to show a dialogic element when she invites the students to reflect on its mathematical properties. Skovsmose's (1994) understanding of negotiation may rather be linked to the different ways of understanding the knowledge claims represented by the graph. The students' reflections can thus be characterised as reflective knowing.

The students were not introduced to uncertainty concepts from post-normal science, but they seemed to recognise that the predictions are associated with uncertainty beyond what can be characterised as technical uncertainty (Hauge and Barwell, 2015). Their reflections are dealing with an understanding that the involved models do not necessarily provide the same predictions or the same uncertainty measurements. This implies that the associated uncertainty is more in line with methodological or epistemic uncertainty.

Again, a clear line cannot be drawn between the two because the students do not comment further on implications of the differences in model output. There are several ways of perceiving the uncertainty relating to the break in the graph. One might picture that the uncertainty is of a methodological sort; that the models are more or less reliable but that there is uncertainty related to connecting components in the climate system. The break can also be perceived as a consequence of ignorance; a lack of knowledge about what affects the climate and how this uncertainty contributes in shaping how the problem of climate change is understood. Similarly, whether the excerpt illustrates technological or reflective knowing is not clear since the students do not reflect further on the significance of the break.

This small classroom study suggests that discussing graphs, has potential for developing critical mathematics education for post-normal science, even though the underlying mathematics and science is to some extent incomprehensible to the participants (Hauge and Barwell, 2015; Hauge et al., 2015). The context of the graph is of course essential for understanding the significance of the associated uncertainties and the reasons for disagreements about climate change. During the classroom discussion, the students offered a range of critical reflections besides those shown above, while still expressing their trust in the IPCC and climate scientists. In fact, they expressed unease about pointing to limitations in the knowledge base of the graph (Hauge et al., 2015).

Nevertheless, the students reflected on a range of aspects of uncertainty, including natural variation, uncertainty in models, irreducible uncertainty, epistemic uncertainty and how to cope with uncertainty. The classroom discussion and the lecture on critical mathematics together offered them ideas and concepts which could help them articulate aspects of mathematics in society that are crucial for critical citizenship.

4.3. Students discussing oil exploitation

The last example is taken from a plenary discussion among fifty 13–14 year olds in their classroom. In Norway there is an ongoing debate on whether the offshore area close to these students' hometown in Lofoten, a rural area, should be opened for petroleum exploitation. The area is considered promising for oil and gas production, but is also a significant area for tourism and a range of fisheries. A research team from Bergen University College prepared the student discussion and visited the school. The students were asked to consider futures with or without oil exploitation and provide arguments for and against (for more information, see Hauge et al., 2014). The activity was not overtly mathematical, but the classroom activity illustrates some qualities relevant for critical mathematics education and post-normal science.

Hauge et al., 2014 described how the classroom discussion exposed the students to uncertainties and the complexity of the issue through their disagreeing opinions. The students introduced a range of topics during a two-hour plenary discussion: fish, scenery, mountains, tourists, oil prize, shopping malls, oil spills, job opportunities, etc. Through arguments and statements, the students made links between these elements, and through doubts and through counter arguments they introduced uncertainties about these links. In this way, the discussion provided an arena where the students together constructed an image of uncertainty and complexity.

The students also disagreed on values and what was on stake. To illustrate this we present two excerpts from the plenary discussion. Melissa is a student and the first speaker when they return after a break:

Melissa: We came up with a point. You talked about getting a shopping mall here. It's the same thing as if you should remove the Eiffel Tower in Paris and rather build a shopping mall. Then tourism there would drop at once, and the same can happen here. That if we get big oil installations and a shopping mall, and we only have industry here, then it will drop here, too.

[several minutes with students discussing]

Roy: I think personally, that change is the best for [their hometown] right now. We need to take a few chances, because now we have lived on fish and dried fish, and tourists, for many, many years – and we are committed to try, at least – to try to become a city – it is a good thing. Maybe people think it's cool. We can- There will be a shopping mall. And then when you want to go out for a walk, there are still mountains, and everything else you can go to. It's not like we're going to remove mountains. We're not going to blast them to create a shopping mall.

Melissa and Roy clearly disagree about preferred futures. The power expressed in Melissa's statement, conveying Lofoten's iconic value, suggests she might not be willing to take any risk of affecting Lofoten's culture and identity. Roy, on the other hand, announces that he wants a different life than Lofoten can offer and calls for a new identity for his hometown. He seems willing to accept associated risks. Conflict of interests and diverging opinions on acceptable risks are common in societal risk debates. The classroom discussion thus allowed the students to experience key elements of public debate: They exposed conflicts of interests, defended their opinions and values and experienced that values and opinions remain conflicting. The conflicting opinions of Melissa and Roy illustrate a knowledge conflict regarding whether a risk is acceptable or not. The classroom discussion thus allowed for a dialogical epistemology in education (Skovsmose, 1994).

Hauge et al. (2014) has exemplified how the students developed arguments, which they refined and further developed because their classmates provided counter arguments. Classroom discussions are recognized as a learning arena for critical citizenship, as students need to practice developing arguments by responding to others, and they learn mutual respect, in spite of conflicting views (see e.g. Johnsen-Høines & Alrø, 2012; Sánchez Aguilar & Molina Zavaleta, 2012; Vithal, 2012).

The activity cannot be labelled 'mathematics education' although the students did use concepts like "probability" and "risk" in their argumentation. Still, the students experienced being exposed to key characteristics of post-normal situations – conflicting values, urgency, uncertainty and complexity – and having to respond to them during the discussion. To learn more about risk, the students could have been made aware of these typical characteristics of confrontational risk issues and with reference to their own discussion. They could also have explored information on the internet, discussed premises and assumptions of presented information and statements and more specifically the question of why experts disagree.

5. Discussion

We have set out an argument for the need for a critical mathematical education as a basis for educating future citizens to participate in extended peer communities. We have focused on uncertainty as a key challenge for extended peer communities. Uncertainty is multi-dimensional and, we argue, can be studied through a critical mathematics education approach. In this section, we start by summarising the main points from the three classroom situations to shed light on what a mathematics classroom activity might look like to prepare students for critical citizenship, inspired by ideas from critical mathematics education and post-normal science. Relevant aspects for mathematics educators to consider include forms of critique, the role of mathematics, and the potential for conflicting values, uncertainty and complexity in the societal issue that is the basis for classroom activities. We use our summary as a starting point for discussing how post-normal science can enrich critical mathematics education and vice versa. Finally we look at what implications this paper might have for the future development of mathematics education and its connections to post-normal science.

In all three projects, the students were given opportunities for critique. In the oil discussion, the students turned critique towards each other through responding to other students' arguments and developing and reconsidering their own reasoning. In the traffic safety project, the students were critical of the teacher's decision on the categories on the count sheet as they worked on quantifying and communicating risks related to the low road barriers. In the second example, the students' discussion of projected temperature change provided an arena for critiquing qualities of assessments and predictions produced by experts, as well as how the public and decision-makers respond to expert knowledge. All three student projects were about choices for the future, they were exploratory in their approach and all three project topics were relevant for the groups of students.

The role of mathematics varies across the three projects. While the students in the traffic project had to complete mathematics tasks in order to develop arguments highlighting the risk of accidents, the students discussing temperature change examined mathematical information and results. The students who discussed oil exploitation did not work with mathematics explicitly, but achieved insight into a real world context where there is plenty of expert information in mathematical form. Indeed, this last example could be considered to be a simulation of how an extended peer community might work. In different ways the classroom activities highlighted the importance of values and uncertainty.

Excerpts from the three classroom situations illustrate how uncertainty concepts from post-normal science can be understood and applied in mathematics education. In the students' discussion on oil exploitation, it was essential in both a critical mathematics perspective and a post-normal science perspective to recognise how conflicting stakes, complexity, decisions and uncertainty were present in their argumentation and how these characteristics were intertwined. The traffic safety project and the temperature change discussion demonstrated students' capabilities to reflect on the impact of uncertainty in mathematical information on risk-related problems. Uncertainty and values were linked by the students in all classroom studies: choices in how to present traffic statistics could influence value perspectives of risks, students reflected on their attitude to risk together with value statements on the future of Lofoten, and differences in model predictions were linked to critical futures in terms of global warming.

Concepts and ideas from post-normal science can be useful for critical mathematics education, including students, teachers and researchers, as they can bring attention to, and awareness of, key characteristics of the role of quantification in society. They can also serve as a guide for what mathematics education should include in preparing students for critical citizenship and, potentially, participation in extended peer communities: handling conflicting views, dealing with complexity and recognising and coping with uncertainty. Together, the classroom activities described in this paper involve some of these elements, but deliberate choices based on post-normal science might have further developed the activities in terms of preparing for critical citizenship. Students could, for instance, be made aware of different sorts of uncertainty that can and cannot be controlled through statistical measures, and post-normal characteristics could be articulated in relation to the topic, such as, for example, conflicting values and stakes, risk, uncertainty, urgency and complexity.

Mathematics education can make an important contribution to post-normal science, particularly in relation to preparing students for critical citizenship and participation in extended peer communities. We have argued that classroom activities based on dialogues in the form of negotiation on uncertainty aspects is central for this purpose. Society will benefit from an educational system that emphasises critical perspectives, although such an approach is far from being realised. As shown earlier in this paper, there are voices from critical mathematics education who emphasise the necessity of education for critical citizenship and lived democracy. There are clear benefits from cooperation between critical mathematics education and post-normal science, yet challenges need to be overcome. Some are related to further exploration of how to apply ideas from post-normal science in critical mathematics education. In post-normal science, the types of uncertainty can be regarded as qualities in expert knowledge. Pointing out the quality of uncertainty serves several purposes. One is to encourage experts to both take the types of uncertainty into account when developing knowledge and to communicate types of uncertainty in advice. Another purpose is to rationalise that post-normal situations call for extended peer communities (Funtowicz & Ravetz, 1990, 1993). Both purposes reflect a theoretical and philosophical stance and imply changes in how science and the role of science in society is understood. However, once the types of uncertainty are used to label knowledge, utterances or student activities, the researcher takes the role of an uncertainty expert. Defining uncertainty may easily become positivistic which in itself contradicts the philosophy of post-normal science. Yet, concepts and ideas from post-normal science, can serve to increase educators' and students' awareness of uncertainty and its possible roles in societal issues. This complements ideas from critical mathematical education related to critical citizenship and has additional valuable perspectives to offer.

As demonstrated in this paper, research on mathematics education in a post-normal science perspective, and vice versa, is in its initial stages. More studies need to be conducted on how to empower students and to develop critical citizenship and how this benefits research on post-normal science.

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